Symmetry enriched U(1) topological orders: symmetry fractionalization of magnetic monopoles

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Outline

1. U(1) quantum spin liquid in 3D: spinon, monopole, photon, and fractionalization

2. Symmetry fractionalization in spinons and spectrum

3. Symmetry fractionalization in monopoles and spectrum
References

“Magnetic Monopole” condensation transition out of spin ice U(1) QSL: application to Pr2Ir2O7
Gang Chen, PRB 94, 205107, (2016)

The spectral periodicity of spinon continuum in quantum spin ice
Gang Chen, PRB 96, 085136, (2017)

Dirac’s “magnetic monopoles” in pyrochlore ice U(1) spin liquids: spectrum and classification
Gang Chen, PRB 96, 195127, (2017)
Fractionalization in FQHE: shot-noise measurement

Etien et al, PRL 79, 2526 (1997)
also see Heiblum et al, Nature (1997)
FQHE is arguably the only existing topological order so far.

Chiral (Abelian) topological order

Fractionalization: fractionalized & deconfined excitation

Chern-Simon gauge structure

with charge U(1) symmetry:
charge conservation

Fractionalized charge excitation

Symmetry makes topological order more visible in experiments.
Pyrochlore spin ice

Over years, there is a lot of activity in spin ice systems. Spin ice is realized in rare earth pyrochlore systems, where the rare earth ions form pyrochlore lattices and host the Ising spins. Because of the crystal field effect, the Ising spins point either into or out of the center of the tetrahedron. The interaction between the Ising spins is AFM, which favors 2 spins in and 2 spins out of the tetrahedra. This is the 2-in-2-out spin ice rule.

Because of the analog relation with H positions in water ice, each O has 4 H near it, 2 close, 2 further.

\[
H = J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z + \text{dipolar}
\]

Castelnovo, Gingras, Moessner, Sondhi, Schiffer, Shannon, Moon, Kim, Savary, Balents, ...
Classical spin ice

Pauling entropy in spin ice, Ramirez, etc, Science 1999

\( \text{Dy}_2\text{Ti}_2\text{O}_7 \)

2-in 2-out spin ice rule

2-in 2-out water ice rule
Toy model and U(1) quantum spin liquid

\[
H = J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z - J_\pm \sum_{\langle i,j \rangle} (S_i^+ S_j^- + S_i^- S_j^+) + \ldots \quad \text{Hermelé, Fisher, Balents, PRB 2004}
\]

- Pretty much one can add any term to create quantum tunneling, as long as it is not too large to induce magnetic order, the ground state is a U(1) quantum spin liquid!

1. But classical spin ice is purely classical and is not a new phase of matter. It is smoothly connected to high temperature paramagnetic phase.
2. In contrast, quantum spin ice is a new quantum phase of matter.
Ring exchange and a unitary transformation

\[ \mathcal{H}_{\text{eff}} = \left( \frac{J_\perp}{J_z} \right) \left( \frac{J_\perp}{J_z} - 1 \right) N_t \]

\[ + J_{\text{ring}} \sum_\Box (S_1^+ S_2^- S_3^+ S_4^- S_5^+ S_6^- + \text{H.c.}), \]

site we can make the transformation \( S^z \rightarrow S^z \) and \( S^{\pm} \rightarrow -S^{\pm} \) by making a \( \pi \) rotation about the \( z \) axis in spin space. One transformation with the desired effect, consisting of \( \pi \) rotations on a pattern of sites, is

\[ S^z_i \rightarrow S^z_i, \]  
\[ S^{\pm}_{R_i} \rightarrow \exp(i \mathbf{Q}_i \cdot \mathbf{R}) S^{\pm}_{R_i}, \]

where \( \mathbf{Q}_0 = \mathbf{Q}_1 = (\mathbf{b}_1 + \mathbf{b}_2)/2 \) and \( \mathbf{Q}_2 = \mathbf{Q}_3 = 0 \).

After this transformation the Hamiltonian takes the form

\[ \mathcal{H}_p = -J_{\text{ring}} \sum_\Box (S_1^+ S_2^- S_3^+ S_4^- S_5^+ S_6^- + \text{H.c.}), \]  

link on diamond lattice

Hermelé, Fisher, Balents, PRB 2004
Theoretical expectation

- **U(1) Spin Liquid**
- **Ising order**
- **Easy-axis Limit**
- **Soluble Point**
- **Transverse spin order**

**U(1)_pi QSL**

\[ J_\perp = 0 \]

Related by unitary transformation (Hermele, Fisher, Balents 2004)
Excitations in the U(1) QSL

- No LRO, no symmetry breaking, **cannot** be understood in Landau’s paradigm!
- The right description is in terms of fractionalization and emergent gauge structure.

Figs from Moessner&Schieller, 2009
### Equivalence of “notations”

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“Magnetic monopole” is probably closer in spirit to **Dirac’s monopole (1931)**. One has to confirm that “magnetic monopole” is emergent excitation, rather than a fictitious particle.

What piece of experimental info indicates these exotic and emergent particles?
Important question

What are sharp physical observables to confirm U(1) QSL?

\[ I(\omega) \sim \omega \]

Nic Shannon, etc 2012, Savary, Balents, 2012

low energy scale suppressed intensity

heat capacity (Savary & Balents):
1000 times larger than phonon!
Our answer:

the spectral periodicity of the spinon/monopole continuum

Enlarged periodicity is like the fractional charge in FQHE.

Gang Chen, PRB 96, 085136, (2017)
PRB 96, 195127, (2017)
2. Symmetry fractionalization in spinons and spectrum
Realistic models

• Usual Kramers’ doublet and non-Kramers’ doublet

\[
H = \sum_{\langle ij \rangle} \{ J_{zz} S_i^z S_j^z - J_\pm (S_i^+ S_j^- + S_i^- S_j^+) \\
+ J_\pm (\gamma_{ij} S_i^+ S_j^+ + \gamma^*_{ij} S_i^- S_j^-) \\
+ J_{z\pm} [S_i^z (\zeta_{ij} S_j^y + \zeta^*_{ij} S_j^z) + i \leftrightarrow j] \}.
\]

S. Onoda, etc, 2009, 2010
SB Lee, Onoda, Balents, 2012

• Dipole-octupole doublet

\[
H = \sum_{\langle ij \rangle} J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_z S_i^z S_j^z \\
+ J_{xz} (S_i^x S_j^z + S_i^z S_j^x).
\]

Y-P Huang, Gang Chen, M Hermele, PRL 2014
Yao-Dong Li, Gang Chen, PRB 2017
Use the XXZ model to illustrate the universal physics

\[ \mathcal{H}_{XXZ} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_\perp (S_i^+ S_j^- + S_i^- S_j^+) \]

Savary, Balents, 2012
Kato, Onoda, 2015
Frustrated regime: early theoretical study

\begin{center}
\begin{tikzpicture}
\node (upi) [circle, fill] \text{U(1)$_{pi}$ QSL};
\node (u0) [right of=upi, circle, fill] \text{U(1)$_{0}$ QSL};
\node (tso) [right of=u0, circle, fill] \text{Transverse spin order};
\node (jperp) [above of=u0, node distance=2cm] \text{\(J_{\perp} = 0\)};
\node (jz) [above of=tso, node distance=2cm] \text{\(J_{z}\)};
\draw [->] (upi) -- (u0);
\draw [->] (u0) -- (tso);
\draw [->] (jperp) -- (tso);
\draw [->] (jz) -- (tso);
\end{tikzpicture}
\end{center}

Related by unitary transformation (Hermele, Fisher, Balents 2004)


\textbf{Generic quantum spin ice}

SungBin Lee, Shigeki Onoda, and Leon Balents

One. We also consider the case of frustrated XY exchange, and find that it favors a \(\pi\)-flux QSL, with an emergent line degeneracy of low-energy spinon excitations. This feature greatly enhances the stability of the QSL with respect to classical ordering.
Besides the quantitative differences, are there sharp distinctions between the U(1)$_{\text{pi}}$ QSL on the left and the U(1)$_0$ QSL on the right?

Related by unitary transformation (Hermele, Fisher, Balents 2004)
The e fields as tertiary hexagon ("model is given in Fig.

TABLE I. Physical properties of the U(1) QSL. This model is defined as

\( \mathcal{H}_{\text{XXZ}} = \sum_{\langle ij \rangle} J_{zz} S^z_i S^z_j - J_{\perp} (S^+_i S^-_j + S^-_i S^+_j) \),

3rd order degenerate perturbation (Hermele, Fisher, Balents 2004)

\( \mathcal{H}_{\text{eff}} = -\frac{12 J^3}{J_{zz}^2} \sum_{\bigodot \Gamma_p} \left( S^+_i S^-_j S^+_k S^-_l S^+_m S^-_n + h.c. \right) \),

\( E_{rr'} \simeq S^z_{rr'} \)

\( e^{iA_{rr'}} \simeq S^\pm_{rr'} \)

\( \mathcal{H}_{\text{LGT}} = -K \sum_{\bigodot \delta} \cos(\text{curl} \ A) + U \sum_{rr'} (E_{rr'} - \frac{\eta_{rr'}}{2})^2 \)

\( K = 24 J^3_{\perp} / J_{zz}^2 \)

Lattice gauge theory on the dual diamond lattice
Pi flux and the spinon translation:

\[ \mathcal{H}_{\text{LGT}} = -K \sum_{\mathcal{O}_d} \cos(\text{curl } A) + U \sum_{rr'} (E_{rr'} - \frac{\eta r}{2})^2 \]

If \( K < 0 \), \( \text{curl } A = \pi \)

If \( K > 0 \), \( \text{curl } A = 0 \)

\[ T_\mu^s T_\nu^s (T_\mu^s)^{-1} (T_\nu^s)^{-1} = \pm 1 \]

Aharonov-Bohm flux experienced by spinon via the 4 translation is identical to the flux in the hexagon.
Pi flux means crystal symmetry fractionalization:

\[ T_\mu^s T_\nu^s = -T_\nu^s T_\mu^s \]

2-spinon scattering state in an inelastic neutron scattering measurement

|a\rangle \equiv |q_a; z_a\rangle,

construct another 3 equal-energy states by translating one spinon by 3 lattice vector

|b\rangle = T_1^s(1)|a\rangle,
|c\rangle = T_2^s(1)|a\rangle,
|d\rangle = T_3^s(1)|a\rangle

\begin{align*}
T_1|b\rangle &= T_1^s(1)T_1^s(2)T_1^s(1)|a\rangle = +T_1^s(1)[T_1|a\rangle], \\
T_2|b\rangle &= T_2^s(1)T_2^s(2)T_1^s(1)|a\rangle = -T_1^s(1)[T_2|a\rangle], \\
T_3|b\rangle &= T_3^s(1)T_3^s(2)T_1^s(1)|a\rangle = -T_1^s(1)[T_3|a\rangle],
\end{align*}

\[ q_b - q_a = 2\pi(100) \]

Xiao-Gang Wen, 2001, 2002,
Andrew Essin, Michael Hermele, 2014
Gang Chen, 1704.02734
Spectral periodicity of the spinon continuum

spectral periodicity for the spinon continuum. The spectral periodicity can be reflected by the spectral intensity $\mathcal{I}(q, E)$, the lower $\mathcal{L}(q)$ and upper excitation edge $\mathcal{U}(q)$ of the spinon continuum. For U$(1)_\pi$ QSL, we have

\[
\mathcal{I}(q, E) = \mathcal{I}(q + 2\pi(100), E) = \mathcal{I}(q + 2\pi(010), E) \\
= \mathcal{I}(q + 2\pi(001), E), \\
\mathcal{L}(q) = \mathcal{L}(q + 2\pi(100)) = \mathcal{L}(q + 2\pi(010)) \\
= \mathcal{L}(q + 2\pi(001)), \\
\mathcal{U}(q) = \mathcal{U}(q + 2\pi(100)) = \mathcal{U}(q + 2\pi(010)) \\
= \mathcal{U}(q + 2\pi(001)).
\]

But elastic neutron scattering will NOT see extra Bragg peak.

Xiao-Gang Wen, 2001, 2002,
Andrew Essin, Michael Hermele, 2014
Gang Chen, 1704.02734
Calculation to demonstrate the above prediction

\[ \mathcal{H}_{\text{XXZ}} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_\perp (S_i^+ S_j^- + S_i^- S_j^+) , \]

FIG. 3. (Color online.) The lower excitation edge of the spinon continuum in U(1)$_{0}$ and U(1)$_{\pi}$ QSLs. Here, $\Gamma_0 \Gamma_1 = 2\pi(\bar{1}11)$, $\Gamma_0 \Gamma_2 = 2\pi(1\bar{1}1)$. We set $J_\perp = 0.12J_{zz}$ for U(1)$_{0}$ QSL in (a) and $J_\perp = -J_{zz}/3$ for U(1)$_{\pi}$ QSL in (b).

Lower excitation edge of spinon continuum within the gauge MFT calculation
3. Symmetry fractionalization in monopoles and spectrum
Equivalence of “notations”

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“Magnetic monopole” is probably closer in spirit to Dirac’s monopole (1931). One has to confirm that “magnetic monopole” is emergent excitation, rather than a fictitious particle.

What piece of experimental info indicates these exotic and emergent particles?
How to observe Dirac’s “magnetic monopole”?

Spinon

Magnetic monopoles

gapless gauge photon

Dirac magnetic monopole

Dirac
The odd number “2”

two-electron -> Cooper pair -> superconductor (odd/even parity)!
1/2 electron -> Majorana fermion -> topo quantum computation
spin-1/2 chain -> gapless,
spin-1 chain -> Haldane gap
topological insulator -> Z2 topological invariant
Z2 topological order, Z2 quantum spin liquid ……
fermion doubling theorem, two Weyl nodes in Weyl semimetal
single-layer graphene vs bilayer-layer graphene…

two (not 3) neutron stars emerge……
Kramers vs Non-Kramers doublet

Kramers doublet: e.g. Yb ion in Yb$_2$Ti$_2$O$_7$

Yb$^{3+}$ ion: 4f$^{13}$ has J=7/2 due to SOC.

\[ \mathcal{T} : S^x \rightarrow -S^x, \quad S^y \rightarrow -S^y, \quad S^z \rightarrow -S^z \]

(usual example is dipole-octupole doublet in Ce$_2$Sn$_2$O$_7$ and Nd$_2$Zr$_2$O$_7$), YP Huang, GC, Hermele, PRL 2014; YD Li, GC, PRB 2016, YD Li, GC, PRB 2017

In contrast, the Tb ion in Tb$_2$Ti$_2$O$_7$, Pr ion in Pr$_2$Ir$_2$O$_7$, Pr$_2$Sn$_2$O$_7$, Pr$_2$Zr$_2$O$_7$, etc, are **non-Kramers doublets**

\[ \mathcal{T} : S_{x,y} \rightarrow S_{x,y}, \quad S^z \rightarrow -S^z. \]
Emergent light: U(1) photon

\[ S_z \sim E \text{ (emergent electric field)} \]

Low energy theory

\[ \text{Im}[E_{-k, -\omega}^\alpha E_k^\beta] \propto [\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{k^2}] \omega \delta(\omega - v|\mathbf{k}|), \]

The well-known result of the photon modes in the INS measurement was obtained by considering the low-energy field theory that describes the long-distance quantum fluctuation within the spin ice manifold. The actual spin dynamics, that is captured by the \( S_z \) correlation in the INS measurement, operates in a broad energy scale up to the exchange energy and certainly contains more information than just the photon mode from the low-energy Maxwell field theory. What is the other informa-

Electromagnetic duality

Duality

Electric loop current $\rightarrow$ Magnetic field
Magnetic loop current $\rightarrow$ Electric field

$S_z \sim E$ (emergent electric field)
Sz correlation = monopole loop current correlation

\[ H_{\text{dual}} = -t \sum_{\langle RR' \rangle} e^{-i2\pi \alpha_{RR'}} \Phi_{R}^\dagger \Phi_{R'} - \mu \sum_{R} \Phi_{R}^\dagger \Phi_{R} + \frac{U}{2} \sum_{O^*} (\text{curl} \alpha - \frac{\eta_r}{2})^2 - K \sum_{\langle RR' \rangle} \cos B_{RR'} + \cdots \]

Monopole always experiences Pi flux
Suggestion 1: combine thermal transport with inelastic neutron scattering

For non-Kramers doublets such as Pr ion in Pr$_2$Zr$_2$O$_7$ and Tb ion in Tb$_2$Ti$_2$O$_7$

Visible in thermal transport

Visible in inelastic neutron scattering
Suggestion 2: effect of the external magnetic field

The weak magnetic field polarizes $S_z$ slightly, and thus modifies the background electric field distribution. This further modulates monopole band structure, creating “Hofstadter” monopole band, which may be detectable in inelastic neutron.

$$H_{Zeeman} = \vec{B} \cdot \sum_i S_i^z \hat{z}_i$$
Classification

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<tr>
<td>Spinon flux</td>
<td>0</td>
<td>$\pi$</td>
</tr>
<tr>
<td>“Monopole” flux</td>
<td>$\pi$</td>
<td>$\pi$</td>
</tr>
<tr>
<td>Spinon continuum</td>
<td>Not enhanced</td>
<td>Enhanced</td>
</tr>
<tr>
<td>‘Mmonopole’ continuum</td>
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![Graph showing the classification of U(1) QSLs](image)
Summary

1. We point out the existence of “magnetic monopole continuum” in the U(1) quantum spin liquid, and monopole is purely quantum origin.

2. We further point out that the “magnetic monopole” always experiences a Pi flux, and thus supports enhanced spectral periodicity with folded Brillouin zone.

In fact, continuum has been observed in Pr$_2$Hf$_2$O$_7$ (R. Sibille, et al, arXiv 1706.03604 Nature Physics).