

Symmetry enriched U(1) quantum spin liquids on a pyrochlore lattice

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Yao-Dong Li, **GC**, arXiv: **1607.02287**,
PRB Rapid Comm, 2017

one is sufficient
transition.

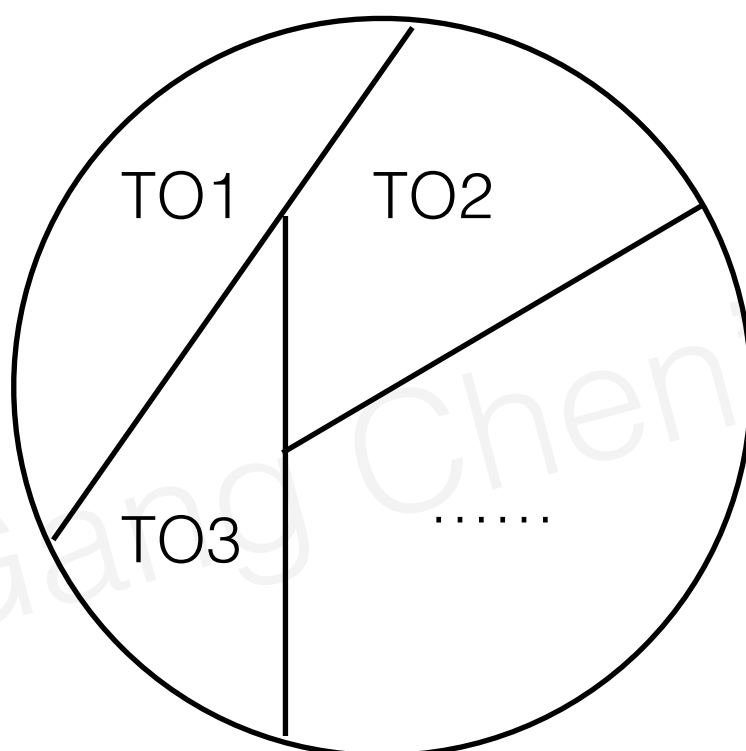


Symmetry enriched topological order

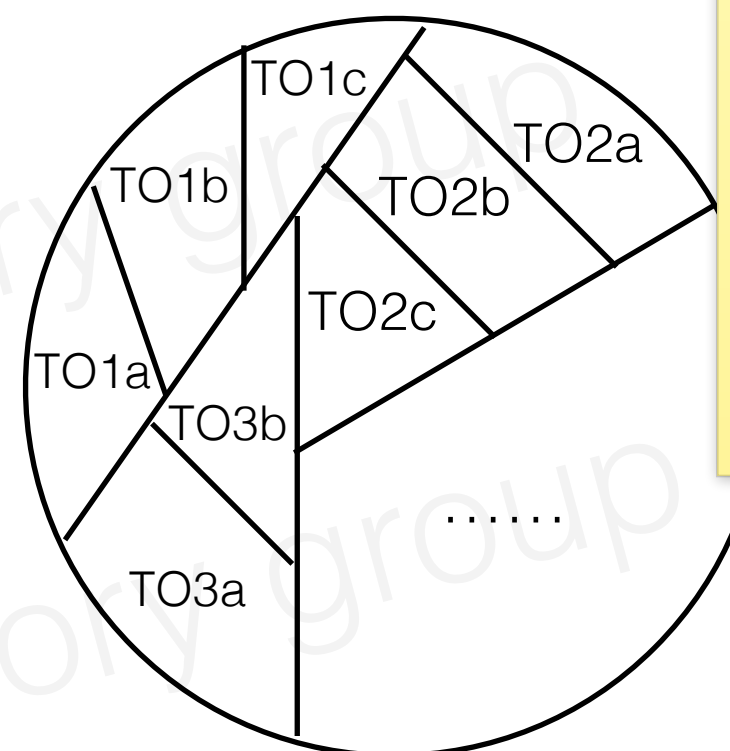


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Ying Ran
Michael Hermele
Meng Cheng
Yuanming Lu
Chenjie Wang
Zhengcheng Gu
Michael Levin
A Kitaev
.....



without symmetry



with symmetry:
the topological phases are richer,
even with the same topological order,
different SET phases cannot transform
into each other with finite-step local unitary
transformation w/o breaking symmetry.

Here, I am using a more general notion of “topological order” here.
I include the exotic phases whose gauge sector is gapless.

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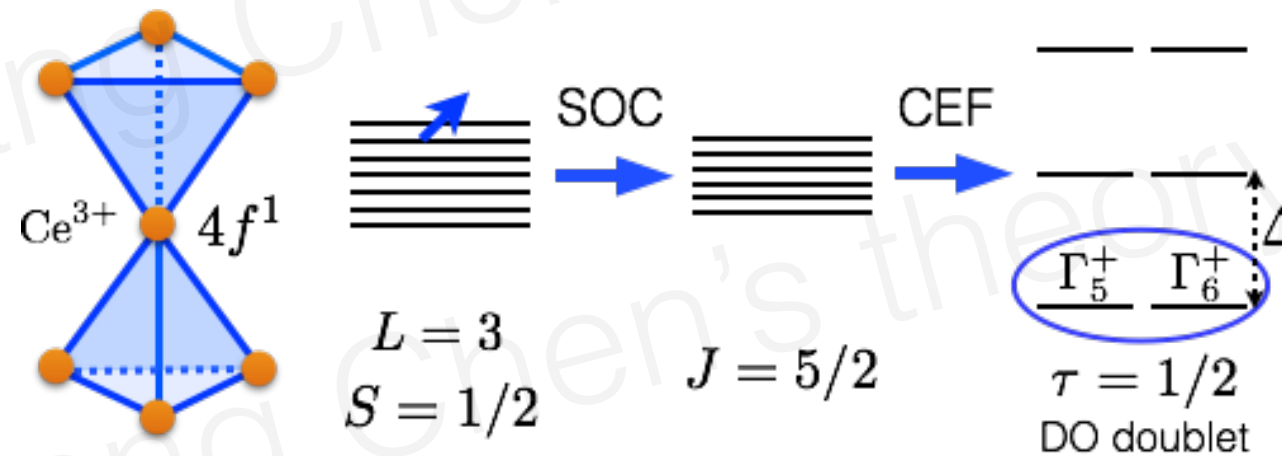
An important question

What are the *physical* degrees of freedom and their interactions to realize these novel quantum phases?

Candidate Quantum Spin Liquid in the Ce^{3+} Pyrochlore Stannate $\text{Ce}_2\text{Sn}_2\text{O}_7$

Romain Sibille,^{1,*} Elsa Lhotel,² Vladimir Pomjakushin,³ Chris Baines,⁴ Tom Fennell,^{3,†} and Michel Kenzelmann¹

$4f^1$ ion in D_{3d} local symmetry to the susceptibility was realized between $T = 1.8$ and 370 K, and the resulting calculation of the single ion magnetic moment is shown in Fig. 2(c). The wave functions of the ground state Kramers doublet correspond to a linear combination of $m_J = \pm 3/2$ states. The fitted coefficients result in energy levels at $50 \pm$



This doublet is **dipole-octupole doublet** !

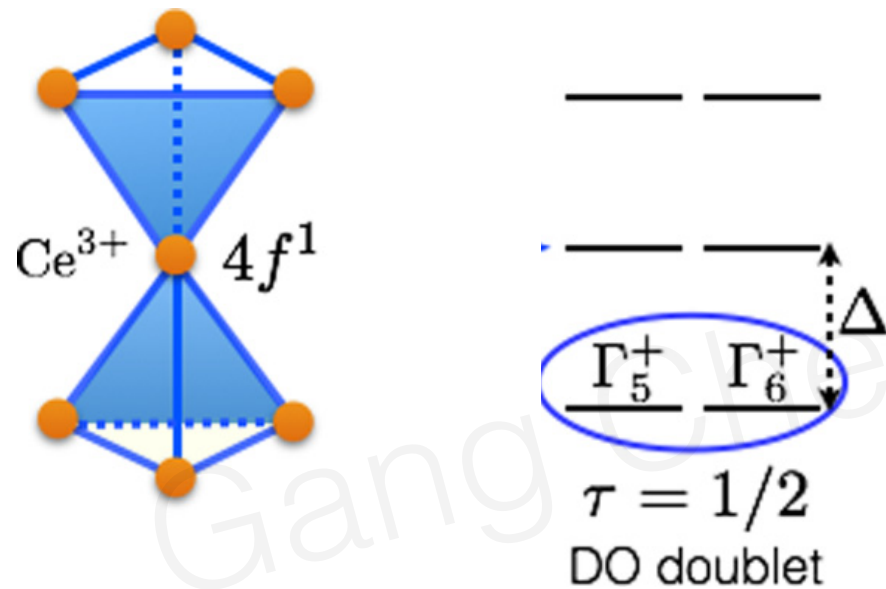
Huang, [GC](#), Hermele, PRL, 112, 167203 (2014), arXiv Nov 2013

Yao-Dong Li, [GC](#), PRB Rapid Comm 2017

Yao-Dong Li, XQ Wang, [GC](#), PRB Rapid Comm 2016

all the theor
sentence.

How does it work? Why so special?



$$|\Psi_+\rangle = a_1 |J^z = \frac{3}{2}\rangle + a_2 |J^z = -\frac{3}{2}\rangle$$

$$|\Psi_-\rangle = a_1^* |J^z = -\frac{3}{2}\rangle + a_2^* |J^z = \frac{3}{2}\rangle$$

$$e^{-i\frac{2\pi}{3}J^z} |\Psi_{\pm}\rangle = e^{-i\frac{2\pi}{3}\frac{3}{2}} |\Psi_{\pm}\rangle = -|\Psi_{\pm}\rangle$$

Huang, [GC](#), Hermele, PRL, 112, 167203 (2014), arXiv Nov 2013

Yao-Dong Li, [GC](#), PRB Rapid 2017

Yao-Dong Li, XQ Wang, [GC](#), arXiv:1608.07008

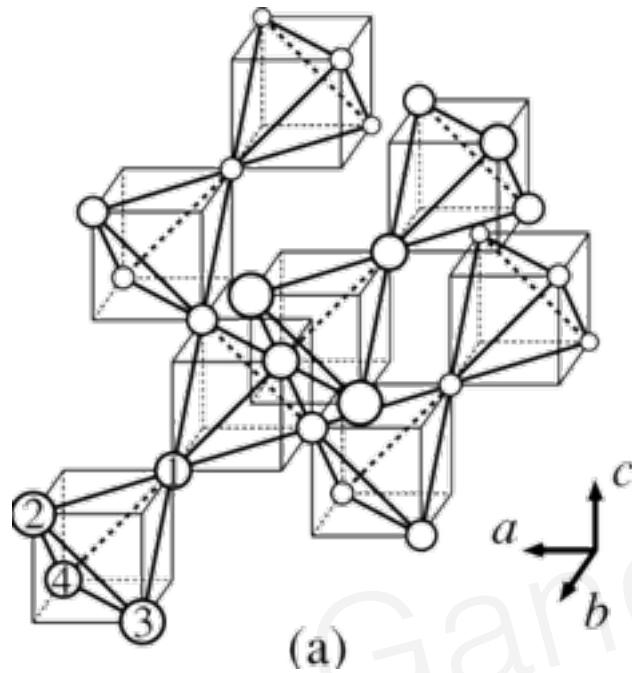
Symmetry Enriched $U(1)$ topological order and Experimental signatures of Symmetry Enrichment

next i will introduce the realistic XYZ model that describe the interaction between these peculiar doublet, and discuss the remarkable physical properties.

this does not look like XYZ,

just because of the spatial uniformity,
one can do a rotation in spin space
without effecting real space,

Generic model: XYZ model



$T_d \times \mathcal{I} \times translations$

$$H = \sum_{\langle ij \rangle} J_z S_i^z S_j^z + J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_{xz} (S_i^x S_j^z + S_i^z S_j^x)$$

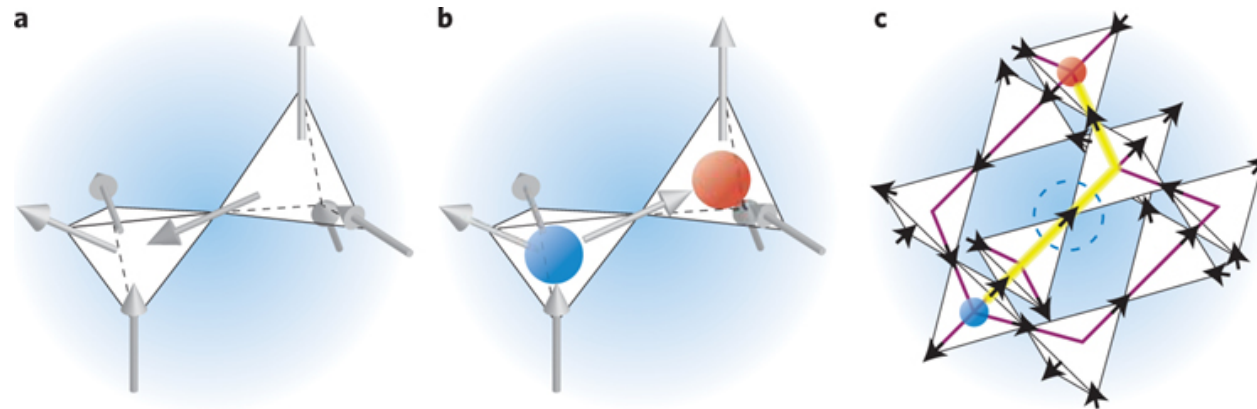


Rotation around the **y axis**
in the effective spin space

$$H_{\text{XYZ}} = \sum_{\langle ij \rangle} \tilde{J}_z \tilde{S}_i^z \tilde{S}_j^z + \tilde{J}_x \tilde{S}_i^x \tilde{S}_j^x + \tilde{J}_y \tilde{S}_i^y \tilde{S}_j^y \quad \text{XYZ model}$$

Important: \mathbf{S}^x and \mathbf{S}^z transform identically (as a dipole),
while \mathbf{S}^y transforms as an octupole moment under *mirror*.

XXZ limit: U(1) QSL of spin ice regime



Figs from Moessner&Schiffer,2009

Spinon deconfinement

Hermele, Fisher, Balents
Moessner, Huse, Sondhi

$$H = J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z - J_{\pm} \sum_{\langle i,j \rangle} (S_i^+ S_j^- + S_i^- S_j^+) + \dots$$

XXZ model can lead to U(1) QSL when J_{zz} is dominant

Emergent electric field

$$S^z \sim E$$

Emergent vector potential

$$S^{\pm} \sim e^{\pm iA}$$

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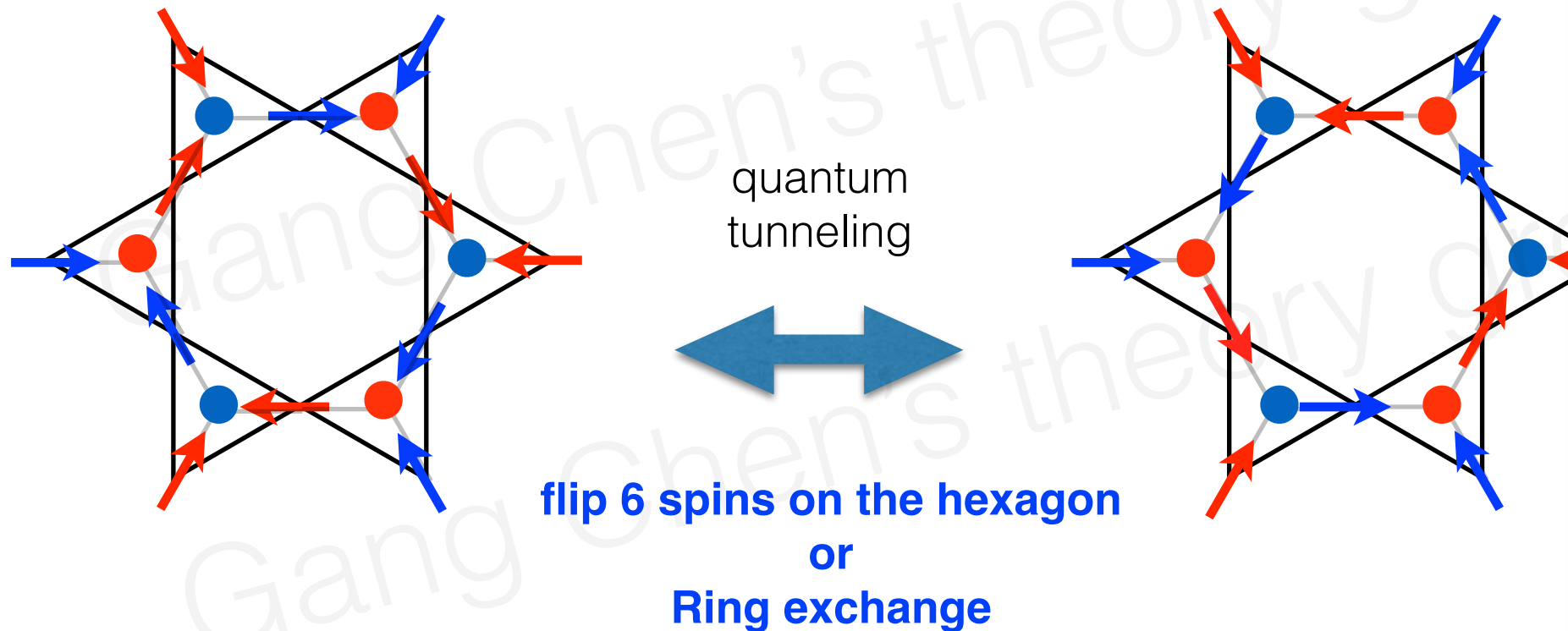
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Quantum fluctuation can lead to U(1) QSL

$$H = J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z - J_{\pm} \sum_{\langle i,j \rangle} (S_i^+ S_j^- + S_i^- S_j^+) + \dots$$

Hermele, Fisher, Balents, Moessner, Isakov, YP Kim



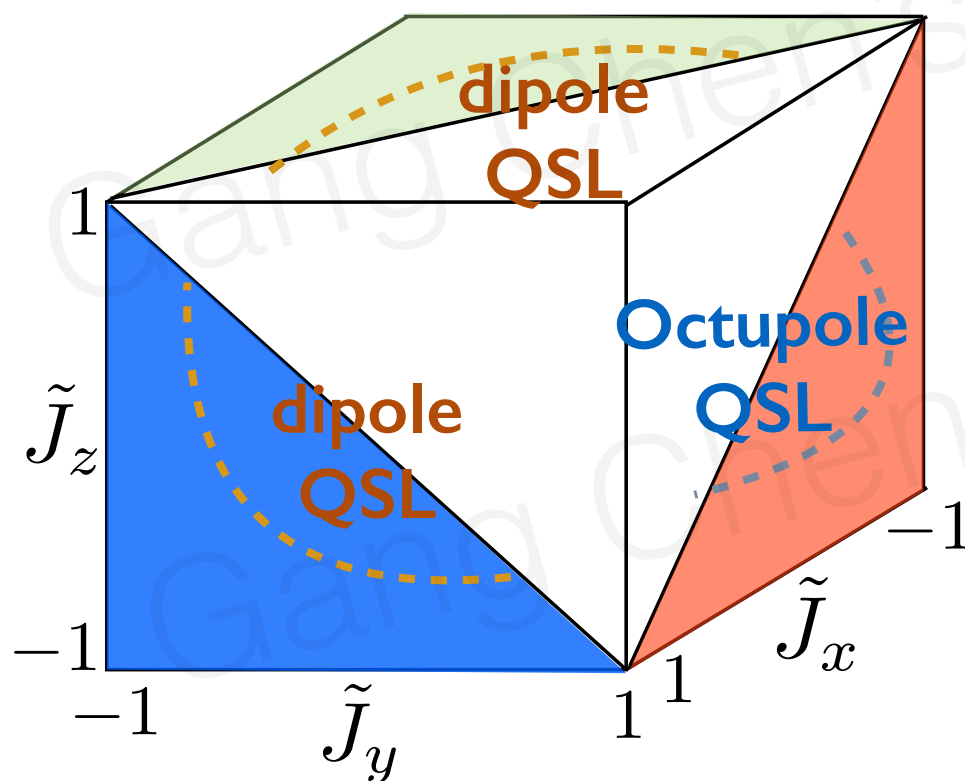
1. But classical spin ice is purely classical and is not a new phase of matter. It is smoothly connected to high temperature paramagnetic state.
2. In contrast, quantum spin ice is a new quantum phase of matter.
3. To obtain quantum spin ice, one simply adds quantum spin flip terms to the Ising Hamiltonian.
4. The quantum term allows the system to tunnel from one spin ice state to another.
5. As long as the term is not too large, the system is disordered, and the ground state is quantum spin ice.

- Pretty much one can add any term to create **quantum** tunneling, as long as it is not too large to induce magnetic order, the **ground state** is a quantum spin ice!

XYZ model is the generic model that describes the interaction between DO doublets.

$$H_{\text{XYZ}} = \sum_{\langle ij \rangle} \tilde{J}_z \tilde{S}_i^z \tilde{S}_j^z + \tilde{J}_x \tilde{S}_i^x \tilde{S}_j^x + \tilde{J}_y \tilde{S}_i^y \tilde{S}_j^y$$

unlike XXZ model, XYZ model is richer



3D phase diagram

Each component (not just S_z) can be emergent electric field, depending on the parameters !

Study phase on a cube: $-1 \leq \tilde{J}_{x,y,z} \leq 1$.

The shady part does not sign problem for quantum Monte Carlo

Infinite anisotropic g-factor

$$H = \sum_{\langle ij \rangle} J_z S_i^z S_j^z + J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_{xz} (S_i^x S_j^z + S_i^z S_j^x) - h \sum_i (\hat{n} \cdot \hat{z}_i) S_i^z$$

Different U(1) QSLs	Heat capacity	Inelastic neutron scattering measurement
Octupolar U(1) QSL for DO doublets	$C_v \sim T^3$	Gapped spinon continuum
Dipolar U(1) QSL for DO doublets	$C_v \sim T^3$	Both gapless gauge photon and gapped spinon continuum
Dipolar U(1) QSL for non-Kramers' doublets [23]	$C_v \sim T^3$	Gapless gauge photon
Dipolar U(1) QSL for usual Kramers' doublets [22]	$C_v \sim T^3$	Both gapless gauge photon and gapped spinon continuum

TABLE I. List of the physical properties of different U(1) QSLs on the pyrochlore lattice. “Usual Kramers doublet” refers to the Kramers doublet that is not a DO doublet. They transform as a two-dimensional irreducible representation under the D_{3d} point group. Although the dipolar U(1) QSL for DO doublets behaves the same as the one for usual Kramers' doublets, their physical origins are rather different [31].

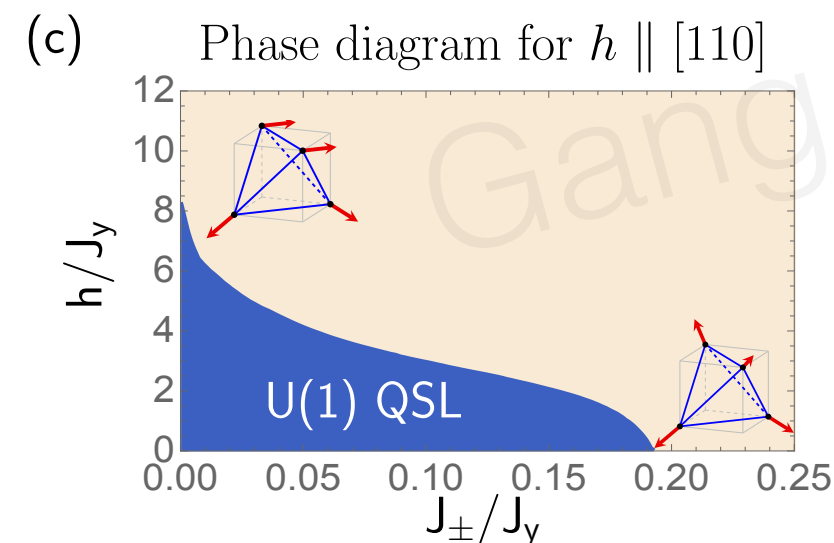
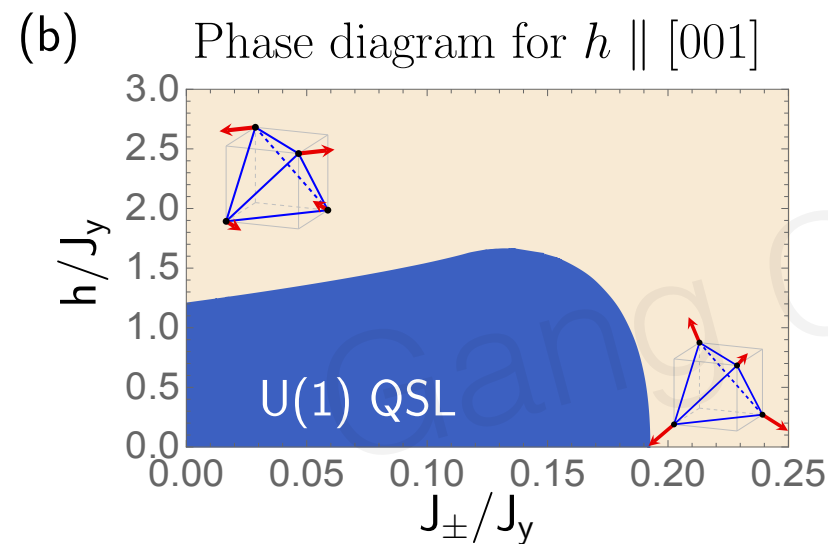
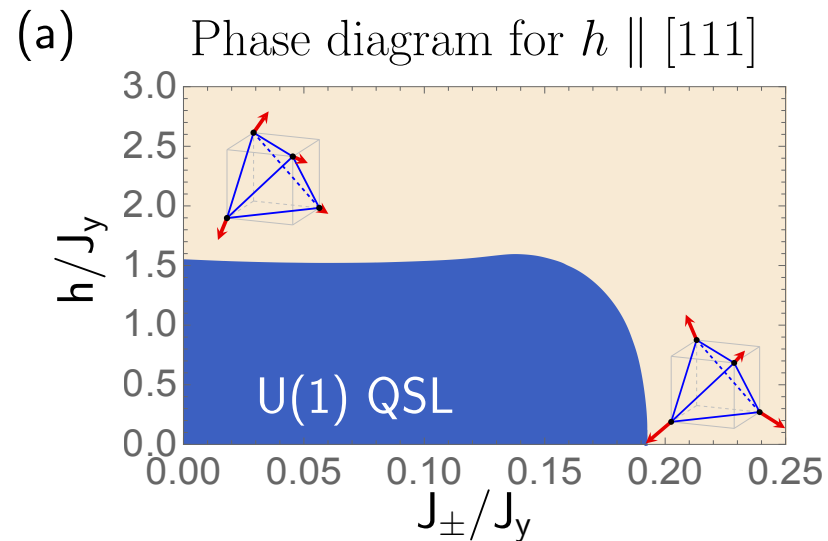
physics is defined with observables.

Field-driven Higgs transition for octupolar U(1) QSL

How to tell if **Ce₂Sn₂O₇** is an octupolar U(1) QSL or not ?

The idea to use a little knob that could simply lead to some clear experimental consequence, very much like the isotope effect of BCS superconductors.

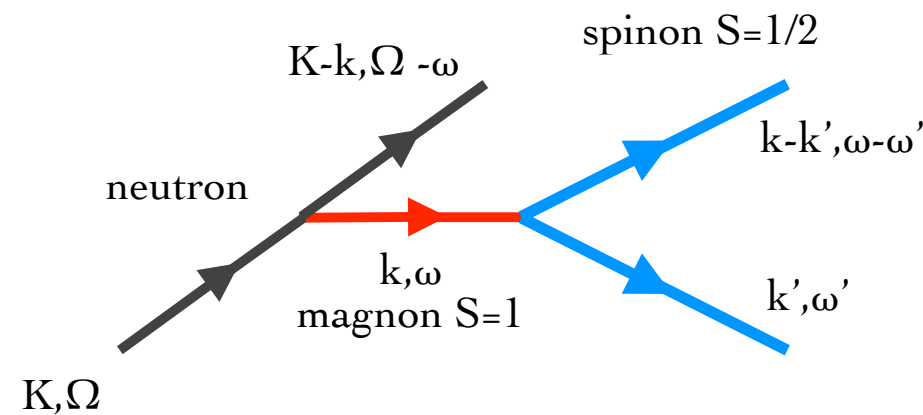
Here we apply external magnetic field, and expect a field-driven Higgs transition to magnetic ordering as the **field only couples to the matter field** (spin) without losing generality. Higgs transition is very different from the Meissner effect in superconductors.



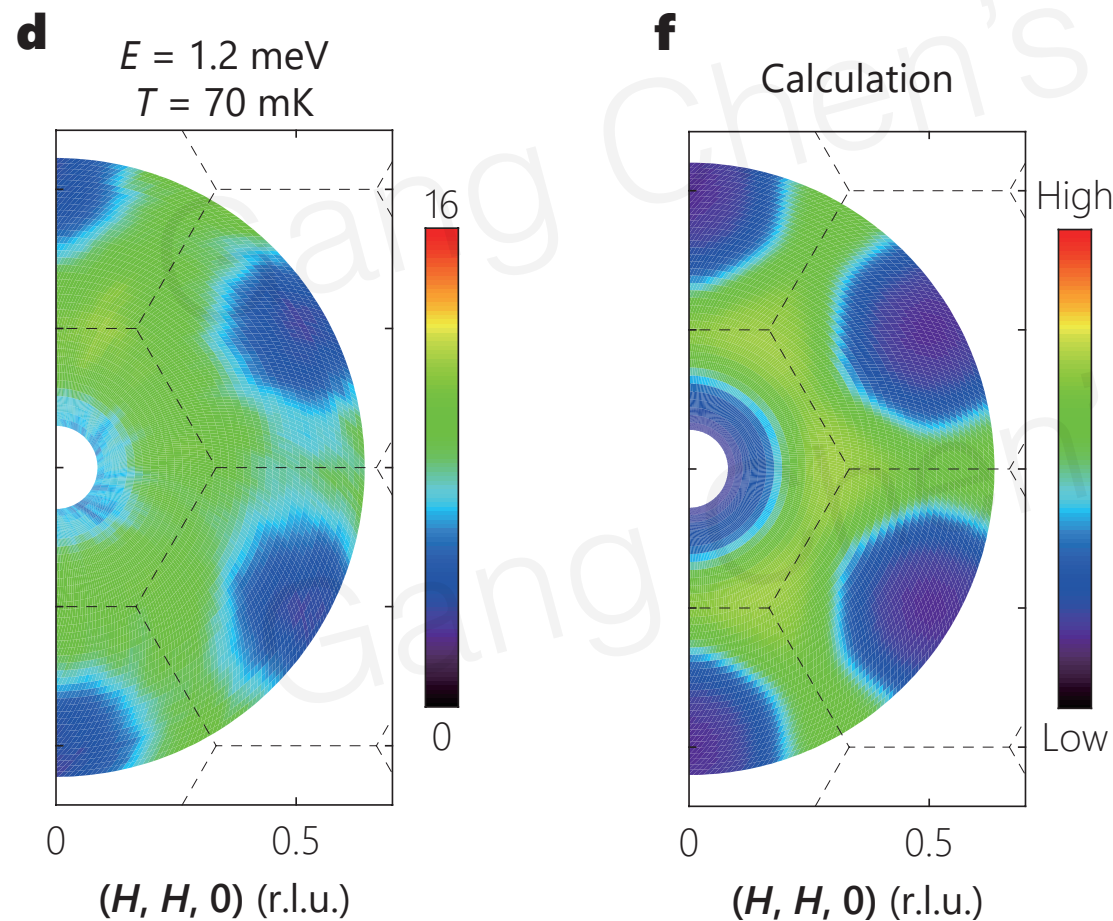
$$H = \sum_{\langle ij \rangle} J_y S_i^y S_j^y - J_{\pm} (S_i^+ S_j^- + S_i^- S_j^+) - h \sum_i (\hat{n} \cdot \hat{z}_i) S_i^z$$

$$S_i^{\pm} \equiv S_i^z \pm i S_i^x$$

Inelastic neutron scattering and spinon continuum



broad peak with
 $\omega = \varepsilon(k') + \varepsilon(k-k')$

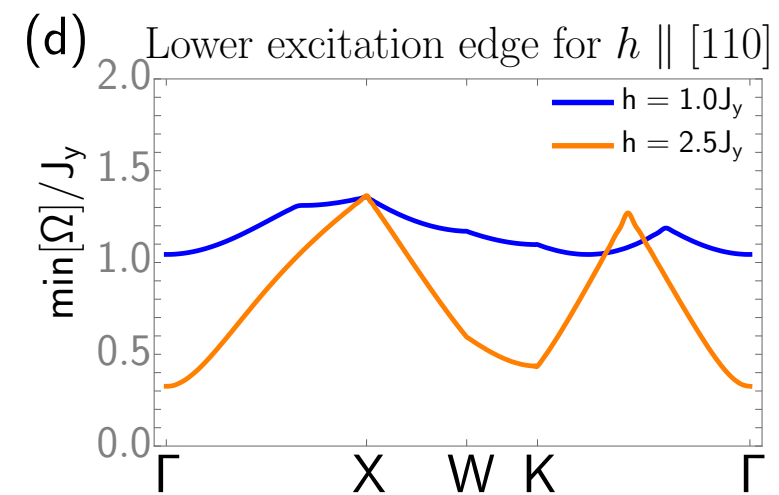
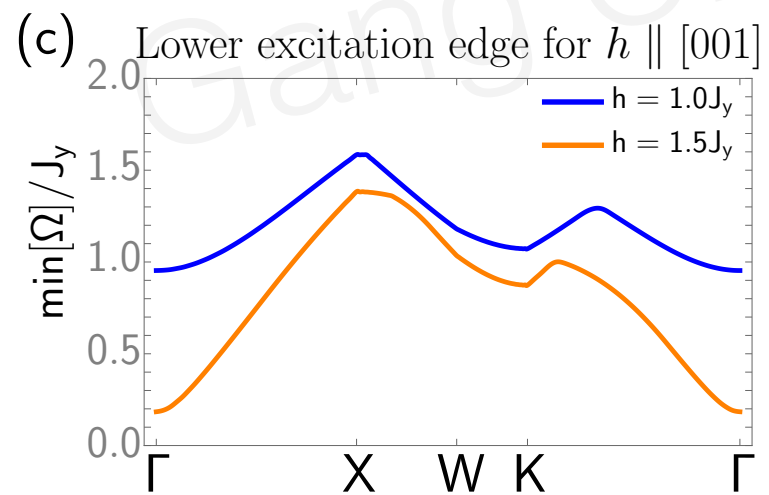
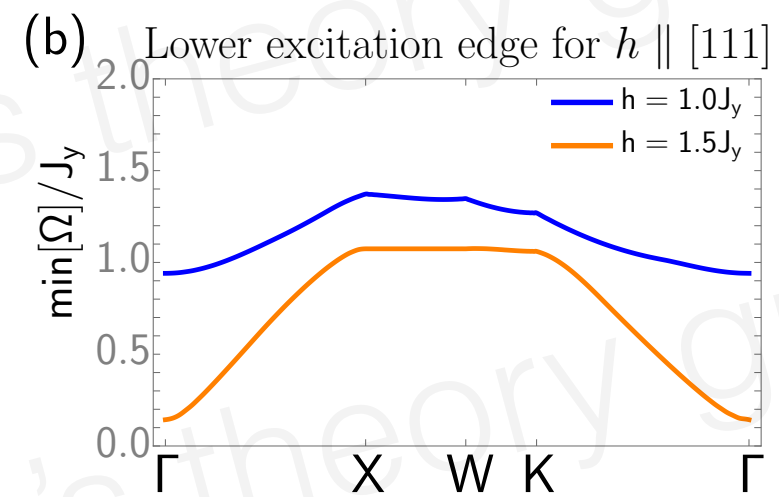
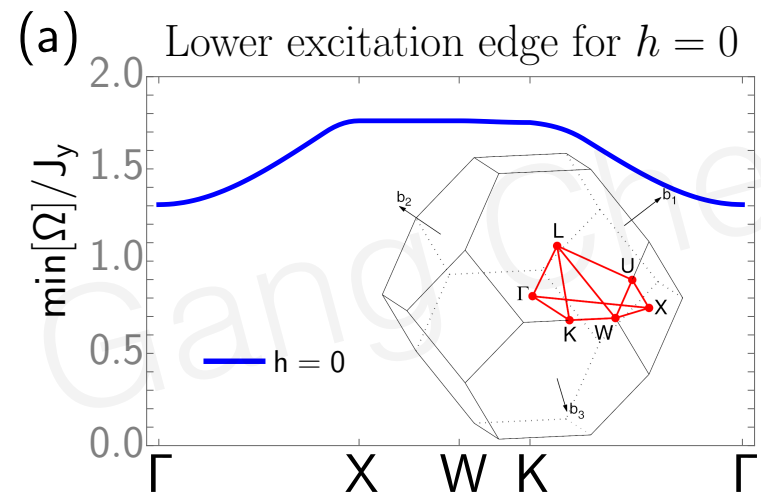


Spinon continuum of spinon Fermi surface
 in **YbMgGaO₄**
 Y Shen, Y-D Li, ..., **GC***, Jun Zhao*
 Nature 2016

Lower excitation edge

$$\mathbf{q} = \mathbf{k}_1 + \mathbf{k}_2,$$

$$\Omega(\mathbf{q}) = \omega_i(\mathbf{k}_1) + \omega_j(\mathbf{k}_2),$$



Material survey: other DO doublet systems

Our doublet can potentially be realized for any Kramers spin moment with $J > 1/2$.

Two well-known systems:

- Pyrochlores $A_2B_2O_7$,

e.g. ,

$Nd_2Ir_2O_7$, $Nd_2Sn_2O_7$, **$Nd_2Zr_2O_7$** , etc

$Dy_2Ti_2O_7$,

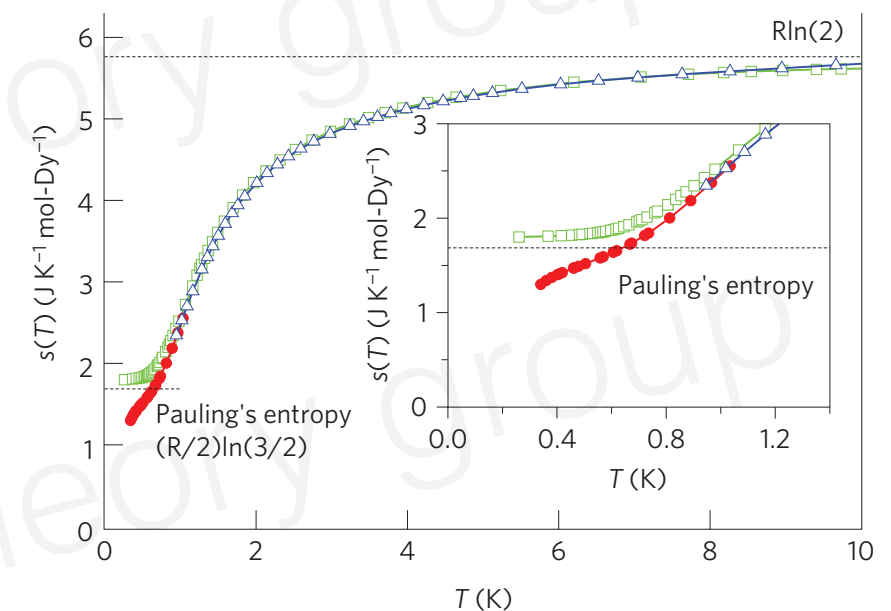
$Cd_2Os_2O_7$, etc

$Ce_2Sn_2O_7$,

- Spinels AB_2X_4 , B=lanthanide?

e.g. $CdEr_2Se_4$

$CdYb_2S_4$



Prof Gaulin's group, $Dy_2Ti_2O_7$, Nat Phys, 2013

Conclusion

- We point out a new doublet dubbed “dipole-octupole” doublet that is realized in the spin liquid material $\text{Ce}_2\text{Sn}_2\text{O}_7$.
- This doublet supports distinct symmetry enriched $U(1)$ spin liquids.
- We **predict** the experimental signatures of distinct symmetry enrichments.