Symmetry enriched U(1) topological orders: symmetry fractionalization of magnetic monopoles

Gang Chen
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Gang Chen's theory group

Outline

1. U(1) quantum spin liquid in 3D: spinon, monopole, photon,
   and fractionalization

2. Symmetry fractionalization in spinons and spectrum

3. Symmetry fractionalization in monopoles and spectrum

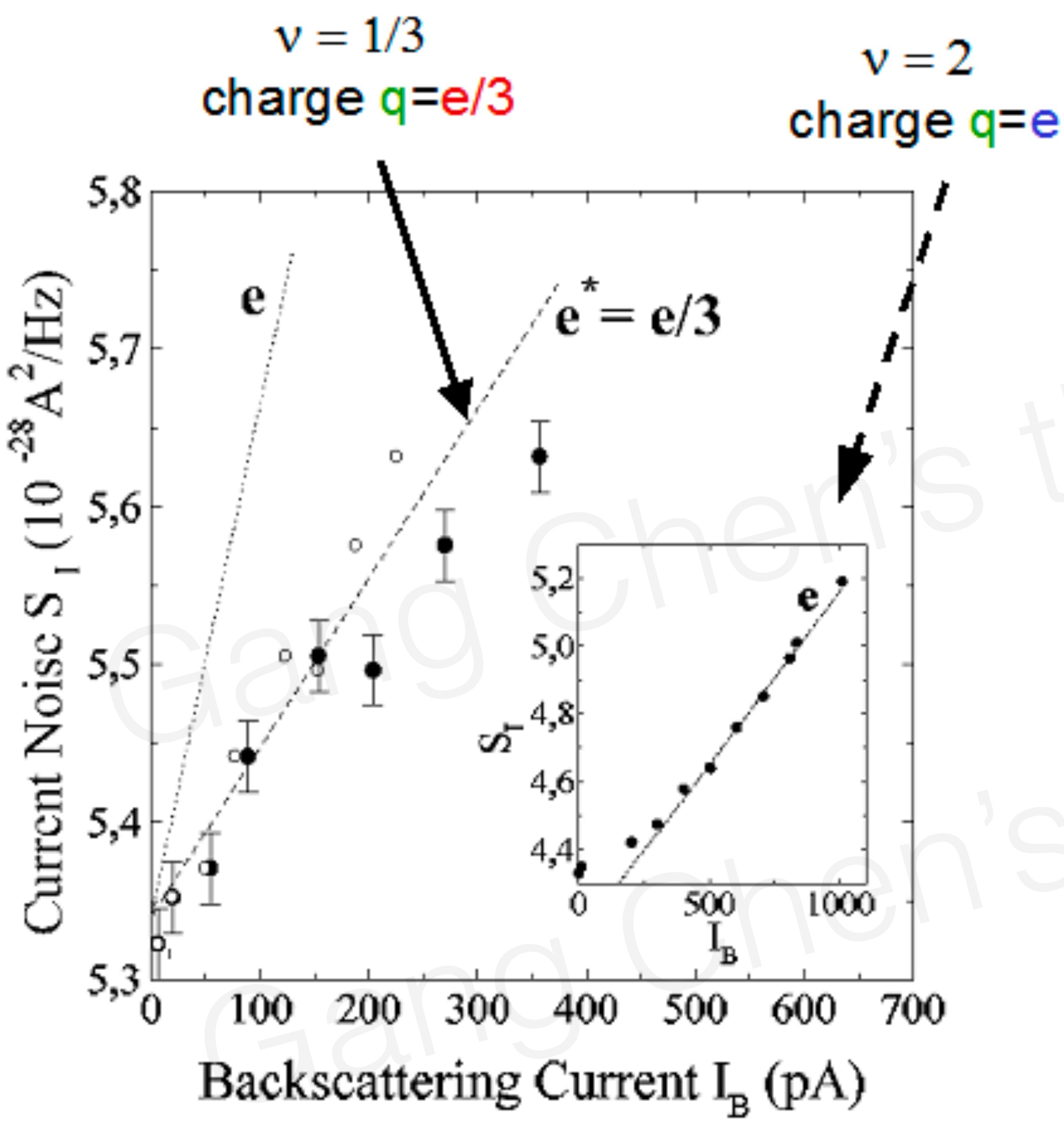
References

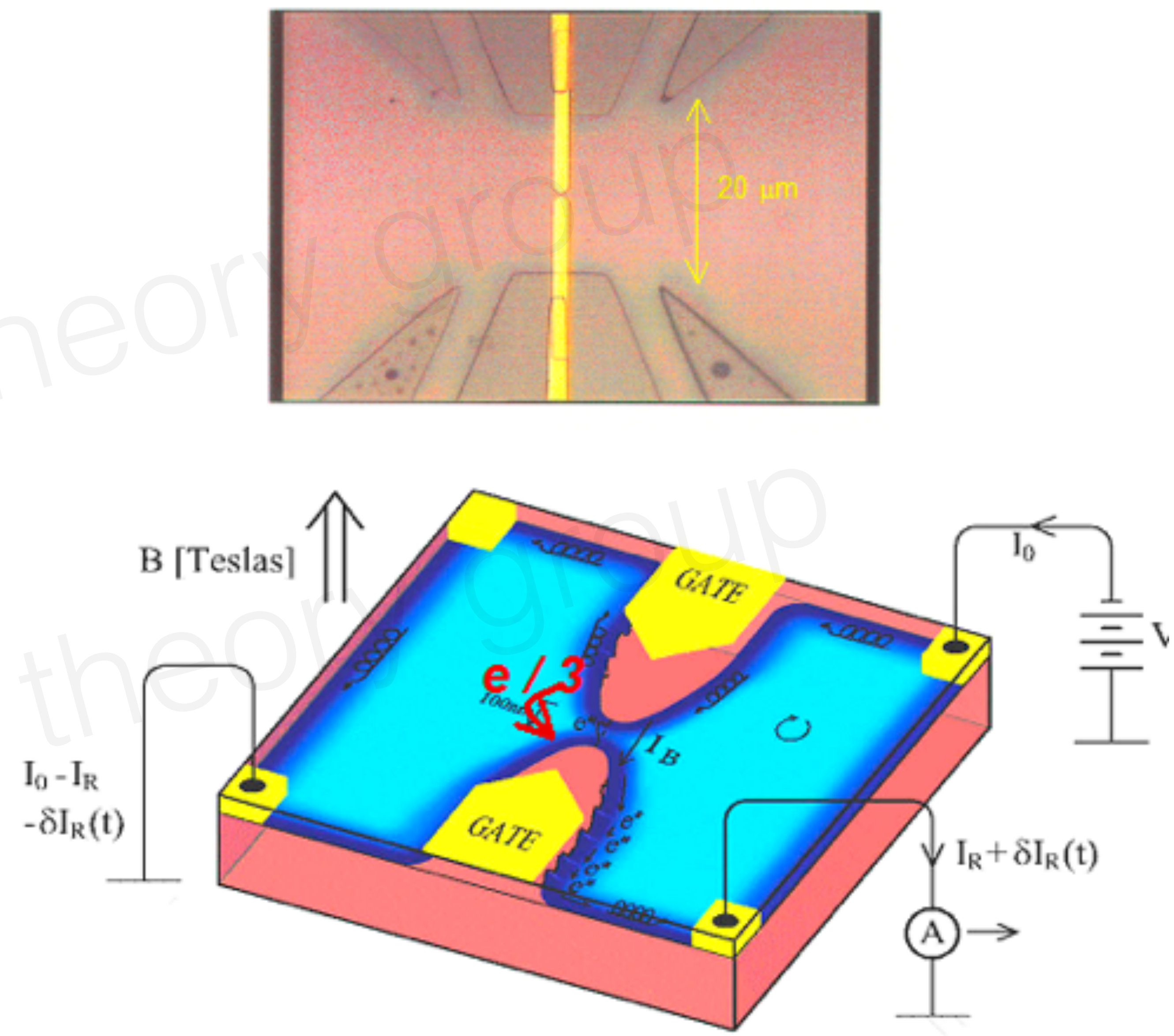
“Magnetic Monopole” condensation transition out of spin ice U(1) QSL: application to $Pr_2Ir_2O_7$
Gang Chen, PRB 94, 205107, (2016)

The spectral periodicity of spinon continuum in quantum spin ice
Gang Chen, PRB 96, 085136, (2017)

Dirac’s “magnetic monopoles” in pyrochlore ice U(1) spin liquids: spectrum and classification
Gang Chen, PRB 96, 195127, (2017)

Fractionalization in FQHE: shot-noise measurement





Etien et al, PRL 79, 2526 (1997)
also see Heiblum et al, Nature (1997)

FQHE is arguably the only existing topological order so far.

Chiral (Abelian) topological order

Fractionalization: fractionalized & deconfined excitation
Chern-Simon gauge structure

with charge U(1) **symmetry**:
charge conservation

Fractionalized charge excitation

Symmetry makes topological order more visible in experiments.

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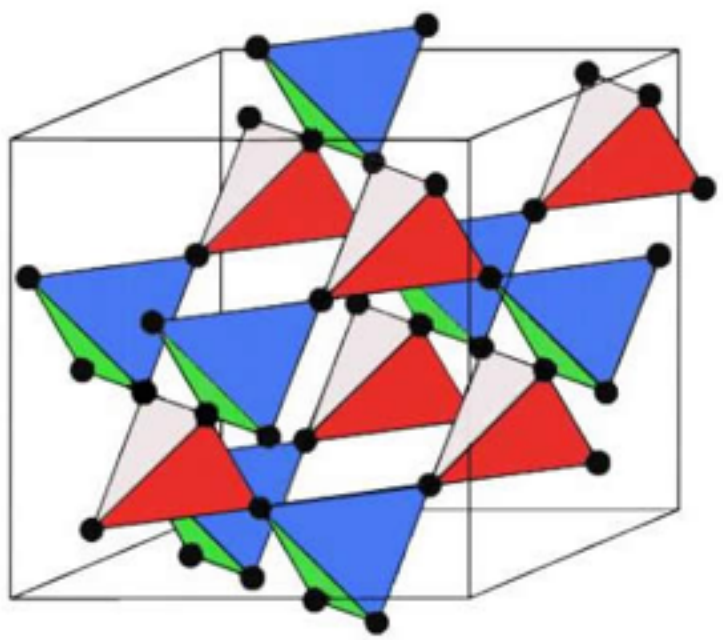
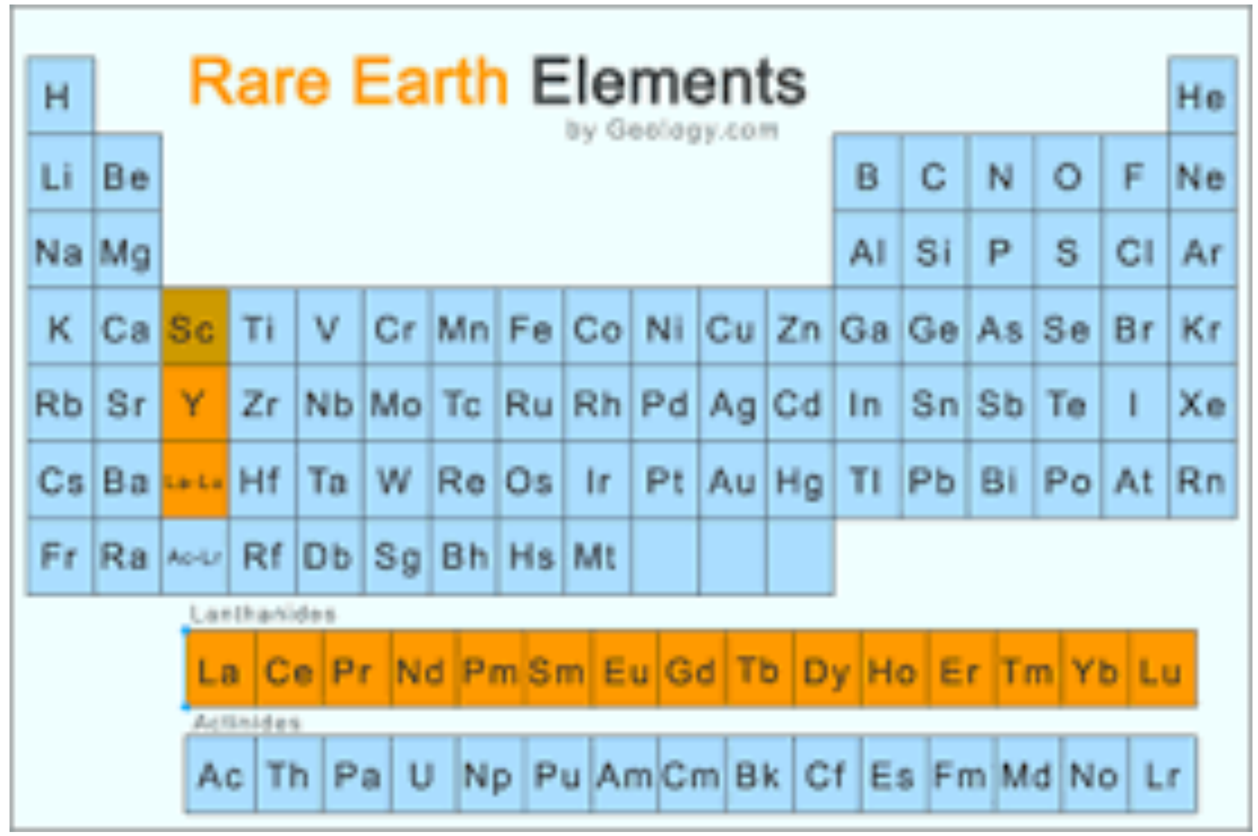
Pyrochlore spin ice



$RE_2M_2O_7$



$$H = J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z \text{ + dipolar}$$

Castelnovo, Gingras, Moessner, Sondhi, Schiffer,
Shannon, Moon, Kim, Savary, Balents, …….

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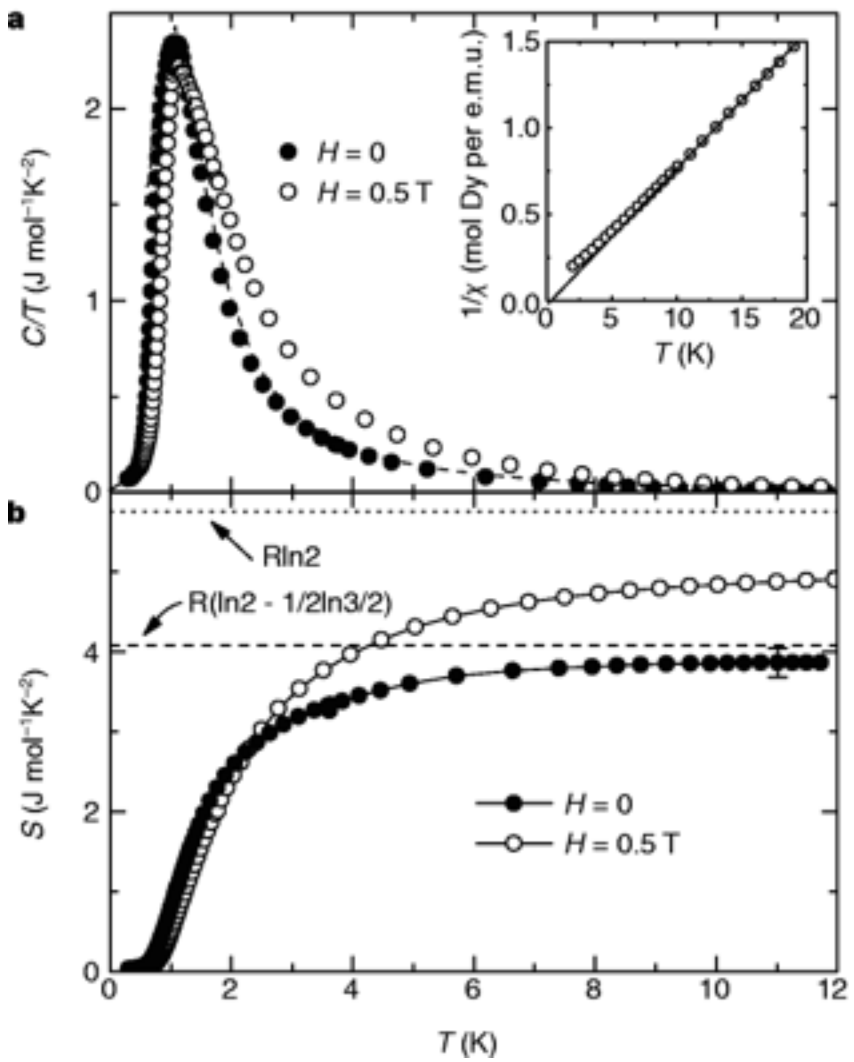
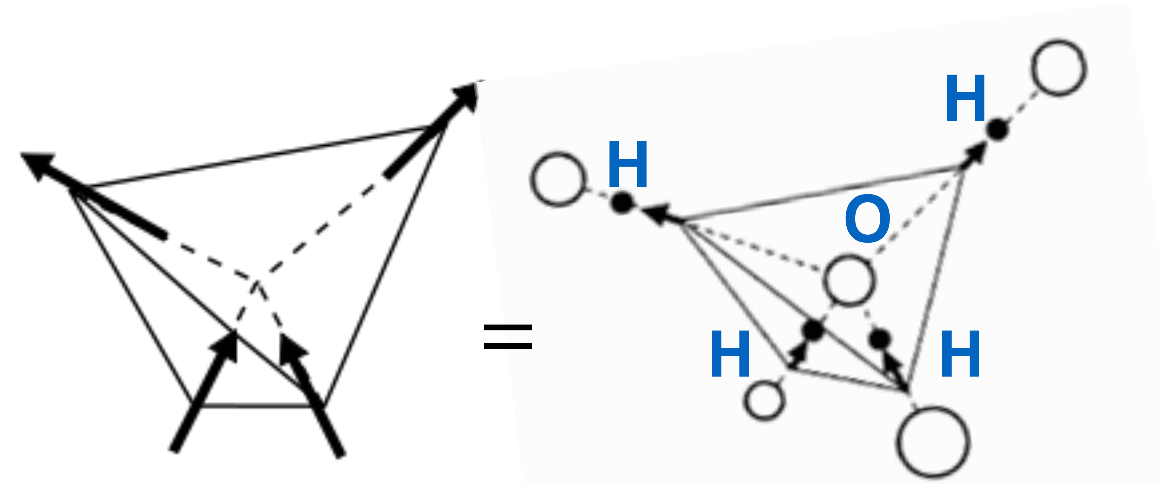
Classical spin ice

$Dy_2Ti_2O_7$

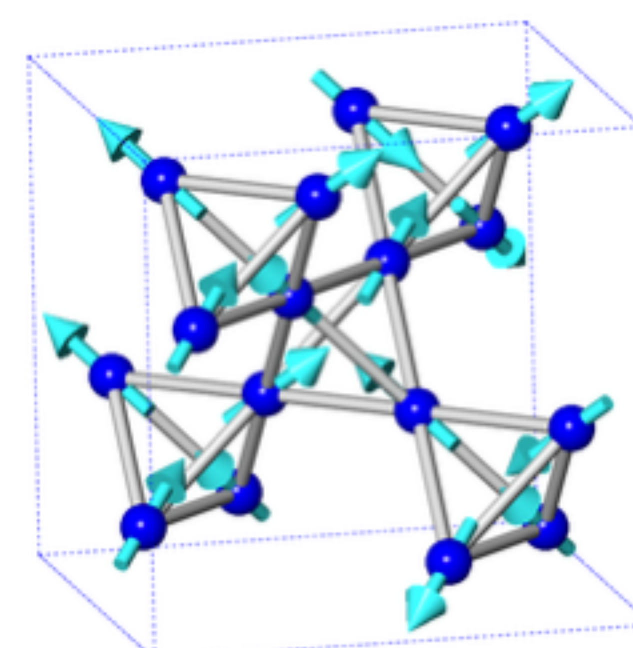


Pauling entropy in spin ice,
Ramirez, etc, Science 1999



2-in 2-out
spin ice rule

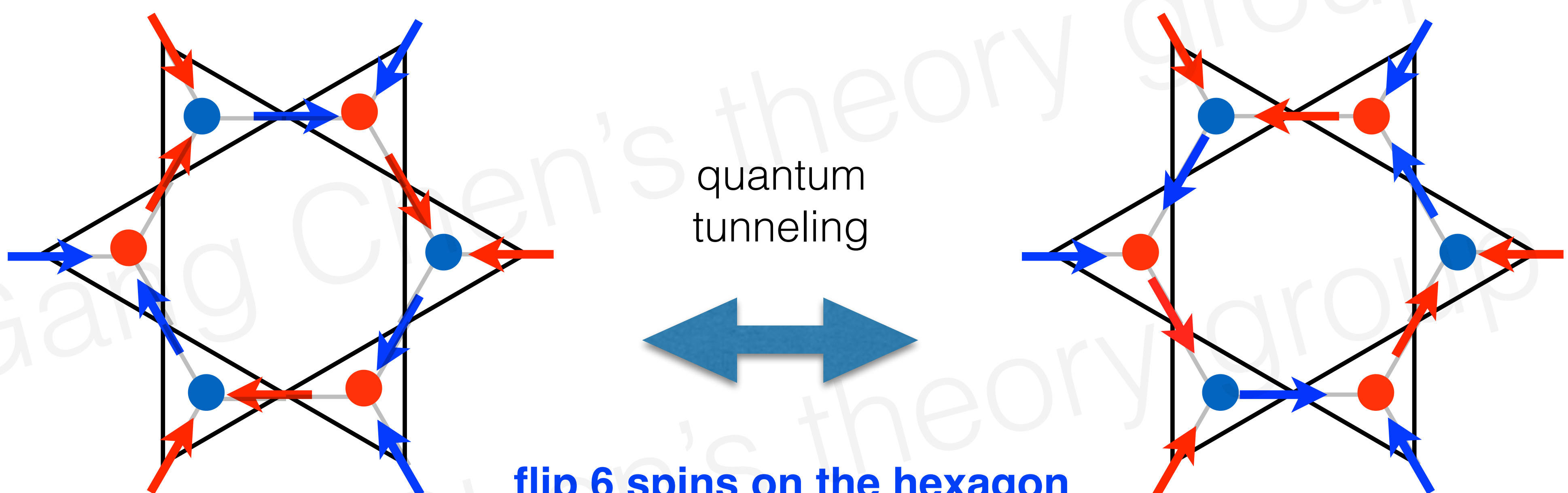
2-in 2-out
water ice rule



Toy model and U(1) quantum spin liquid

$$H = J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z \boxed{- J_{\pm} \sum_{\langle i,j \rangle} (S_i^+ S_j^- + S_i^- S_j^+)} + \cdots\cdots$$

Hermele, Fisher, Balents, PRB 2004



flip 6 spins on the hexagon
or
Ring exchange

- Pretty much one can add any term to create **quantum** tunneling, as long as it is not too large to induce magnetic order, the **ground state** is a U(1) quantum spin liquid !

1. But classical spin ice is purely classical and is not a new phase o matter. It is smoothly connected to high temperature paramagnetic ph

2. In contrast, quantum spin ice is new quantum phase of matter.

Ring exchange and a unitary transformation

$$\mathcal{H}_{eff}=(J_\perp^2/J_z)(J_\perp/J_z-1)N_t$$

$$+J_{ring}\sum_{\hexagon}(S_1^+S_2^-S_3^+S_4^-S_5^+S_6^-+\text{H.c.}),$$

site we can make the transformation $S^z \to S^z$ and $S^\pm \to -S^\pm$ by making a π rotation about the z axis in spin space. One transformation with the desired effect, consisting of π rotations on a pattern of sites, is

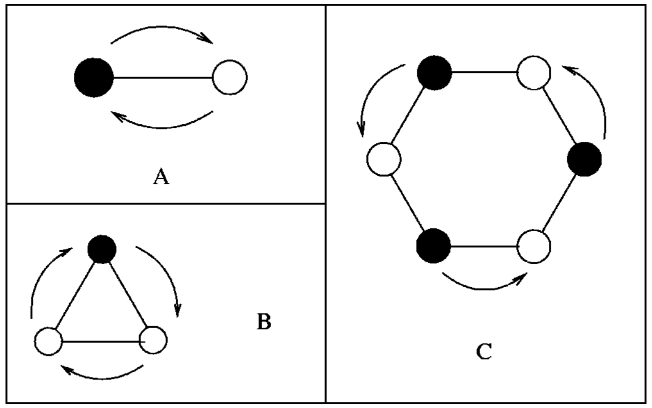
$$S_i^z \to S_i^z, \tag{7}$$

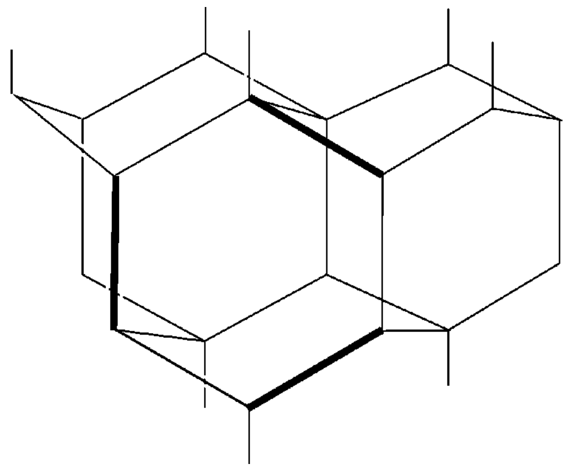
$$S_{\mathbf{R}i}^\pm \to \exp(i\mathbf{Q}_i \cdot \mathbf{R}) S_{\mathbf{R}i}^\pm, \tag{8}$$

where $\mathbf{Q}_0=\mathbf{Q}_1=(\mathbf{b}_1+\mathbf{b}_2)/2$ and $\mathbf{Q}_2=\mathbf{Q}_3=0$.

After this transformation the Hamiltonian takes the form

$$\mathcal{H}_p=-J_{ring}\sum_{\hexagon}(S_1^+S_2^-S_3^+S_4^-S_5^+S_6^-+\text{H.c.}), \tag{9}$$

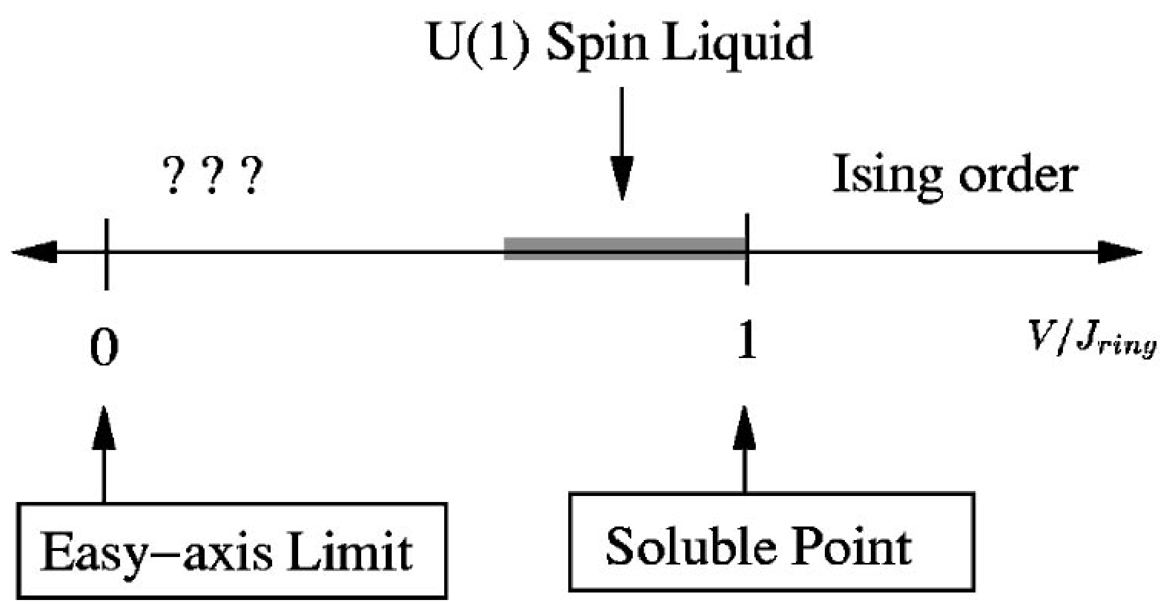


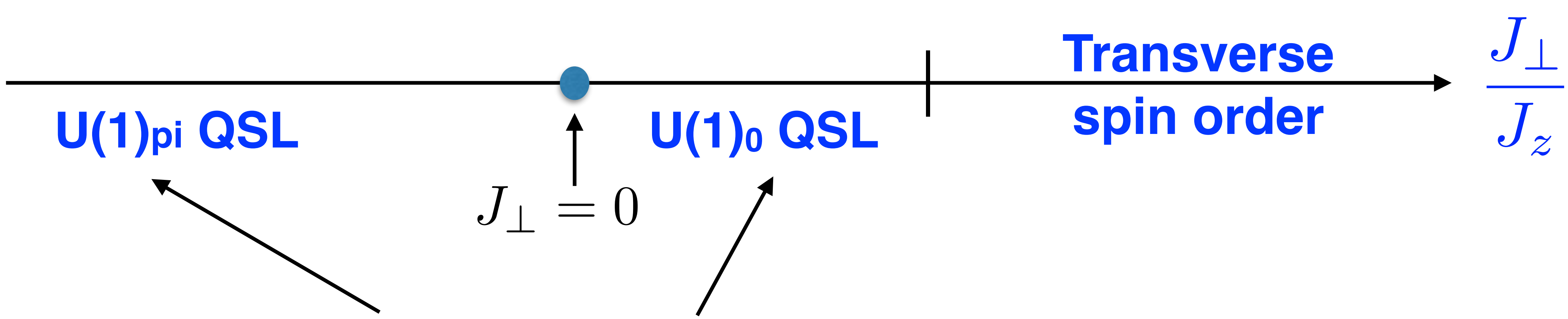


link on diamond lattice

Hermele, Fisher, Balents, PRB 2004

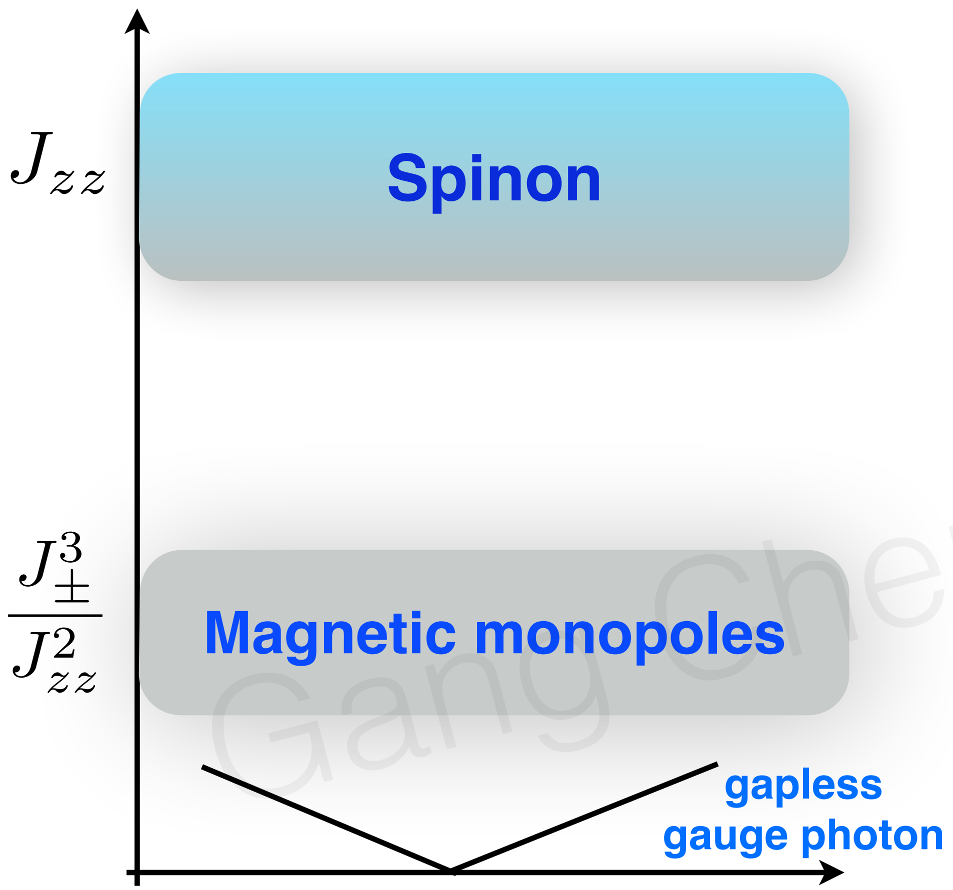
Theoretical expectation

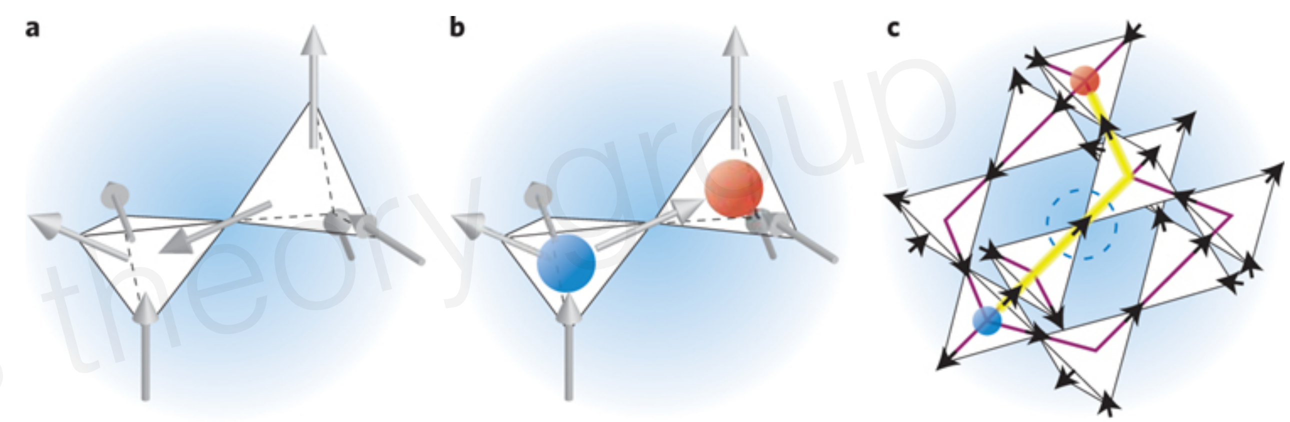




Related by unitary transformation
(Hermele, Fisher, Balents 2004)

Excitations in the U(1) QSL





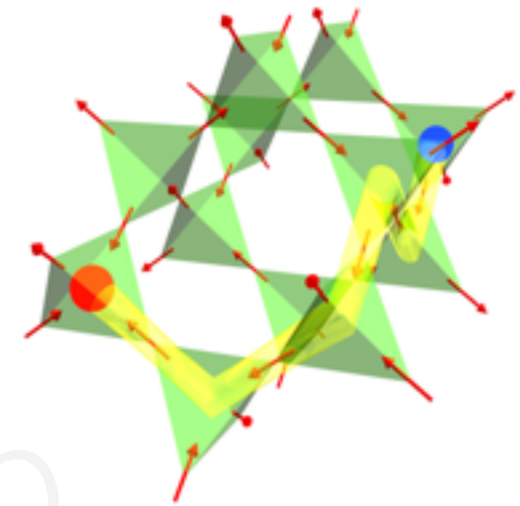
Figs from Moessner&Schiffer,2009

Spinon deconfinement

- No LRO, no symmetry breaking, **cannot** be understood in Landau's paradigm!
- The right description is in terms of fractionalization and emergent gauge structure.

as quantum spin ice is a disordered state,
there is no long range order, no symmeetry b
a new phase of matter and cannot be unders
in the landau's paradigm of symmetry breakin

Equivalence of “notations”



Excitations (notation 1)	Excitations (notation 2)
Spinon	Magnetic monopole
“Magnetic monopole”	Electric monopole
Gauge photon	Gauge photon

has classical analogue

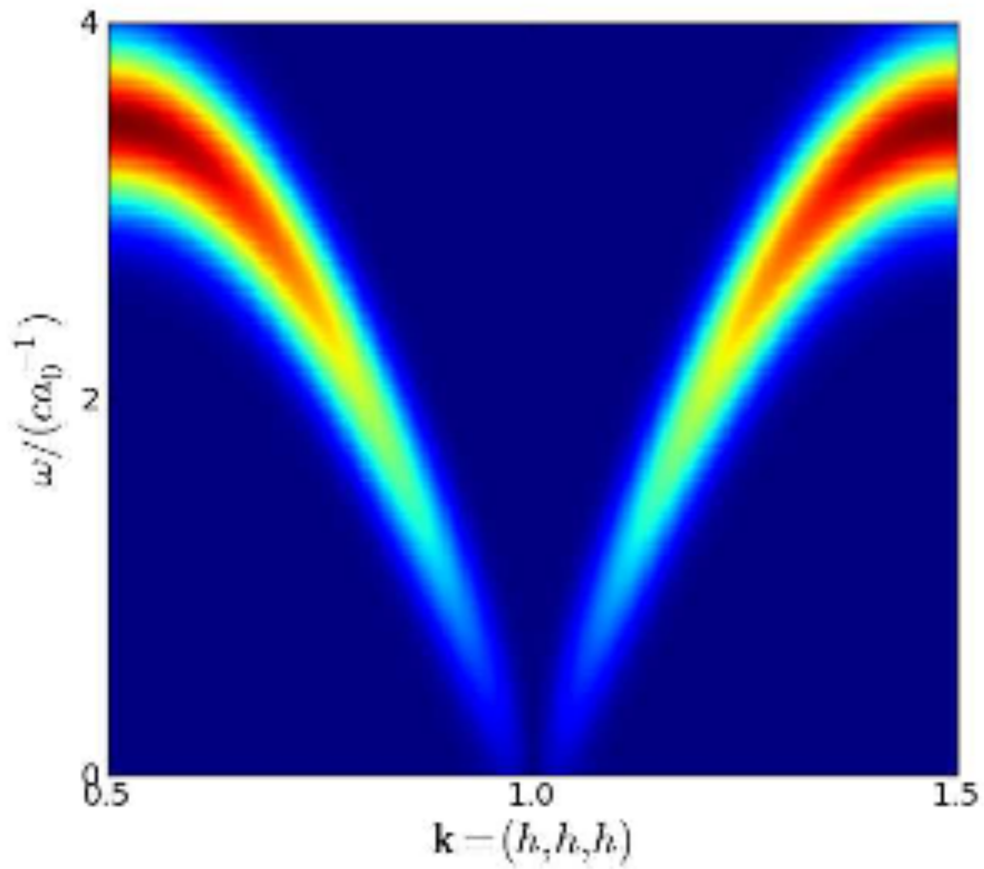
purely quantum, no classical analogue

“Magnetic monopole” is probably closer in spirit to **Dirac’s monopole (1931).**
One has to confirm that “magnetic monopole” is emergent excitation,
rather than a fictitious particle.

What piece of experimental info indicates these exotic and emergent particles?

Important question

What are sharp physical observables to confirm U(1) QSL ?



$$I(\omega) \sim \omega$$

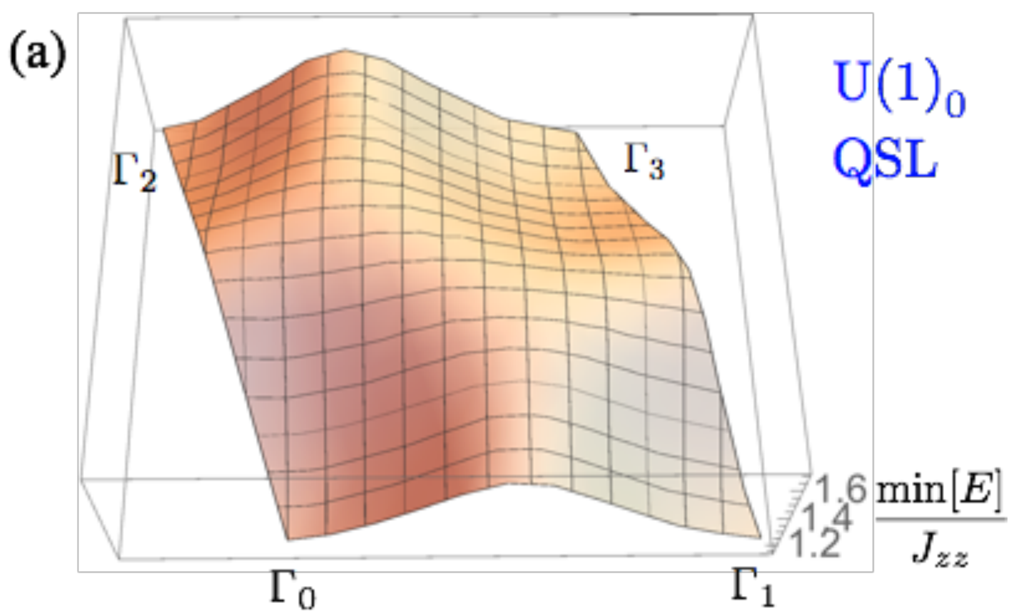
Nic Shannon, etc 2012,
Savary, Balents, 2012

low energy scale suppressed intensity

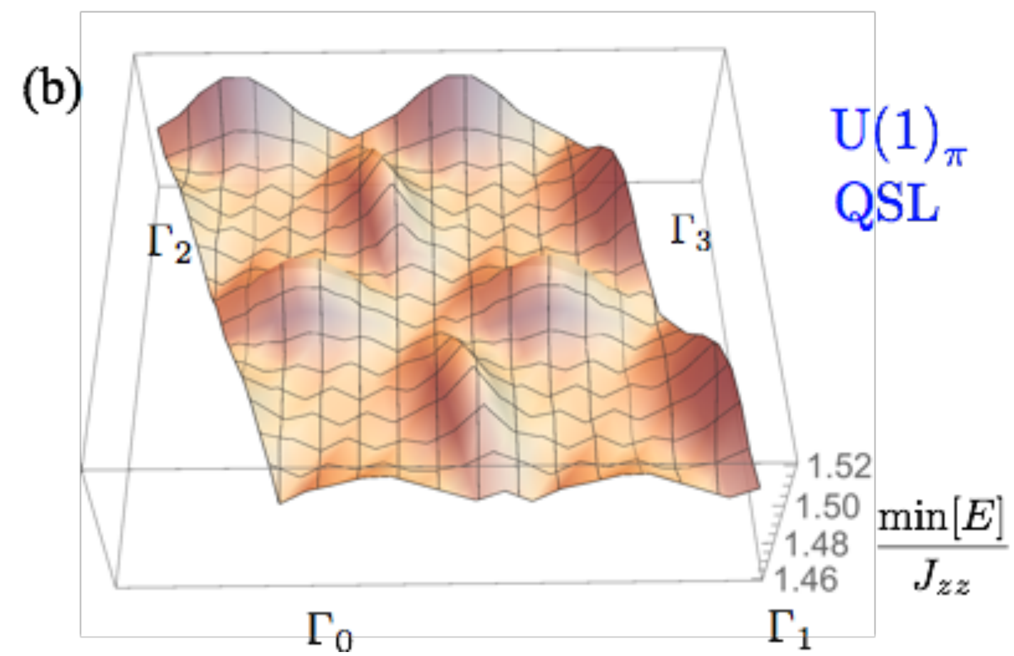
heat capacity (Savary & Balents):
1000 times larger than phonon!

Our answer:

the spectral periodicity of the spinon/monopole continuum



regular periodicity



enlarged periodicity

Enlarged periodicity is like the fractional charge in FQHE.

Gang Chen, PRB 96, 085136, (2017)
PRB 96, 195127, (2017)

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2. Symmetry fractionalization in spinons and spectrum

Realistic models

- Usual Kramers' doublet and non-Kramers' doublet

$$H = \sum_{\langle ij \rangle} \{ J_{zz} \mathsf{S}_i^z \mathsf{S}_j^z - J_{\pm} (\mathsf{S}_i^+ \mathsf{S}_j^- + \mathsf{S}_i^- \mathsf{S}_j^+)$$
$$+ J_{\pm\pm} (\gamma_{ij} \mathsf{S}_i^+ \mathsf{S}_j^+ + \gamma_{ij}^* \mathsf{S}_i^- \mathsf{S}_j^-)$$
$$+ J_{z\pm} [\mathsf{S}_i^z (\zeta_{ij} \mathsf{S}_j^+ + \zeta_{ij}^* \mathsf{S}_j^-) + i \leftrightarrow j] \},$$

S. H. Curnoe, PRB (2008).

Savary, Balents, PRL 2012

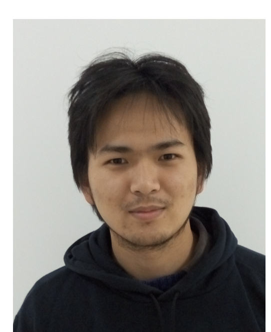
S. Onoda, etc, 2009, 2010

SB Lee, Onoda, Balents, 2012

- Dipole-octupole doublet

$$H = \sum_{\langle ij \rangle} J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_z S_i^z S_j^z$$
$$+ J_{xz} (S_i^x S_j^z + S_i^z S_j^x).$$

Yi-Ping Huang

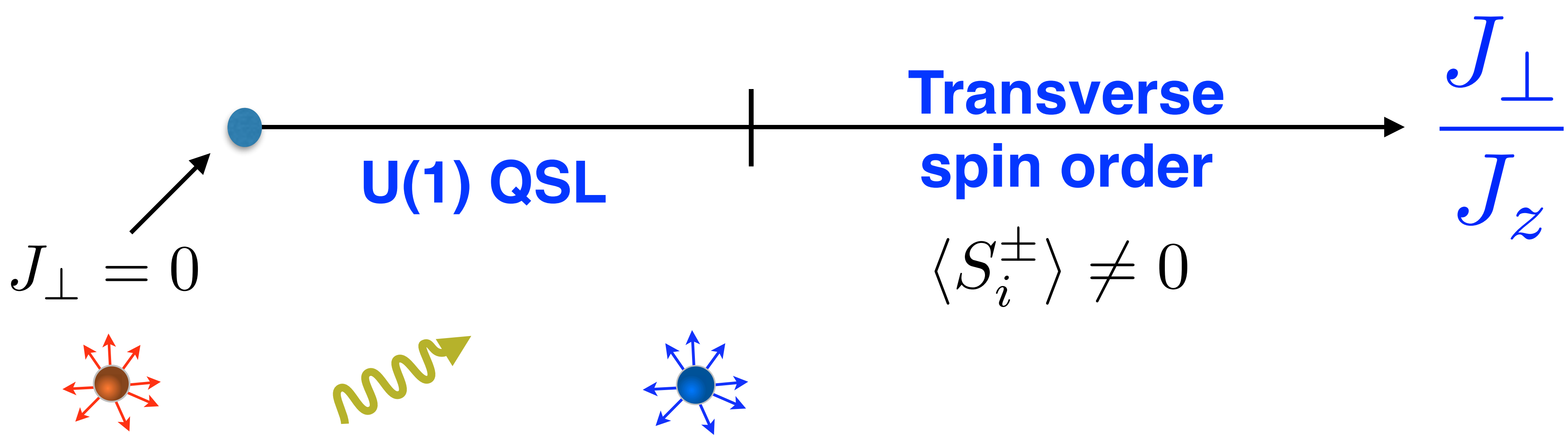


Yao-Dong Li
(Fudan -> UCSB)

Y-P Huang, **Gang Chen**, M Hermele, PRL 2014
Yao-Dong Li, **Gang Chen**, PRB 2017

Use the XXZ model to illustrate the universal physics

$$\mathcal{H}_{\mathrm{XXZ}} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_{\perp}(S_i^+ S_j^- + S_i^- S_j^+),$$



Hermele, Fisher, Balents, 2004,
Banerjee, Isakov, Demle, YongBaek Kim 2008
Savary, Balents, 2012
Kato, Onoda, 2015
Nic Shannon, et al, 2012

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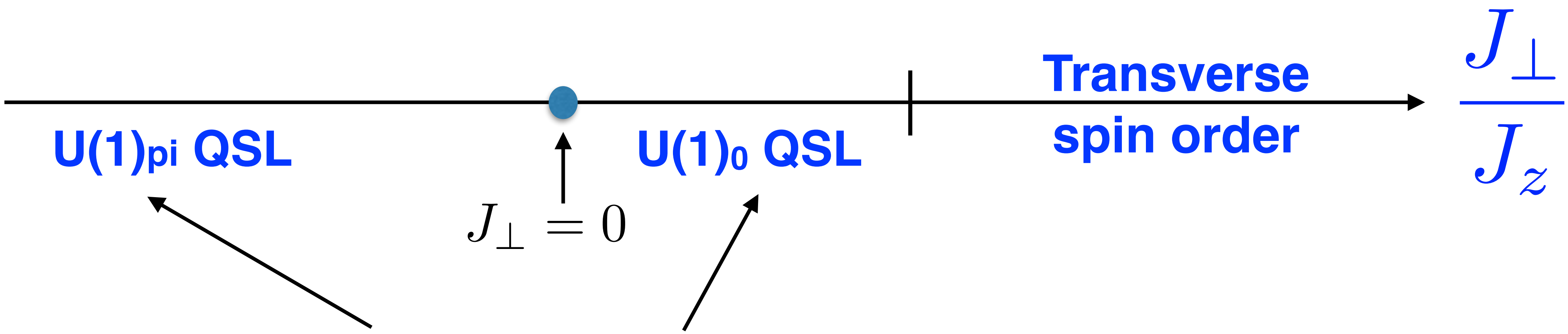
Frustrated regime: early theoretical study



Related by unitary transformation (Hermele, Fisher, Balents 2004)

PHYSICAL REVIEW B **86**, 104412 (2012)

Generic quantum spin ice

SungBin Lee,[1] Shigeki Onoda,[2] and Leon Balents[3]

one. We also consider the case of frustrated *XY* exchange, and find that it favors a π-flux QSL, with an emergent line degeneracy of low-energy spinon excitations. This feature greatly enhances the stability of the QSL with respect to classical ordering.

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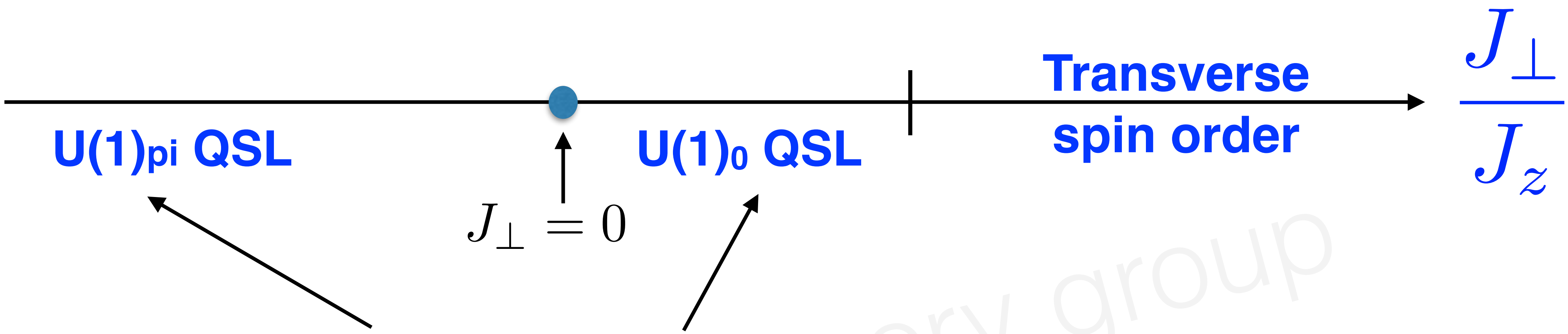


Related by unitary transformation (Hermele, Fisher, Balents 2004)

Besides the quantitative differences, are there sharp distinctions between the $U(1)_{pi}$ QSL on the left and the $U(1)_0$ QSL on the right?

Lattice gauge theory

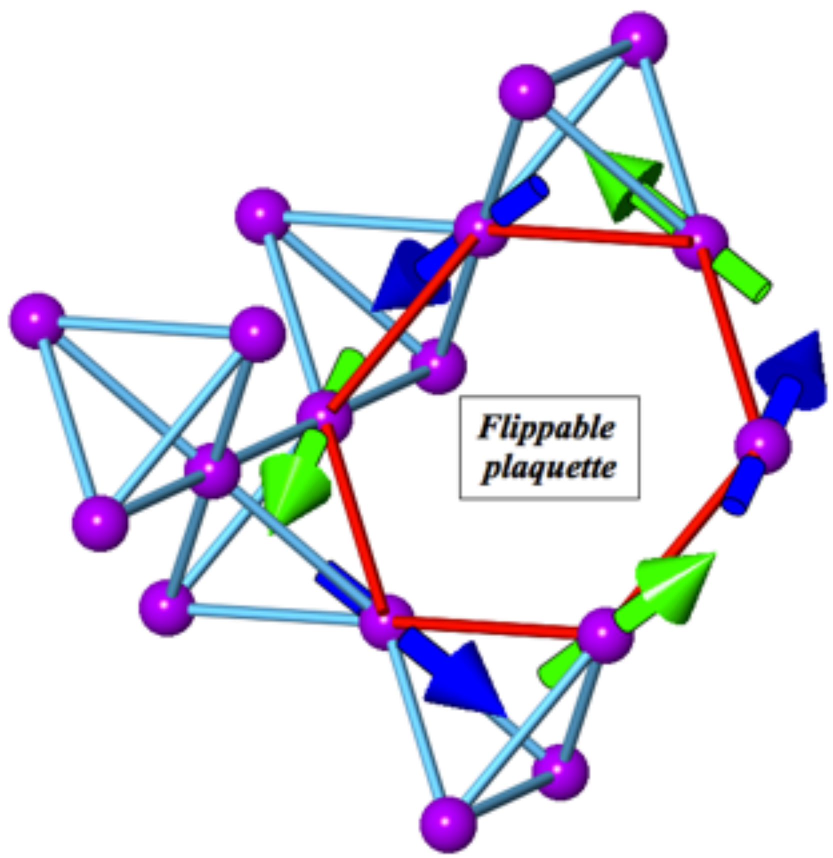


Figure from Michel Gingras' paper

$$\mathcal{H}_{\mathrm{XXZ}} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_\perp (S_i^+ S_j^- + S_i^- S_j^+),$$

3rd order degenerate perturbation
(Hermele, Fisher, Balents 2004)

$$\mathcal{H}_{\mathrm{eff}} = -\frac{12 J_\perp^3}{J_{zz}^2} \sum_{\hexagon_{\mathrm{p}}} (S_i^+ S_j^- S_k^+ S_l^- S_m^+ S_n^- + h.c.),$$

$$E_{\boldsymbol{r}\boldsymbol{r}'} \simeq S^z_{\boldsymbol{r}\boldsymbol{r}'}$$
$$e^{iA_{\boldsymbol{r}\boldsymbol{r}'}} \simeq S^{\pm}_{\boldsymbol{r}\boldsymbol{r}'}$$

Lattice gauge theory
on the dual diamond lattice

$$\mathcal{H}_{\mathrm{LGT}} = -K \sum_{\hexagon_{\mathrm{d}}} \cos(\mathrm{curl}\, A) + U \sum_{\boldsymbol{r}\boldsymbol{r}'} (E_{\boldsymbol{r}\boldsymbol{r}'} - \frac{\eta_{\boldsymbol{r}}}{2})^2$$

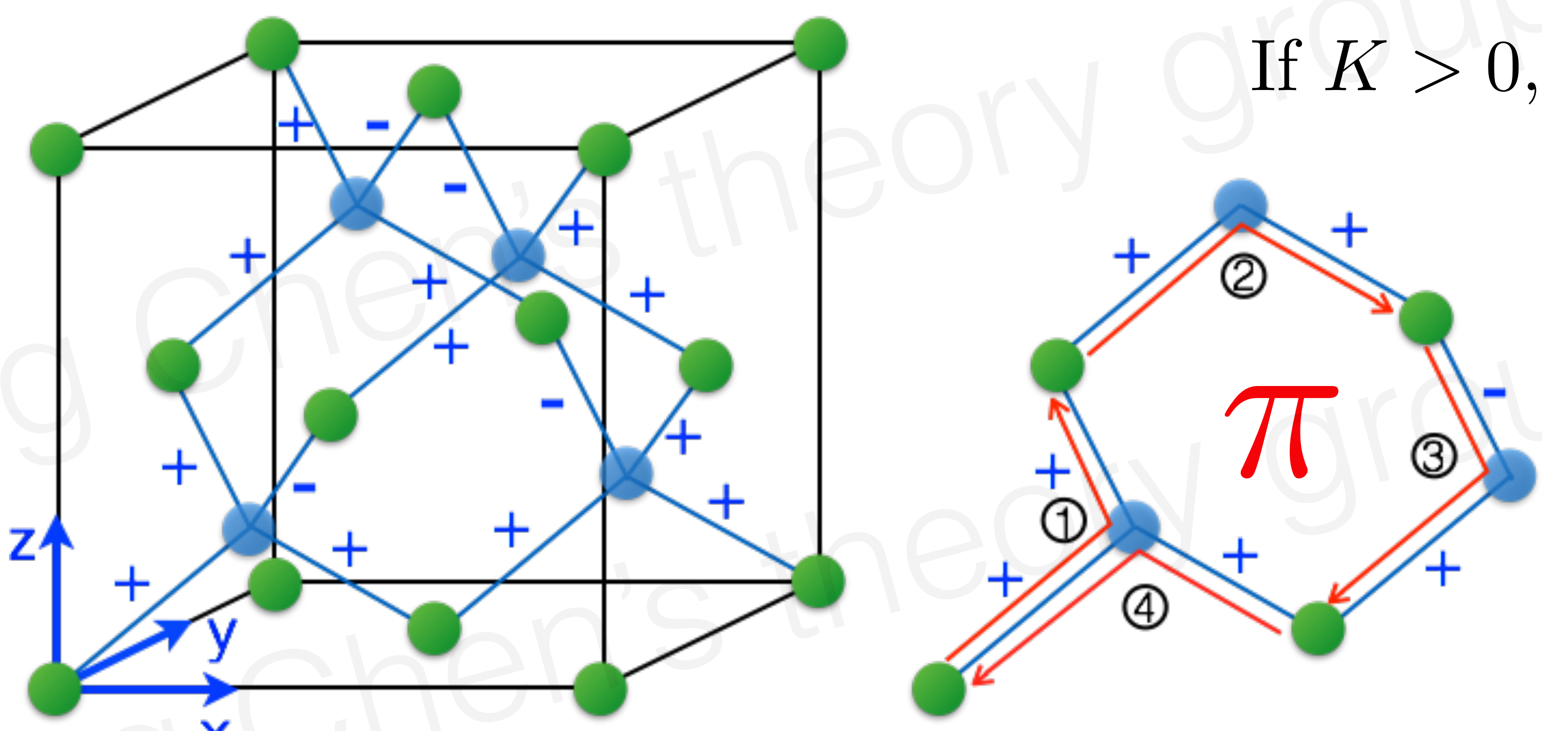
$$K = 24 J_\perp^3 / J_{zz}^2$$

Pi flux and the spinon translation

$$\mathcal{H}_{\rm LGT} = -K \sum_{\hexagon_{\rm d}} \cos(\text{curl}\, A) + U \sum_{\boldsymbol{r}\boldsymbol{r}'} (E_{\boldsymbol{r}\boldsymbol{r}'} - \frac{\eta_{\boldsymbol{r}}}{2})^2$$

$$\text{If } K < 0, \;\; curl A = \pi$$

$$\text{If } K > 0, \;\; curl A = 0$$



(a) (b)

$$T^s_\mu T^s_\nu (T^s_\mu)^{-1} (T^s_\nu)^{-1} = \pm 1$$

Aharonov-Bohm flux experienced by spinon via the 4 translation
is identical to the flux in the hexagon.

Pi flux means crystal symmetry fractionali

with definitive momentum and other quantum number

It is like symmetry breaking.

$$T^s_\mu T^s_\nu = -T^s_\nu T^s_\mu$$

2-spinon scattering state in an inelastic neutron scattering measurement

$$|a\rangle \equiv |\boldsymbol{q}_a; z_a\rangle,$$

construct another 3 equal-energy states by translating one spinon by 3 lattice vector

$$|b\rangle = T^s_1(1)|a\rangle, \quad |c\rangle = T^s_2(1)|a\rangle, \quad |d\rangle = T^s_3(1)|a\rangle$$

$$\begin{aligned}
T_1|b\rangle &= T^s_1(1)T^s_1(2)T^s_1(1)|a\rangle = +T^s_1(1)[T_1|a\rangle],\\
T_2|b\rangle &= T^s_2(1)T^s_2(2)T^s_1(1)|a\rangle = -T^s_1(1)[T_2|a\rangle],\\
T_3|b\rangle &= T^s_3(1)T^s_3(2)T^s_1(1)|a\rangle = -T^s_1(1)[T_3|a\rangle],
\end{aligned}
\quad \Longrightarrow \quad \mathbf{q}_b - \mathbf{q}_a = 2\pi(100)$$

Xiao-Gang Wen, 2001, 2002,
Andrew Essin, Michael Hermele, 2014
Gang Chen, 1704.02734

Spectral periodicity of the spinon continuum

spectral periodicity for the spinon continuum. The spectral periodicity can be reflected by the spectral intensity $\mathcal{I}(\boldsymbol{q}, E)$, the lower $\mathcal{L}(\boldsymbol{q})$ and upper excitation edge $\mathcal{U}(\boldsymbol{q})$ of the spinon continuum. For $\mathrm{U}(1)_\pi$ QSL, we have

$$
\begin{aligned}
\mathcal{I}(\boldsymbol{q}, E) &= \mathcal{I}(\boldsymbol{q} + 2\pi(100), E) = \mathcal{I}(\boldsymbol{q} + 2\pi(010), E) \\
&= \mathcal{I}(\boldsymbol{q} + 2\pi(001), E), \\
\mathcal{L}(\boldsymbol{q}) &= \mathcal{L}(\boldsymbol{q} + 2\pi(100)) = \mathcal{L}(\boldsymbol{q} + 2\pi(010)) \\
&= \mathcal{L}(\boldsymbol{q} + 2\pi(001)), \\
\mathcal{U}(\boldsymbol{q}) &= \mathcal{U}(\boldsymbol{q} + 2\pi(100)) = \mathcal{U}(\boldsymbol{q} + 2\pi(010)) \\
&= \mathcal{U}(\boldsymbol{q} + 2\pi(001)).
\end{aligned}
$$

But elastic neutron scattering will NOT see extra Bragg peak.

Xiao-Gang Wen, 2001, 2002,
Andrew Essin, Michael Hermele, 2014
Gang Chen, 1704.02734

Calculation to demonstrate the above prediction

$$\mathcal{H}_{\mathrm{XXZ}} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_{\perp}(S_i^+ S_j^- + S_i^- S_j^+),$$

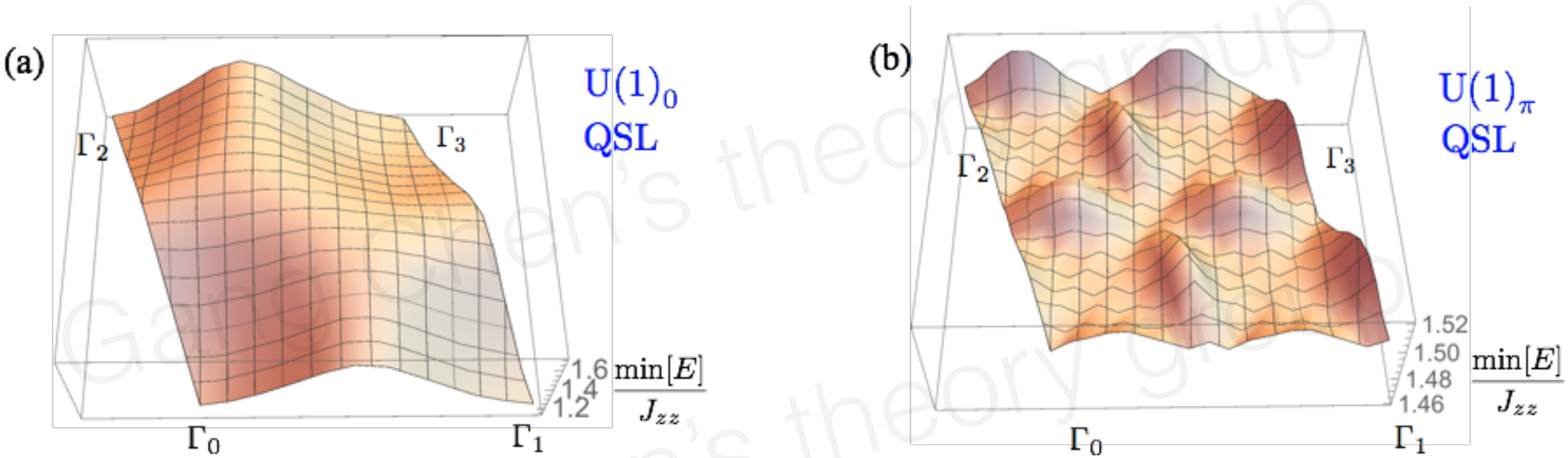


FIG. 3. (Color online.) The lower excitation edge of the spinon continuum in $\mathrm{U}(1)_0$ and $\mathrm{U}(1)_\pi$ QSLs. Here, $\Gamma_0\Gamma_1 = 2\pi(\bar{1}11), \Gamma_0\Gamma_2 = 2\pi(1\bar{1}1)$. We set $J_\perp = 0.12 J_{zz}$ for $\mathrm{U}(1)_0$ QSL in (a) and $J_\perp = -J_{zz}/3$ for $\mathrm{U}(1)_\pi$ QSL in (b).

Lower excitation edge of spinon continuum within the gauge MFT calculation

3. Symmetry fractionalization in monopoles and spectrum

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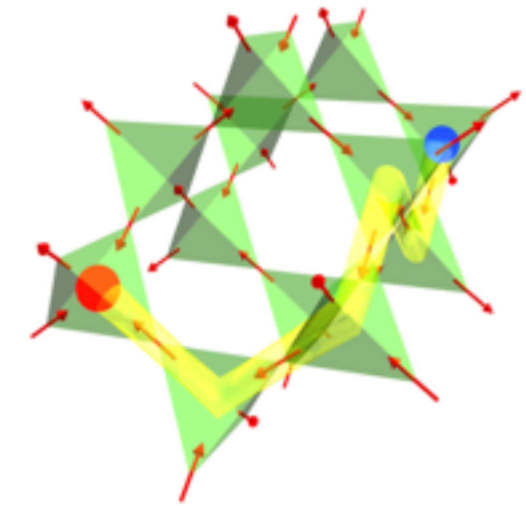
Equivalence of “notations”



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Spinon	Magnetic monopole
“Magnetic monopole”	Electric monopole
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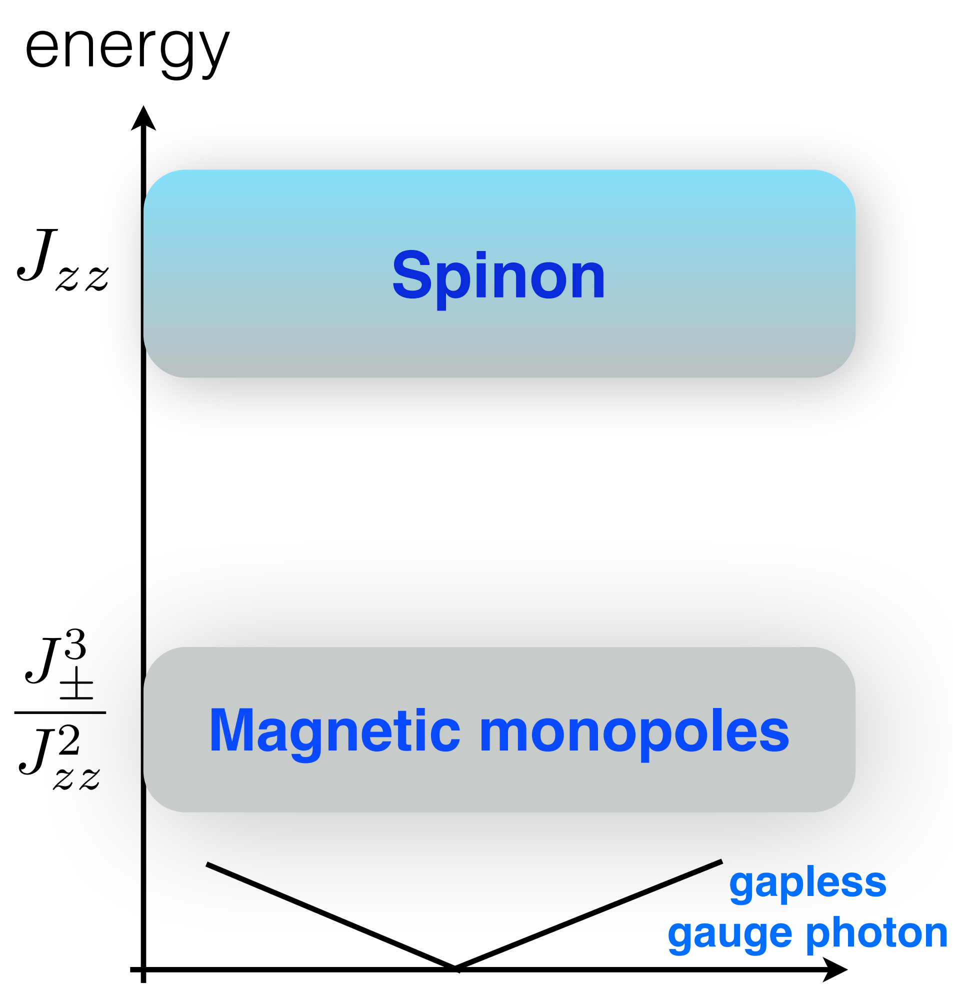
has classical analogue

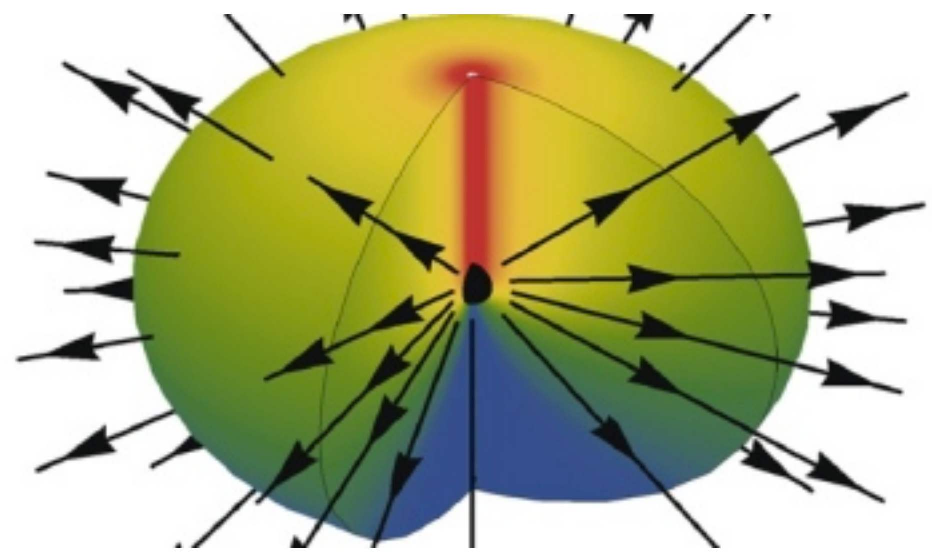
purely quantum, no classical analogue

“Magnetic monopole” is probably closer in spirit to **Dirac’s monopole (1931).**
One has to confirm that “magnetic monopole” is emergent excitation,
rather than a fictitious particle.

What piece of experimental info indicates these exotic and emergent particles?

How to observe Dirac's "magnetic monopole"?

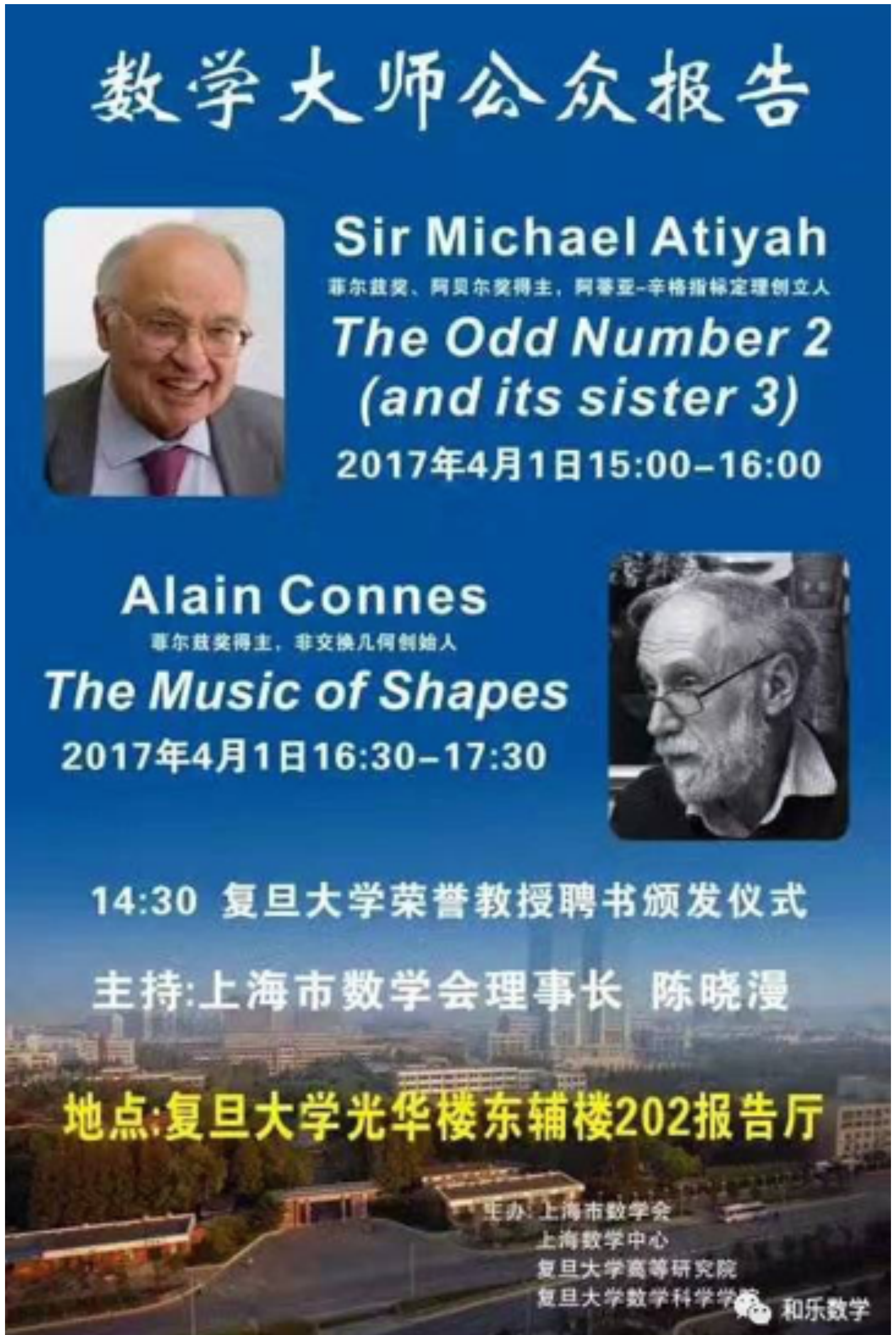




Dirac magnetic monopole

Dirac

The odd number “2”



Chenjie Wang (王晨杰)

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two-electron -> Cooper pair -> superconductor (odd/even parity)!
1/2 electron -> Majorana fermion -> topo quantum computation
spin-1/2 chain -> gapless,
spin-1 chain -> Haldane gap
topological insulator -> Z2 topological invariant
Z2 topological order, Z2 quantum spin liquid ……
fermion doubling theorem, two Weyl nodes in Weyl semimetal
single-layer graphene vs bilayer-layer graphene…

two (not 3) neutron stars emerge……

2-electron -> coope
superconductor,

Z2 topological invar
topological insulato

haldane chain,

s-wave
p-wave
odd/even parity sup

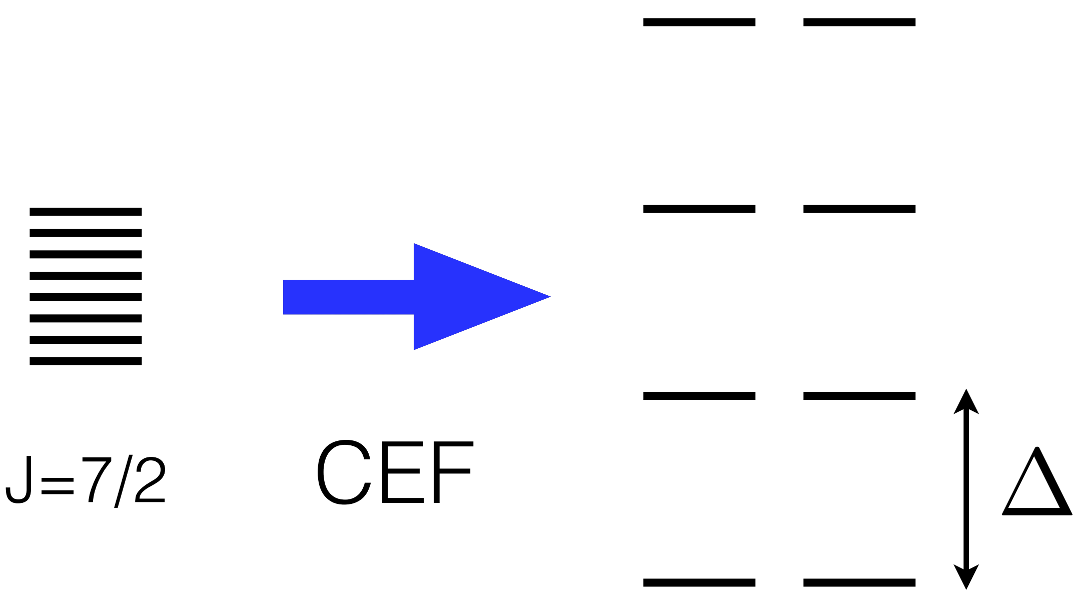
前几天 报告的 2个中
3个一起合并就是fin

Kramers vs Non-Kramers doublet

Kramers doublet: e.g. Yb ion in $Yb_2Ti_2O_7$

Yb^{3+} ion: $4f^{13}$ has J=7/2 due to SOC.

$$\mathcal{T}: S^x \rightarrow -S^x, S^y \rightarrow -S^y, S^z \rightarrow -S^z$$



(unusual example is dipole-octupole doublet in $Ce_2Sn_2O_7$ and $Nd_2Zr_2O_7$),
YP Huang, GC, Hermele, PRL 2014; YD Li, GC, PRB2016, YD Li, GC, PRB 2017

In contrast, the Tb ion in $Tb_2Ti_2O_7$, Pr ion in $Pr_2Ir_2O_7$, $Pr_2Sn_2O_7$, $Pr_2Zr_2O_7$, etc, are **non-Kramers doublets**

$$\mathcal{T}: \qquad S^{x,y} \rightarrow S^{x,y}, \quad S^z \rightarrow -S^z.$$

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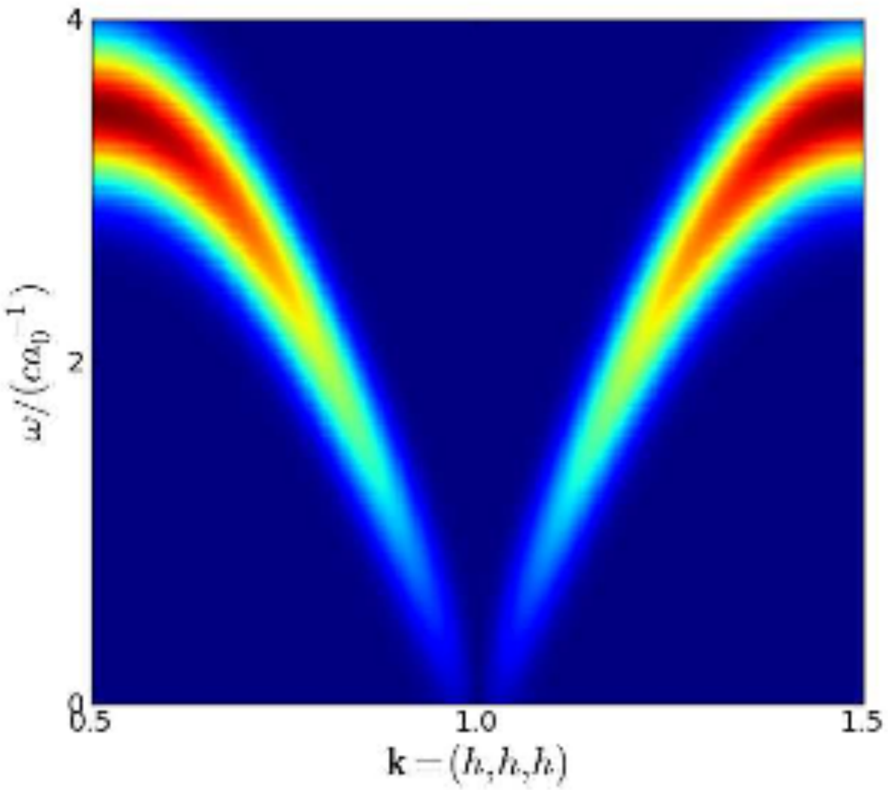
Emergent light: U(1) photon



$$I(\omega) \sim \omega$$

emergent U(1) photon in U(1) QSL

Hermele etc 2004
N Shannon etc 2012,
L Savary etc 2012

$$S_z \sim E \text{ (emergent electric field)}$$

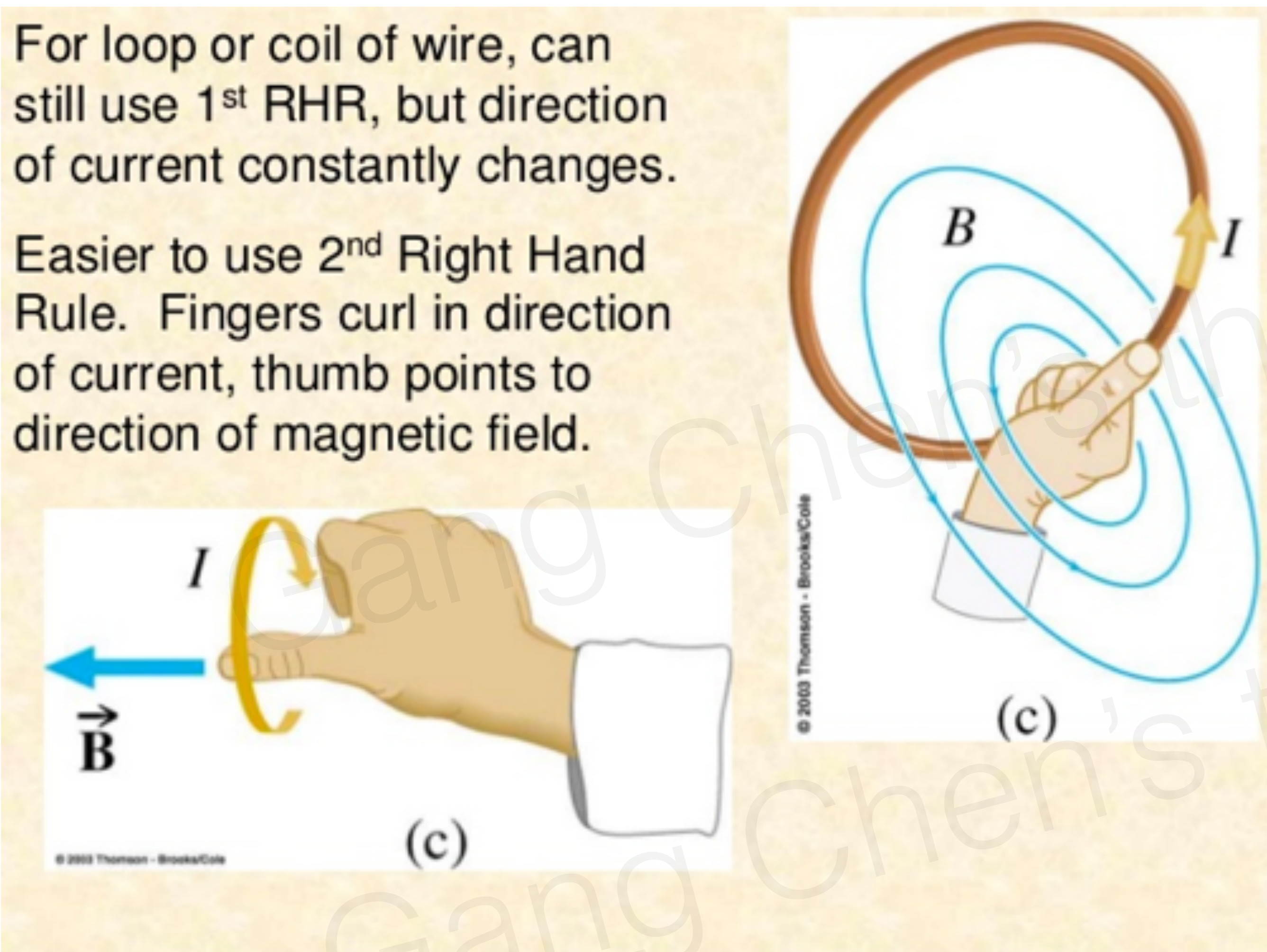
Low energy theory

$$\mathrm{Im}[E^{\alpha}_{-\mathbf{k},-\omega} E^{\beta}_{\mathbf{k},\omega}] \propto [\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{\mathbf{k}^2}]\, \omega\, \delta(\omega - v|\mathbf{k}|),$$

The well-known result of the photon modes in the INS measurement was obtained by considering the low-energy field theory that describes the long-distance quantum fluctuation within the spin ice manifold. The actual spin dynamics, that is captured by the S^z correlation in the INS measurement, operates in a broad energy scale up to the exchange energy and certainly contains more information than just the photon mode from the low-energy Maxwell field theory. What is the other informa-

Gang Chen, arXiv:1706.04333

Electromagnetic duality

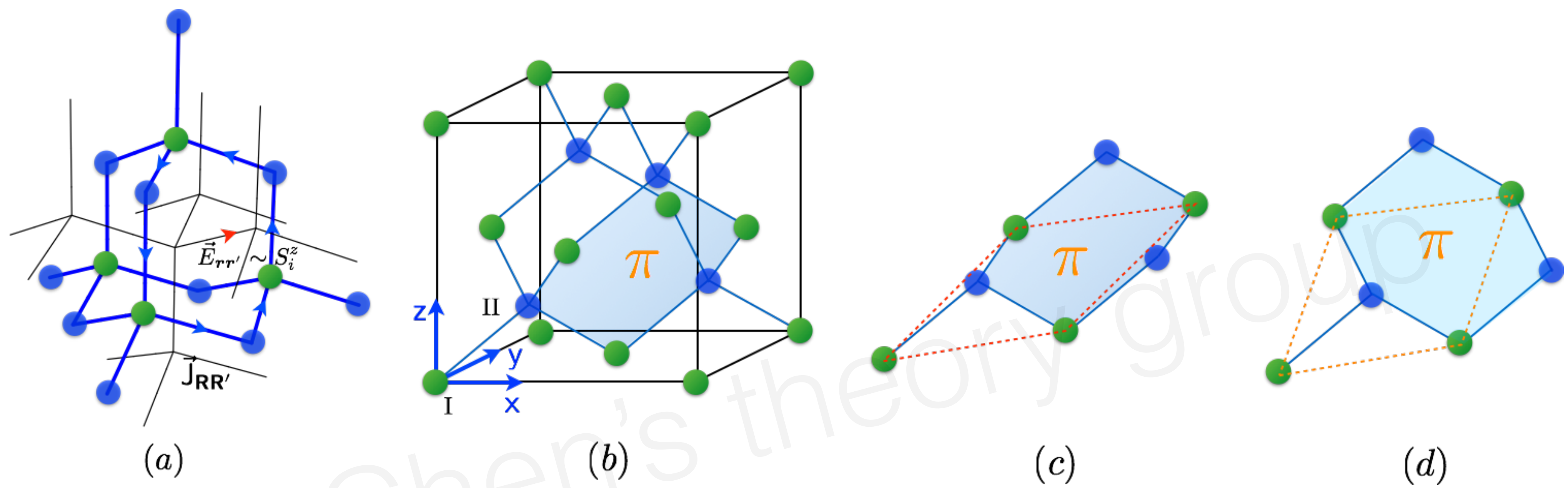


Duality

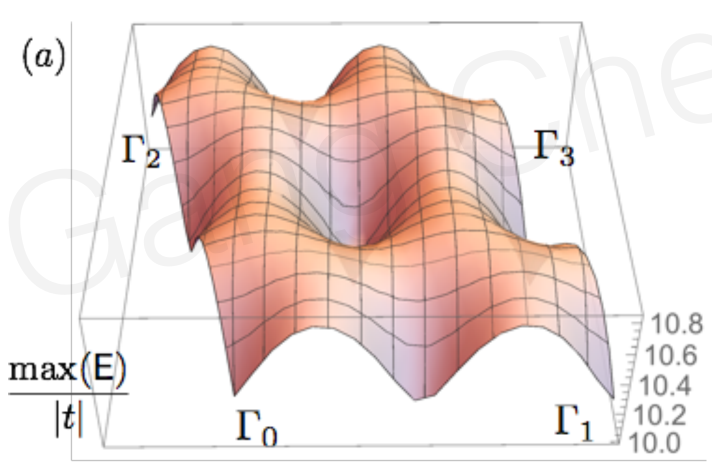
Electric loop current -> Magnetic field
Magnetic loop current -> Electric field

$$S_z \sim E \text{ (emergent electric field)}$$

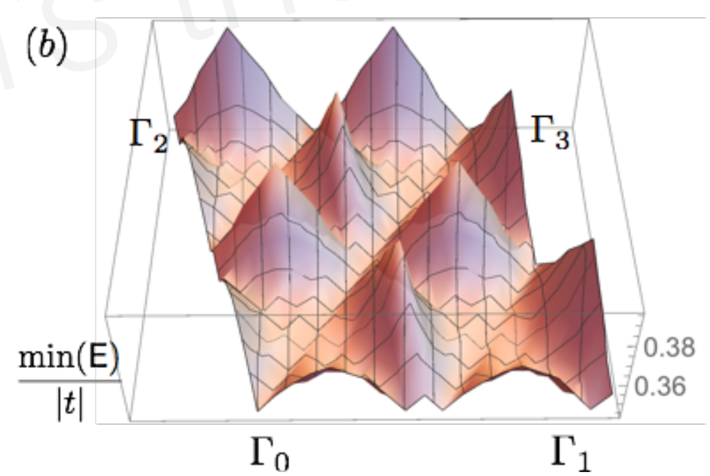
Sz correlation = monopole loop current correlation



$$H_{\mathrm{dual}} = -t \sum_{\langle \mathbf{RR'} \rangle} e^{-i2\pi\alpha_{\mathbf{RR'}}} \Phi^\dagger_{\mathbf{R}} \Phi_{\mathbf{R'}} - \mu \sum_{\mathbf{R}} \Phi^\dagger_{\mathbf{R}} \Phi_{\mathbf{R}} + \frac{U}{2} \sum_{\hexagon^*} (curl\alpha - \frac{\eta_{\mathbf{r}}}{2})^2 - K \sum_{\langle \mathbf{RR'} \rangle} \cos B_{\mathbf{RR'}} + \cdots$$



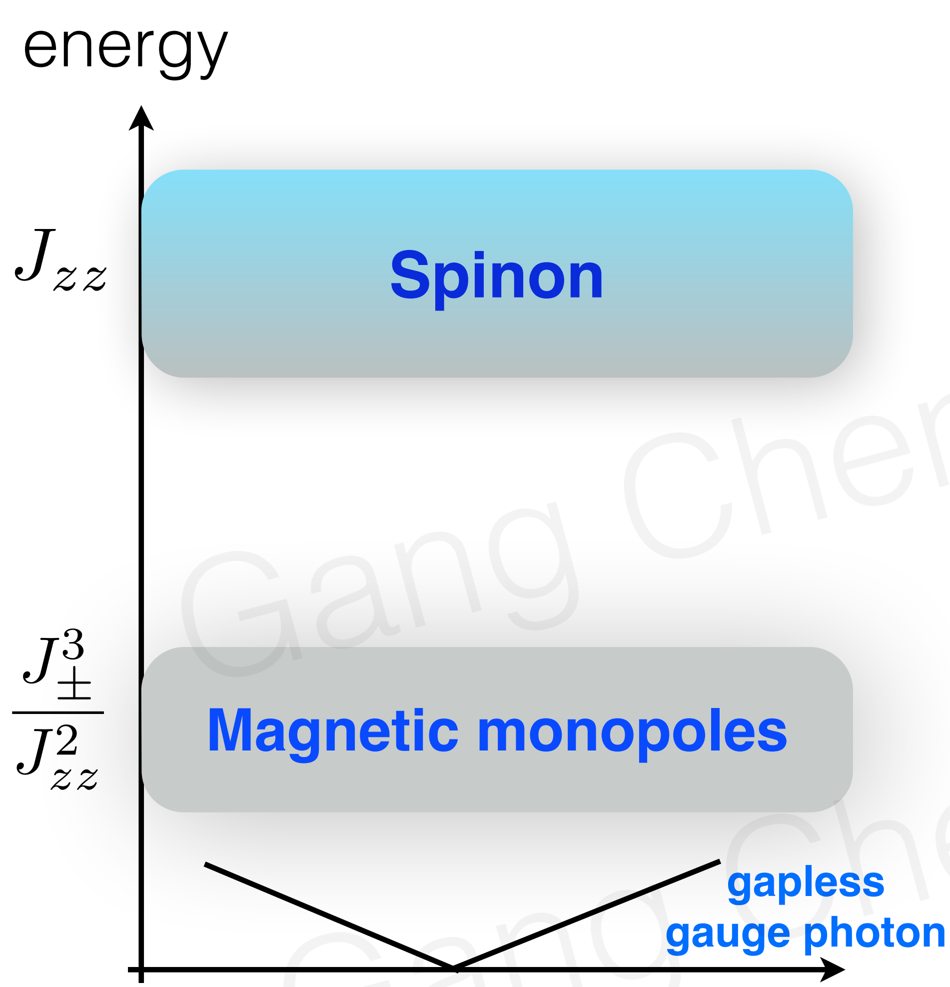
the upper edge



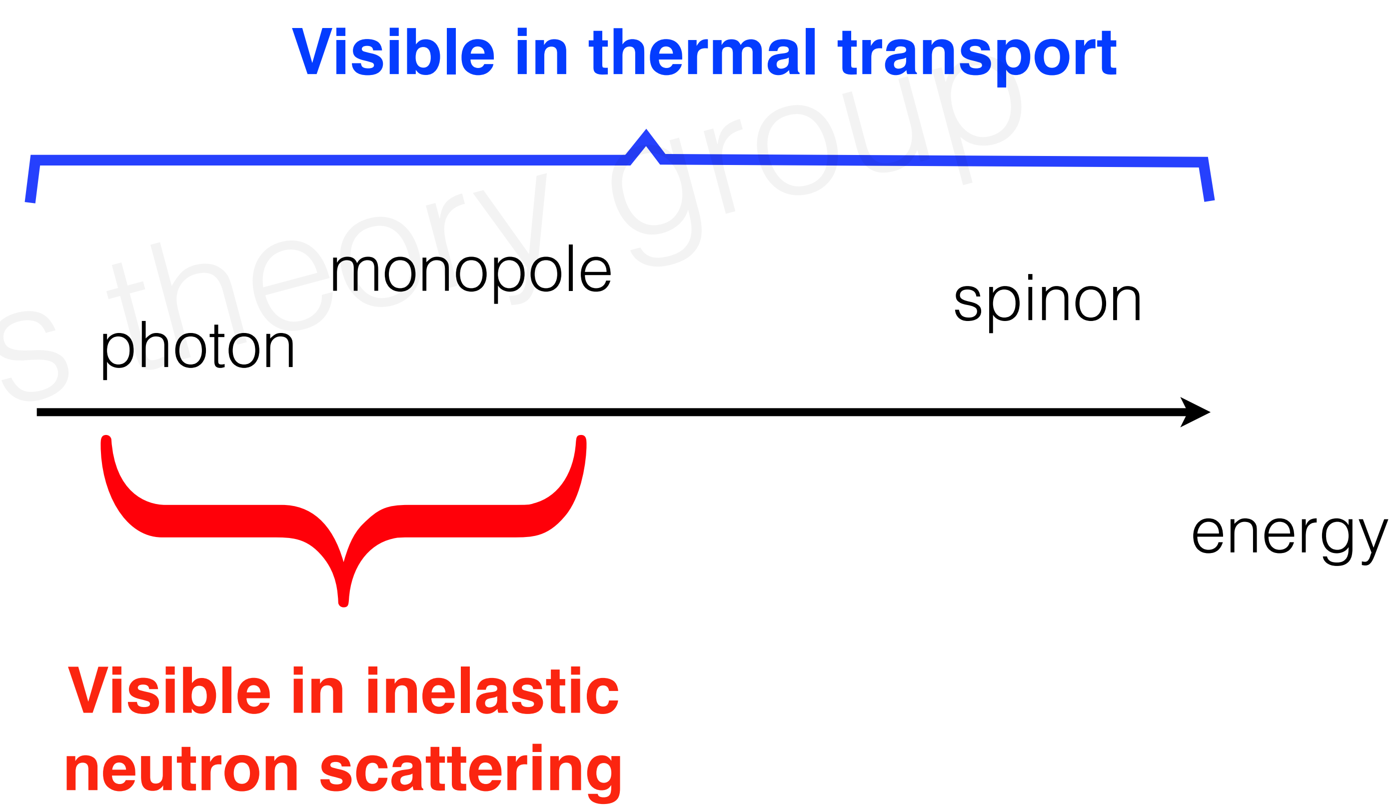
the lower edge

Monopole always experiences Pi flux

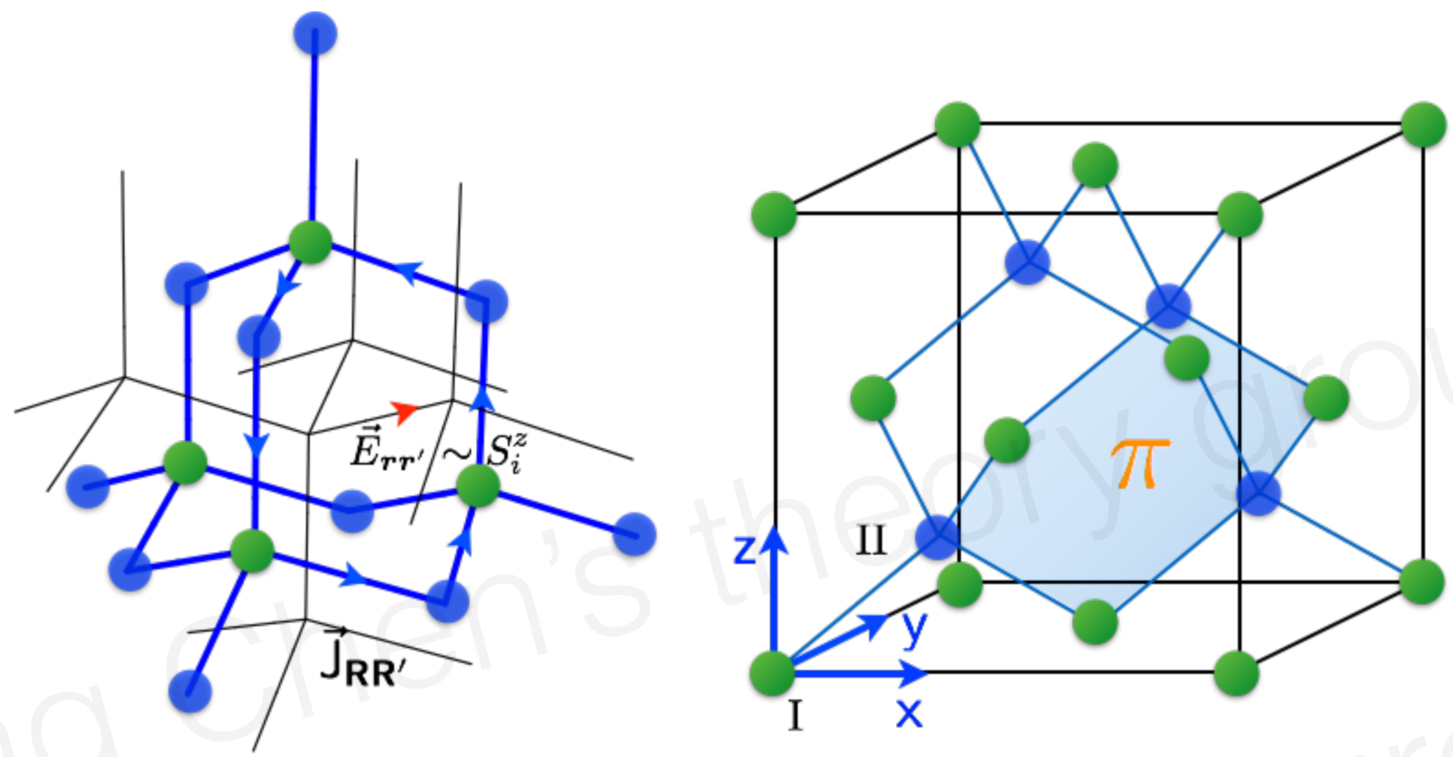
Suggestion 1: combine thermal transport with inelastic neutron



For **non-Kramers doublets** such as Pr ion in $Pr_2Zr_2O_7$ and Tb ion in $Tb_2Ti_2O_7$



Suggestion 2: effect of the external magnetic field



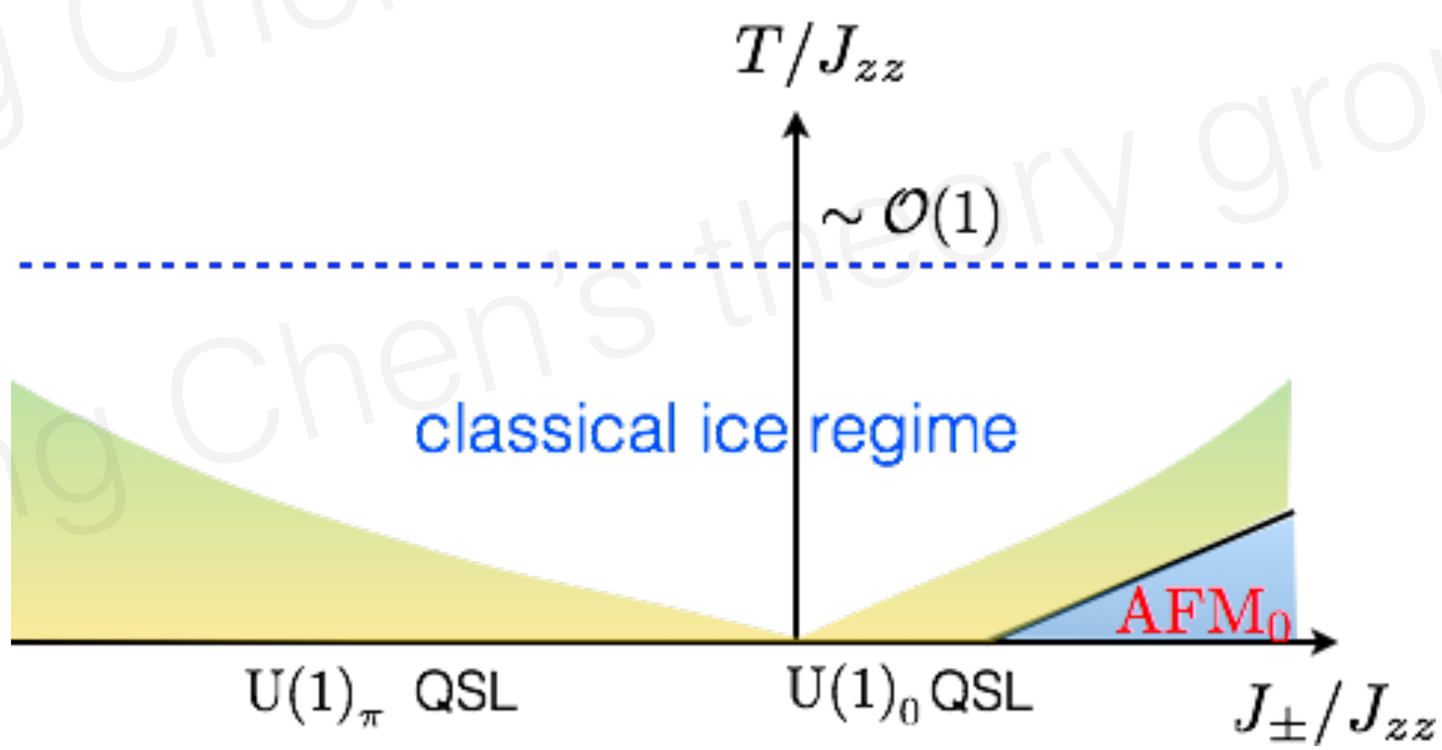
(a) (b)

$$H_{Zeeman} = \vec{B} \cdot \sum_i S_i^z \hat{z}_i$$

The weak magnetic field polarizes Sz slightly, and thus modifies the background electric field distribution. This further modulates monopole band structure, creating "**Hofstadter**" monopole band, which may be detectable in inelastic neutron.

Classification

Properties	$U(1)_{0,\pi}$ QSL	$U(1)_{\pi,\pi}$ QSL
Spinon flux	0	π
“Monopole” flux	π	π
Spinon continuum	Not enhanced	Enhanced
‘Mmonopole” continuum	Enhanced	Enhanced

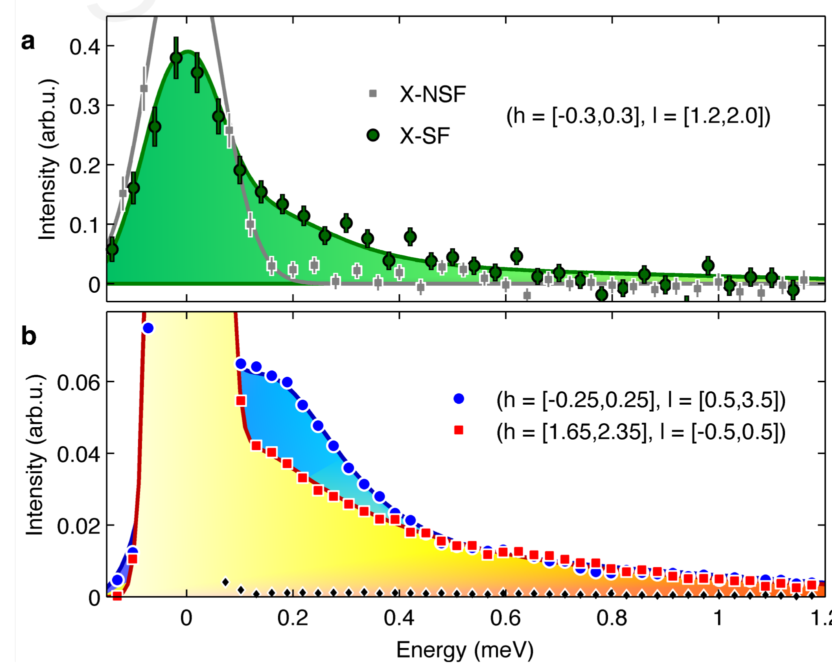


Summary

1. We point out the existence of "magnetic monopole continuum" in the U(1) quantum spin liquid, and monopole is purely **quantum origin**.

2. We further point out that the "magnetic monopole" always experiences a Pi flux, and thus supports enhanced spectral periodicity with **folded Brillouin zone**.

In fact, continuum has been observed in $Pr_2Hf_2O_7$ (R. Sibille, et al, arXiv 1706.03604 Nature Physics).



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