Synthetic-Gauge-Field Stabilization of The Chiral-Spin-Liquid Phase

Gang Chen Fudan University





Outline

1. Chiral spin liquid from SU(N) Hubbard model on square lattice

GC, K Hazzard, AM Rey, M Hermele, PRA(R), 93, 061601 (2016)

2. Quantum Paramagnet and frustrated quantum criticality on a diamond lattice

GC, PRB(R), 96, 020412 (2017)



SU(N) symmetry of alkaline-earth atoms

Two-orbital SU(N) magnetism with ultracold alkaline-earth atoms

NatPhys

A. V. Gorshkov^{1*}, M. Hermele², V. Gurarie², C. Xu¹, P. S. Julienne³,

J. Ye⁴, P. Zoller⁵, E. Demler^{1,6}, M. D. Lukin^{1,6}, and A. M. Rey⁴

¹Physics Department, Harvard University, Cambridge, MA 02138

²Department of Physics, University of Colorado, Boulder, CO 80309

³Joint Quantum Institute, NIST and University of Maryland, Gaithersburg, MD 20899-8423

⁴JILA, NIST, and Department of Physics, University of Colorado, Boulder, CO 80309

⁵Institute for Theoretical Physics, University of Innsbruck, A-6020 Innsbruck,

Austria and Institute for Quantum Optics and Quantum Information of the Austrian Academy of Sciences, A-6020 Innsbruck, Austria

⁶Institute for Theoretical Atomic, Molecular and Optical Physics,

Harvard-Smithsonian Center of Astrophysics, Cambridge, MA 02138 and

**e-mail:* gorshkov@post.harvard.edu (Dated: September 26, 2009)

Fermionic alkaline-earth atoms have unique properties that make them attractive candidates for the realization of novel atomic clocks and degenerate quantum gases. At the same time, they are attracting considerable theoretical attention in the context of quantum information processing. Here we demonstrate that when such atoms are loaded in optical lattices, they can be used as quantum simulators of unique many-body phenomena. In particular, we show that the decoupling of the nuclear spin from the electronic angular momentum can be used to implement many-body systems with an unprecedented degree of symmetry, characterized by the SU(N) group with N as large as 10. Moreover, the interplay of the nuclear spin with the electronic degree of freedom provided by a stable optically excited state allows for the study of spin-orbital physics. Such systems may provide valuable insights into strongly correlated physics of transition metal oxides, heavy fermion materials, and spin liquid phases.





SU(N) Mott insulator

An SU(6) Mott insulator of an atomic Fermi gas realized by large-spin Pomeranchuk cooling NatPhys

Shintaro Taie¹*, Rekishu Yamazaki^{1,2}, Seiji Sugawa¹ and Yoshiro Takahashi^{1,2}

The Hubbard model accounts for many of the diverse phenomena observed in solid-state materials, despite incorporating only nearest-neighbour hopping and on-site interactions for correlated electrons. One interesting extension to the model involves enlarging its spin symmetry to SU(N > 2), which describes systems with orbital degeneracy. Here we report a successful formation of the SU(6) symmetric Mott-insulator state with an atomic Fermi gas of ytterbium (¹⁷³Yb) atoms in a three-dimensional optical lattice. In addition to the suppression of compressibility and the charge-excitation gap characteristic of a Mott-insulating phase, we reveal that the SU(6) system can achieve lower temperatures than the SU(2) state, owing to differences in the entropy carried by an isolated spin. The mechanism is analogous to Pomeranchuk cooling in solid ³He and will be helpful for investigating exotic quantum phases of the SU(N) Hubbard system at extremely low temperatures.

even magnetic Feshbash resonance is not available...



SU(N) Heisenberg model on square lattice

Fundamentally different from the SU(2) spin-S Heisenberg model





Quantum fluctuation is **much much stronger** for SU(N) matter, so exotic (unconventional) phases are expected.

Large-N approach of the SU(N) Heisenberg model

$$\mathcal{H} = J \sum_{\langle \boldsymbol{rr}' \rangle} S_{\alpha\beta}(\boldsymbol{r}) S_{\beta\alpha}(\boldsymbol{r}'), \ S_{\alpha\beta}(\boldsymbol{r}) = f_{\boldsymbol{r}\alpha}^{\dagger} f_{\boldsymbol{r}\beta},$$

$$S_{\alpha\beta}(\boldsymbol{r}) = f_{\boldsymbol{r}\alpha}^{\dagger} f_{\boldsymbol{r}\beta}, \qquad f_{\boldsymbol{r}\alpha}^{\dagger} f_{\boldsymbol{r}\alpha} = \boldsymbol{m} = \boldsymbol{1}$$

$$\mathcal{H}_{\mathrm{MFT}} = \tilde{\mathcal{H}}_{\mathrm{MFT}} + \sum_{\boldsymbol{r}} \mu_{\boldsymbol{r}} (\boldsymbol{m} - f_{\boldsymbol{r}\alpha}^{\dagger} f_{\boldsymbol{r}\alpha}),$$

$$\tilde{\mathcal{H}}_{\mathrm{MFT}} = (N/\mathcal{J}) \sum_{\langle \boldsymbol{rr}' \rangle} |\chi_{\boldsymbol{rr}'}|^2 + \mathcal{H}_K$$

$$\mathcal{H}_K = \sum_{\langle \boldsymbol{rr}' \rangle} (\chi_{\boldsymbol{rr}'} f_{\boldsymbol{r}\alpha}^{\dagger} f_{\boldsymbol{r}'\alpha} + \mathrm{H.c.}).$$

$$\chi_{\boldsymbol{rr}'} = -\frac{\mathcal{J}}{N} \langle f_{\boldsymbol{r}'\alpha}^{\dagger} f_{\boldsymbol{r}\alpha} \rangle \ (\mathrm{a}), \qquad \boldsymbol{m} = \langle f_{\boldsymbol{r}\alpha}^{\dagger} f_{\boldsymbol{r}\alpha} \rangle$$

Hermele, etc PRL 2010

k	ĉ	CSL	ICSL	LC
5	5	-0.043080	-0.043070	-0.042987
6	5	-0.033069	-0.03299	-0.032961
7	7	-0.026130	-0.02597	-0.025730
8	3	-0.021138	-0.02102	-0.020897

TABLE I: Energies of CSL and competing states, in units of $N\mathcal{J}N_s = N^2 J N_s$, for $5 \leq k \leq 8$. ICSL is the lowest-energy inhomogeneous CSL that was found. LC is the lowest competing state that cannot be interpreted as an inhomogeneous CSL. Note that the energy difference between CSL and LC is larger for k = 7, 8 than for k = 5, 6.

"Break" the spin into halves, and glue them back by gauge field, this is exact in the large-N limit.

> Hermele, etc PRL 2010, XG Wen 1990, Affleck, Baskaran, Anderson 1988.



Chiral spin liquid of the SU(N) Heisenberg model

N>4, there will be Chiral spin liquid.



- 1. Spontaneously break T by developing flux for spinons.
- 2. At MFT, spinon experience flux and form Landau level, actually fill the lowest Landau level.
- The system is fully gapped, supporting anyonic excitation with fractional statistics. There are topologically protected edge states.

Experimental issue:

- 1. cooling to spin liquid regime
- 2. How to do braiding?
- 3. How to measure edge states?



Extension to the SU(N) Hubbard model





Slave rotor approach

This approach is designed to be smoothly connected to the parton-gauge construction in the Heisenberg limit.

(3)

$$c_{\alpha,j} = e^{-i\theta_j} f_{\alpha,j}.$$

In order to reproduce the original Hilbert space, we must impose the constraint

$$L_j = \sum_{\alpha} f^{\dagger}_{\alpha,j} f_{\alpha,j} - 1 \tag{4}$$

that the rotor angular momentum L_j is uniquely determined by the particle number. Here, L_j satisfies $[\theta_j, L_j] = i$. We rewrite the Hamiltonian in terms of these new degrees of freedom, giving

$$H = -t \sum_{\langle i,j \rangle,\alpha} e^{i\phi_{ij}} e^{i(\theta_i - \theta_j)} f^{\dagger}_{\alpha,i} f_{\alpha,j} + \frac{U}{2} \sum_i L_i^2.$$

$$H_r = -\sum_{\langle i,j \rangle} J_{ij} e^{i\theta_i - i\theta_j} + \sum_i \frac{U}{2} L_i^2 + h_i (L_i + 1), \quad (6)$$

$$H_f = -\sum_{\langle i,j\rangle,\alpha} \tilde{t}_{ij} e^{i\phi_{ij}} f^{\dagger}_{\alpha,i} f_{\alpha,j} - \sum_{i,\alpha} h_i f^{\dagger}_{\alpha,i} f_{\alpha,i}, \qquad (7)$$

where h_i is a Lagrange multiplier that enforces on average the constraint Eq. (4), $\tilde{t}_{ij} \equiv t \langle e^{i\theta_i - i\theta_j} \rangle_r$, and $J_{ij} \equiv t e^{i\phi_{ij}} \sum_{\alpha} \langle f_{\alpha,i}^{\dagger} f_{\alpha,j} \rangle_f$. Here the sub-index r (f) refers



Phase diagram of Hubbard model with no gauge flux





Phase diagram of Hubbard model with gauge flux



FIG. 2. The excitation gap of the CSL phase, Δ , as a function of interaction strength, U, both in units of the tunnelling t.



Different phases

Phases	$\langle e^{i\theta} \rangle$	rotor flux	spinon gap	spinon flux
FL	$\neq 0$	0	0	0
SFS	0	0	0	
CSL	0	$-2\pi/N$	$\neq 0$	$2\pi/N$
SU(3)-VBS	0	$5 -\pi$	$\neq 0$	π
SU(4)-VBS	0	0	$\neq 0$	aroup
IQH	$\neq 0$	0	$\neq 0$	$2\pi/N$
CSL	001	J'Soll'	$\neq 0$	$2\pi/N$
SU(3)-VBS	0	$\pi/3$	$\neq 0$	π
SU(4)-VBS	0	$\pi/2$	$\neq 0$	0
	Phases FL SFS CSL CSL SU(3)-VBS SU(4)-VBS SU(3)-VBS SU(3)-VBS SU(4)-VBS	Phases $\langle e^{i\theta} \rangle$ FL $\neq 0$ SFS0CSL0SU(3)-VBS0SU(4)-VBS0IQH $\neq 0$ CSL0SU(3)-VBS0SU(4)-VBS0SU(4)-VBS0	Phases $\langle e^{i\theta} \rangle$ rotor fluxFL $\neq 0$ 0SFS00CSL0 $-2\pi/N$ SU(3)-VBS0 $-\pi$ SU(4)-VBS00IQH $\neq 0$ 0SU(3)-VBS0 $\pi/3$ SU(4)-VBS0 $\pi/3$ SU(4)-VBS0 $\pi/2$	Phases $\langle e^{i\theta} \rangle$ rotor fluxspinon gapFL $\neq 0$ 00SFS000CSL0 $-2\pi/N$ $\neq 0$ SU(3)-VBS0 $-\pi$ $\neq 0$ SU(4)-VBS00 $\neq 0$ IQH $\neq 0$ 0 $\neq 0$ SU(3)-VBS0 $\pi/3$ $\neq 0$ SU(4)-VBS0 $\pi/3$ $\neq 0$ SU(4)-VBS0 $\pi/2$ $\neq 0$



Open theoretical questions

1. Direct transition between the CSL to magnetic ordered states. (Gang Chen, unpublished)



destroy the Chiral Abelian topological order and induce magnetic order at the same time?

F Mila, etc PRL 2010

2. Direct transition between the SU(3) VBS to magnetic ordered states. (Gang Chen, unpublished)







FIG. 10. \mathbb{Z}_4 vortex in the 6-site cluster state.

Summary of this part

1. SU(N) Hubbard model is more realistic than SU(N) Heisenberg model.

2. SU(N) Hubbard model with and without gauge flux could stabilize the Chiral spin liquid in a much larger parameter space, and provide a larger energy gap for experimental observation.



2. Quantum Paramagnet and frustrated quantum criticality on a diamond lattice

GC, PRB(R), 96, 020412 (2017)



Spin-one Haldane chain

Due to Berry phase effect, spin-1/2 chain is gapless, spin-1 Heisenberg chain is gapped.



Duncan Haldane



Building degree of freedom is S=1, but at there is S=1/2 edge state.





Symmetry Protected Topological Phase

Xiao-Gang Wen

Symmetry	d = 0	d = 1	<i>d</i> = 2	<i>d</i> = 3
$U(1) \rtimes Z_2^T$	Z	Z_2	Z_2	Z ₂ ²
Z_2^T	Z ₁	Z ₂	$\mathbf{Y}_{\mathbf{z}_{1}}$	Z ₂
<i>U</i> (1)	Z	Z ₁	Z	Z ₁
<i>SO</i> (3)		Z ₂	Z	Z1
$SO(3) \times Z_2'$	Z ₁	Z_2^2	Z ₂	Z_2^3
Zn	Zn	Z ₁	Zņ	Z1
$Z_2' \times D_2 = D_{2h}$	Z ₂ ²	Z ₂ ⁴	Z ₂ ⁶	Z ₂ ⁹

Table for **boson SPTs**

classified with group cohomology from symmetry and dimension.

It turns out, the well-known topological insulator is a fermion SPT that is protected by time reversal symmetry. Boson SPT must be stabilized by interaction.

重要的问题:理解导致SPT的物理机制,以及在什么物理体系中可以找到。

Senthil's suggestion

PhysRevB, 2015

Topological Paramagnetism in Frustrated Spin-One Mott Insulators

Chong Wang, Adam Nahum, and T. Senthil

Department of Physics, Massachusetts Institute of Technology, Cambridge, MA 02139, USA (Dated: January 7, 2015)

Time reversal protected three dimensional (3D) topological paramagnets are magnetic analogs of the celebrated 3D topological insulators. Such paramagnets have a bulk gap, no exotic bulk excitations, but non-trivial surface states protected by symmetry. We propose that frustrated spin-1 quantum magnets are a natural setting for realising such states in 3D. We describe a physical picture of the ground state wavefunction for such a spin-1 topological paramagnet in terms of loops of fluctuating Haldane chains with non-trivial linking phases. We illustrate some aspects of such loop gases with simple exactly solvable models. We also show how 3D topological paramagnets can be very naturally accessed within a slave particle description of a spin-1 magnet. Specifically we construct slave particle mean field states which are naturally driven into the topological paramagnet upon including fluctuations. We propose bulk projected wave functions for the topological paramagnet based on this slave particle description. An alternate slave particle construction leads to a stable U(1) quantum spin liquid from which a topological paramagnet may be accessed by condensing the emergent magnetic monopole excitation of the spin liquid.



T. Senthil

The frustrated diamond lattice model appears to describe well [56] the physics of the spinel oxide materials $MnAl_2O_4$ and $CoAl_2O_4$ [58] which belong to a general family of materials of the form AB_2O_4 . The A site forms

There is no sharp question in 1D any more. So what is the 3D analogue of Haldane spin-1 phase?



APS March Meeting 2017

Monday–Friday, March 13–17, 2017; New Orleans, Louisiana

Session B48: Frustrated Magnetism: Spinels, Pyrochlores, and Frustrated 3D Magnets I

11:15 AM-2:15 PM, Monday, March 13, 2017 Room: 395

Sponsoring Units: GMAG DMP Chair: Martin Mourigal, Georgia Tech

Abstract: B48.00006 : S = 1 on a Diamond Lattice in NiRh2O4

12:15 PM-12:27 PM

Preview Abstract

Authors:

Juan Chamorro (Johns Hopkins University)

Tyrel McQueen (Johns Hopkins University)

An S = 1 system has the potential of rich physics, and has been the subject of intense theoretical work. Extensive work has been done on onedimensional and two-dimensional S = 1 systems, yet three dimensional systems remain elusive. Experimental realizations of three-dimensional S = 1, however, are limited, and no system to date has been found to genuinely harbor this. Recent theoretical work suggests that S = 1 on a diamond lattice would enable a novel topological paramagnet state, generated by fluctuating Haldane chains within the structure, with topologically protected end states. Here we present data on NiRh2O4, a tetragonal spinel that has a structural phase transition from cubic to tetragonal at T = 380 K. High resolution XRD shows it to have a tetragonally distorted spinel structure, with Ni2+ (d8, S = 1) on the tetrahedral, diamond sublattice site. Magnetic susceptibility and specific heat measurements show that it does not order magnetically down to T = 0.1 K. Nearest neighbor interactions remain the same despite the cubic to tetragonal phase transition. Comparison to theoretical models indicate that this system might fulfill the requirements necessary to have both highly entangled and topological behaviors.





T. McQueen

MathJax On | Off + Abstract +

Minimal spin model



Immediate experimental consequence

$$\Theta_{\rm CW}^{z} = -\frac{D_z}{3} - \frac{S(S+1)}{3}(z_1J_1 + z_2J_2),$$

$$\Theta_{\rm CW}^{\perp} = +\frac{D_z}{6} - \frac{S(S+1)}{3}(z_1J_1 + z_2J_2),$$



The phase diagram



Deep in quantum paramagnet, the ground state is a trivial product state. The state is trivial, but excitation and phase transition out of it can be non-trivial.

$$|\Psi\rangle = \prod_{\boldsymbol{r}} |S_{\boldsymbol{r}}^z = 0\rangle$$







Unconventional magnetic excitation



These are bosonic excitations. What is relevant for bosons is the lowest energy mode. Usually, the lowest energy modes occur at certain discrete momenta. But here, the lowest energy modes occur at a surface in the reciprocal space.



Frustrated Quantum Criticality: collapse of boson surface



These degenerate surfaces are NOT Fermi surface !

But at low temperature, the fluctuation of the system is governed by the surface, i.e. low-energy fluctuations are near the 2D surface. We obtain a linear-T heat capacity **Cv** ~ **T**, which is like a Fermi surface.



Degeneracy breaking in the ordered side



Here, because infinite number of boson modes are condensed, the system does not know which order to select.

So quantum fluctuation will pick up the order that gives the lowest quantum zero-point energy.



Summary

- 1. We point out that NiRh₂O₄ spin-1 diamond lattice antiferromagnet is NOT the topological quantum paramagnet.
- 2. Through a minimal model, we find that the ground state can be a trivial quantum paramagnet. But due to the frustrated interaction, the excitations with respect to this trivial state develop an extensively degenerate minima in the reciprocal space.
- 3. Moreover, as the system approaches the phase transition to a magnetic order, these extensively degenerate low-energy bosonic modes condense at the same time, leading to an unusual critical behavior.





Magnetic excitation in the kx-ky plane

Gang Chen, arXiv 1701.05634