Fractionalization in spin liquids

Gang Chen
Fudan University

Works of 2017 in my group

1. Quantum paramagnet and frustrated quantum criticality in a **spin-one diamond lattice antiferromagnet**
   GC, Phys. Rev. B (R) 96, 020412 (2017)

2. **Symmetry enriched U (1) topological orders**
   Y-D Li, **GC** , Phys. Rev. B (R) 95, 041106 (2017)

3. **Detecting spin fractionalization** in a spinon Fermi surface spin liquid
   Y-D Li, **GC**, Phys. Rev. B 96, 075105 (2017)

4. The spinon Fermi surface U(1) spin liquid in a **spin-orbit-coupled triangular lattice** Mott insulator YbMgGaO4
   Y-D Li, Y-M Lu, **GC** Phys. Rev. B 96, 054445 (2017)

5. **Kitaev materials** beyond iridates
   F-Y Li, Y-D Li, Y Yu, A Paramekanti, **GC**, Phys. Rev. B 95, 085132 (2017)

6. Tripartite entangled plaquette state in a **cluster magnet**
   J Carrasquilla, **GC**, RG Melko, Phys. Rev. B 96, 054405 (2017)

7. **Fractionalized excitations** in the partially magnetized spin liquid candidate YbMgGaO4
   Y Shen, Y-D Li, …, **GC, J Zhao** arXiv preprint 1708.06655

8. Emergent orbitals in the **cluster Mott insulator** on a breathing Kagome lattice
   GC, PA Lee, arXiv preprint 1709.09789

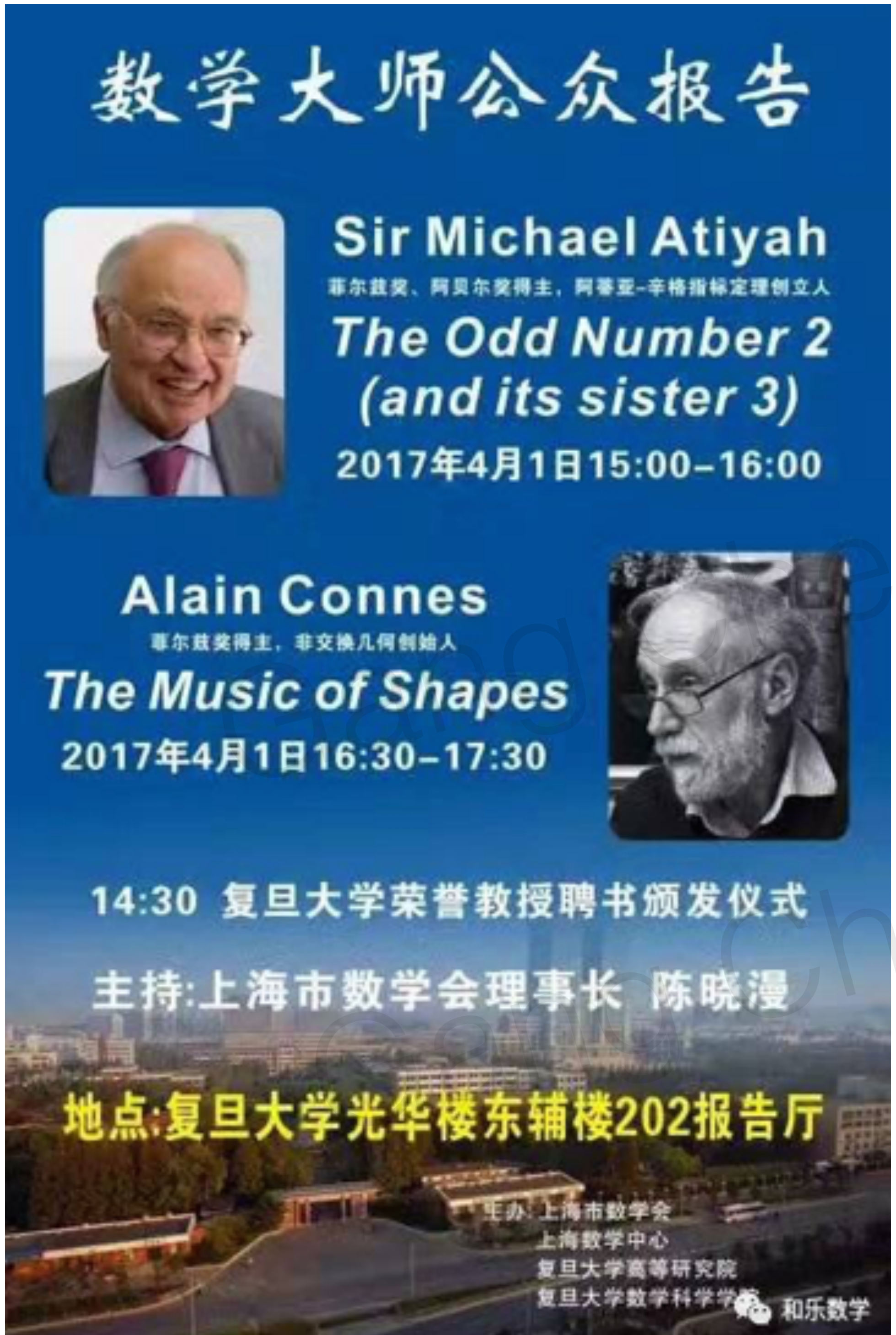
Works of 2017 in my group

9. Spectral periodicity of the spinon continuum in quantum spin ice
Gang Chen
Phys. Rev. B 96, 085136 (2017)

10. What does inelastic neutron scattering measure in quantum spin ices?
Gang Chen
arXiv preprint 1706.04333

Recent interest has shifted to **metallic systems** and **non-Fermi liquids**, some works are pending. I will share with you late the year.

The odd number “2”



Chenjie Wang (王晨杰)

City University of Hong Kong, China

two-electron -> Cooper pair -> superconductor (odd/even parity)!
1/2 electron -> Majorana fermion -> topo quantum computation
spin-1/2 chain -> gapless,
spin-1 chain -> Haldane gap
topological insulator -> Z2 topological invariant
Z2 topological order, Z2 quantum spin liquid ……
fermion doubling theorem, two Weyl nodes in Weyl semimetal
single-layer graphene vs bilayer-layer graphene…

two (not 3) neutron stars emerge……

2-electron -> coope
superconductor,

Z2 topological invar
topological insulato

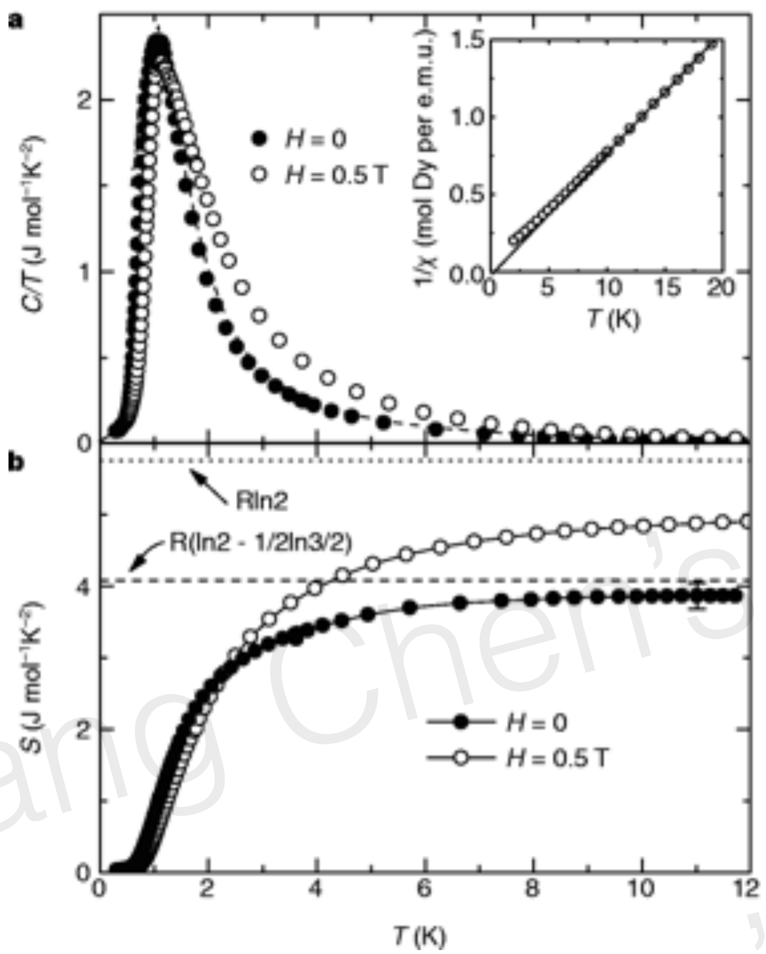
haldane chain,

s-wave
p-wave
odd/even parity sup

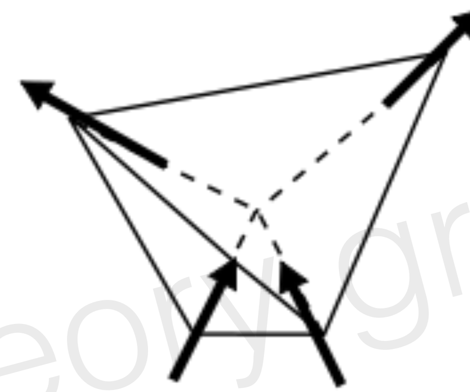
前几天 报告的 2个中
3个一起合并就是fin

Classical spin ice (NOT a spin liquid !)

$Dy_2Ti_2O_7$

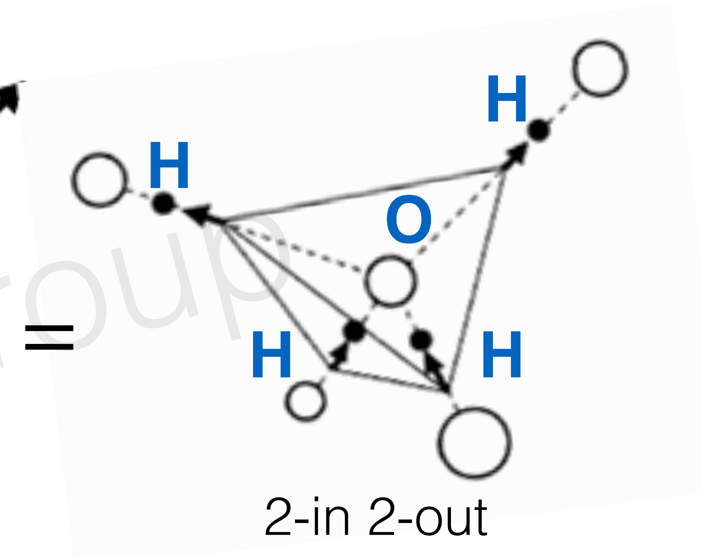


Ramirez, etc, Science 1999

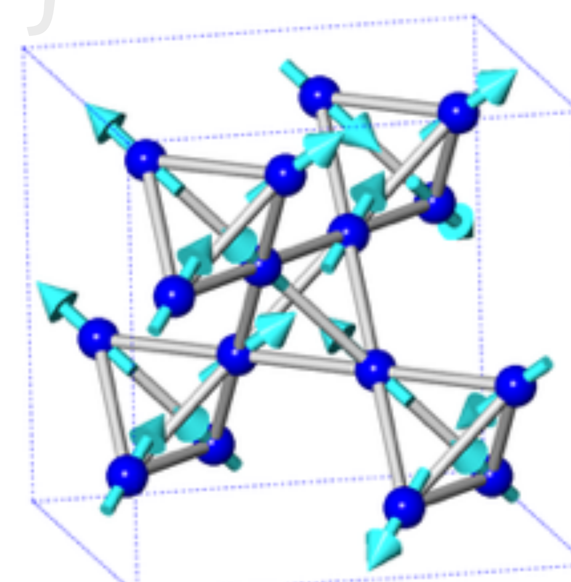


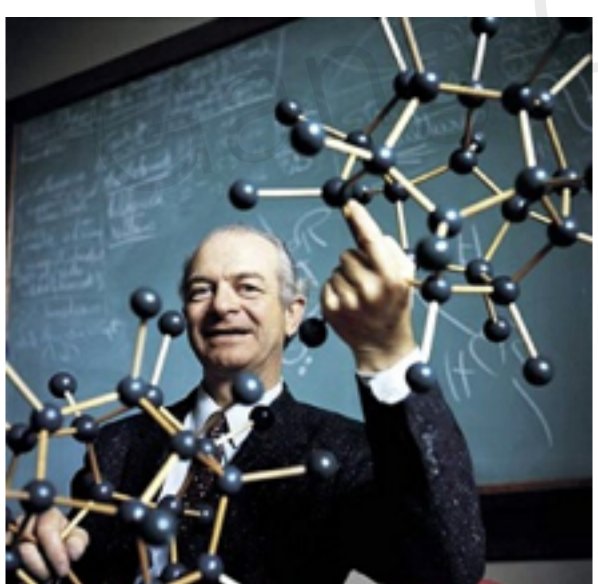
2-in 2-out
spin ice rule

=



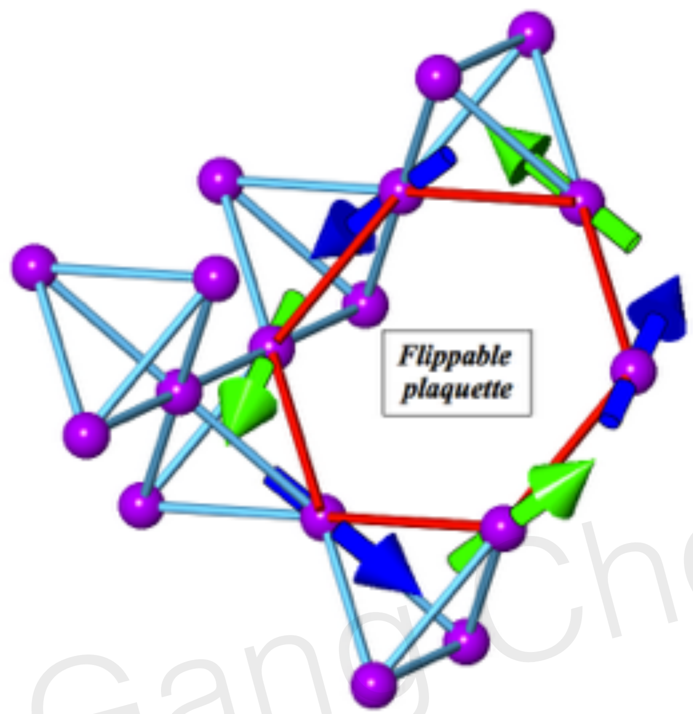
2-in 2-out
water ice rule





Pauling entropy in spin ice

Lattice gauge theory for U(1) spin liquid



Lattice gauge theory on the diamond lattice

$$\mathcal{H}_{\text{XXZ}} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_\perp (S_i^+ S_j^- + S_i^- S_j^+),$$

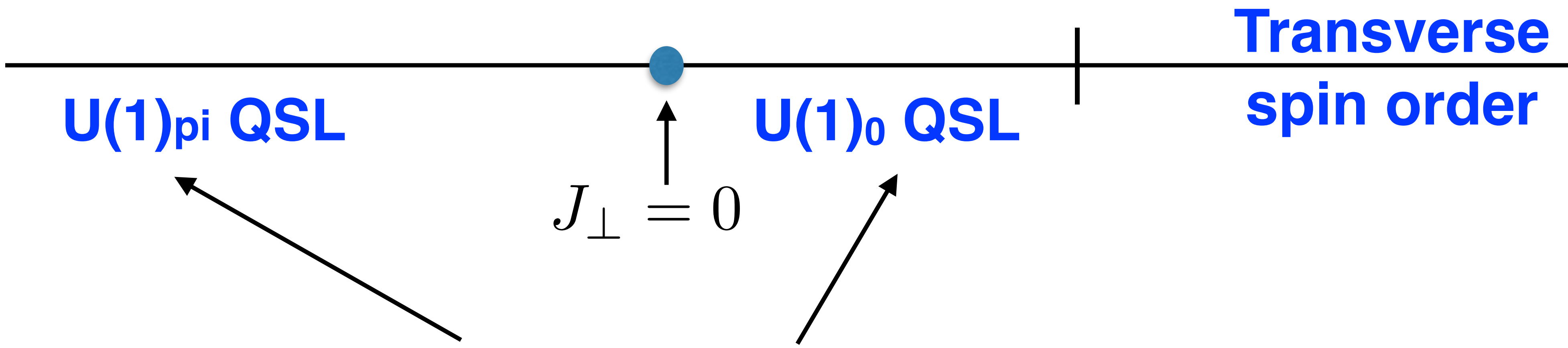
3rd order degenerate perturbation (Hermele, Fisher, Balents 2004)

$$\mathcal{H}_{\text{eff}} = -\frac{12 J_\perp^3}{J_{zz}^2} \sum_{\hexagon_{\text{p}}} (S_i^+ S_j^- S_k^+ S_l^- S_m^+ S_n^- + h.c.),$$

$$E_{\boldsymbol{r}\boldsymbol{r}'} \simeq S^z_{\boldsymbol{r}\boldsymbol{r}'}$$
$$e^{iA_{\boldsymbol{r}\boldsymbol{r}'}} \simeq S^\pm_{\boldsymbol{r}\boldsymbol{r}'}$$

$$\mathcal{H}_{\text{LGT}} = -K \sum_{\hexagon_{\text{d}}} \cos(\text{curl}\, A) + U \sum_{\boldsymbol{r}\boldsymbol{r}'} (E_{\boldsymbol{r}\boldsymbol{r}'} - \frac{\eta_{\boldsymbol{r}}}{2})^2$$

$$K = 24 J_\perp^3 / J_{zz}^2$$



Fuchun老师一直强调物理应该有

string-net condensed phase.

这些数学的东西可以push了很远

reality 可能更重要。 more import

物理，我们需要例子 ！

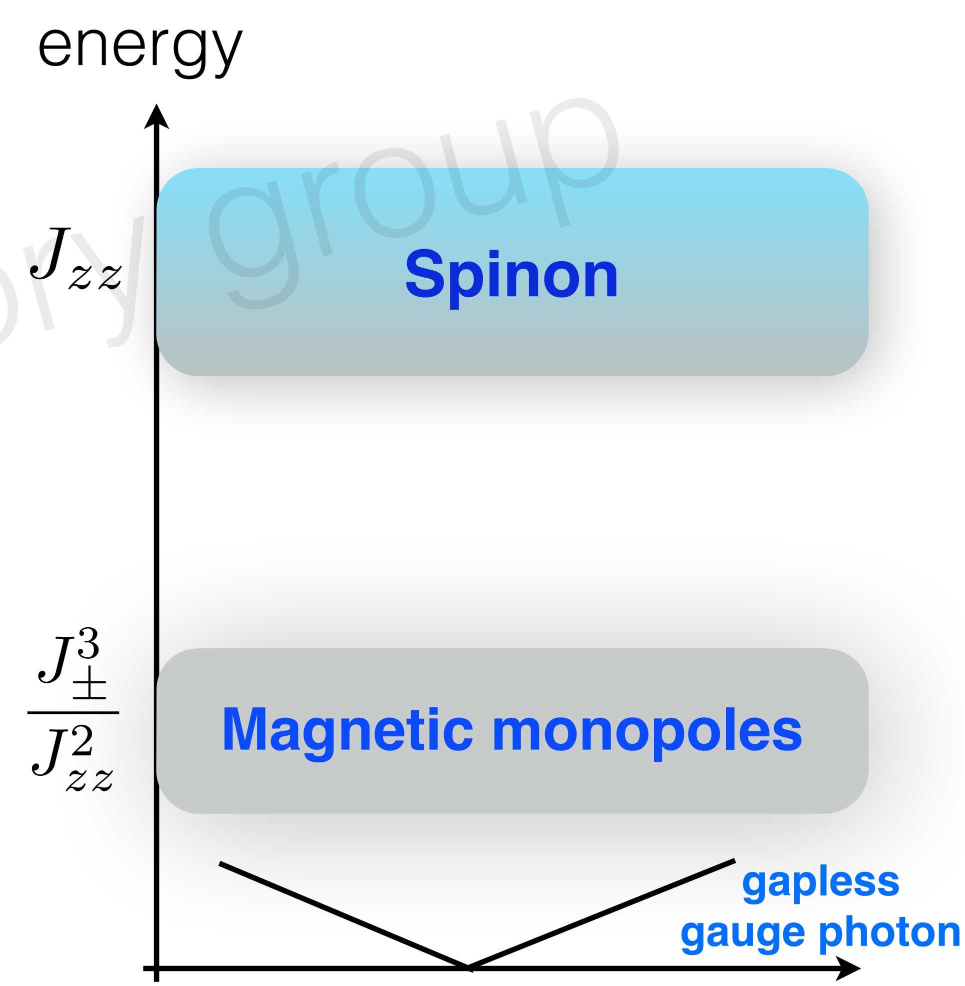
我不喜欢

质朴的语言来刻画物理！！

Related by unitary transformation (Hermele, Matthew Fisher, Balents, PhysRevB, 2004)

$$\mathcal{H}_{\rm XXZ} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_\perp (S_i^+ S_j^- + S_i^- S_j^+),$$

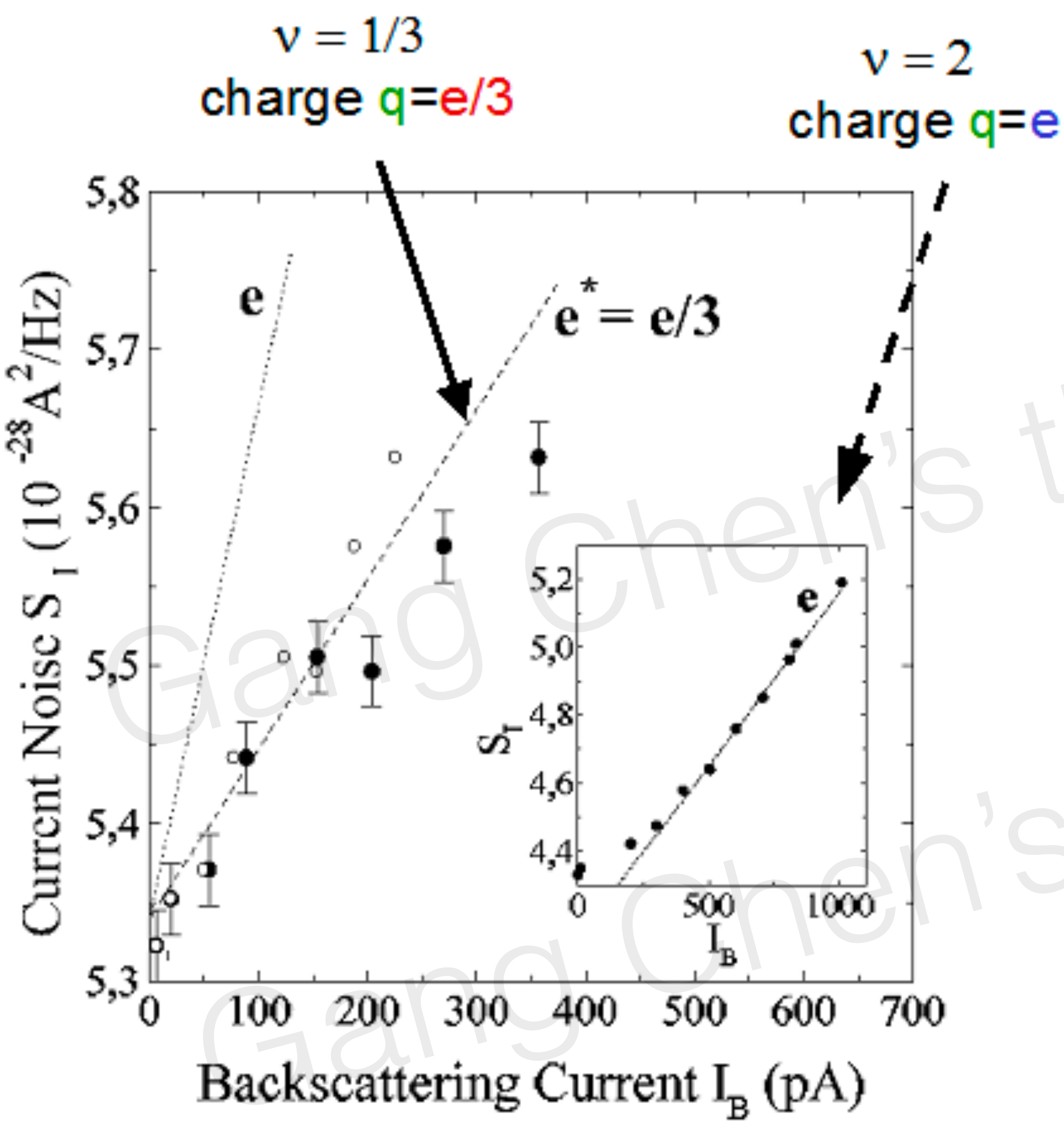
Besides the quantitative differences, are there sharp distinctions between the U(1)pi QSL on the left and the U(1)0 QSL on the right?

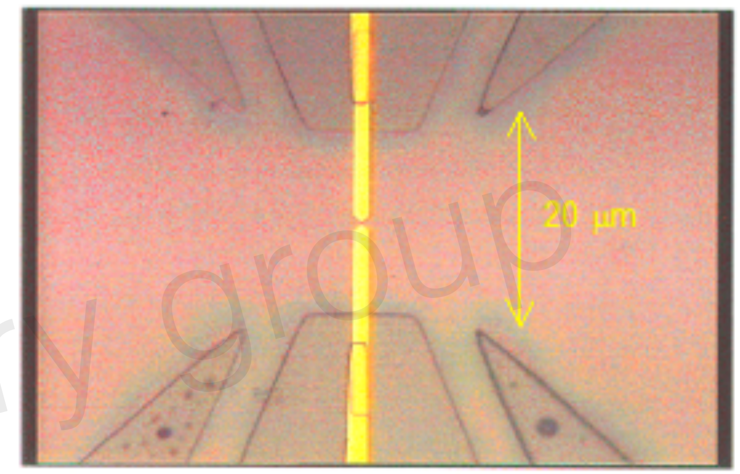


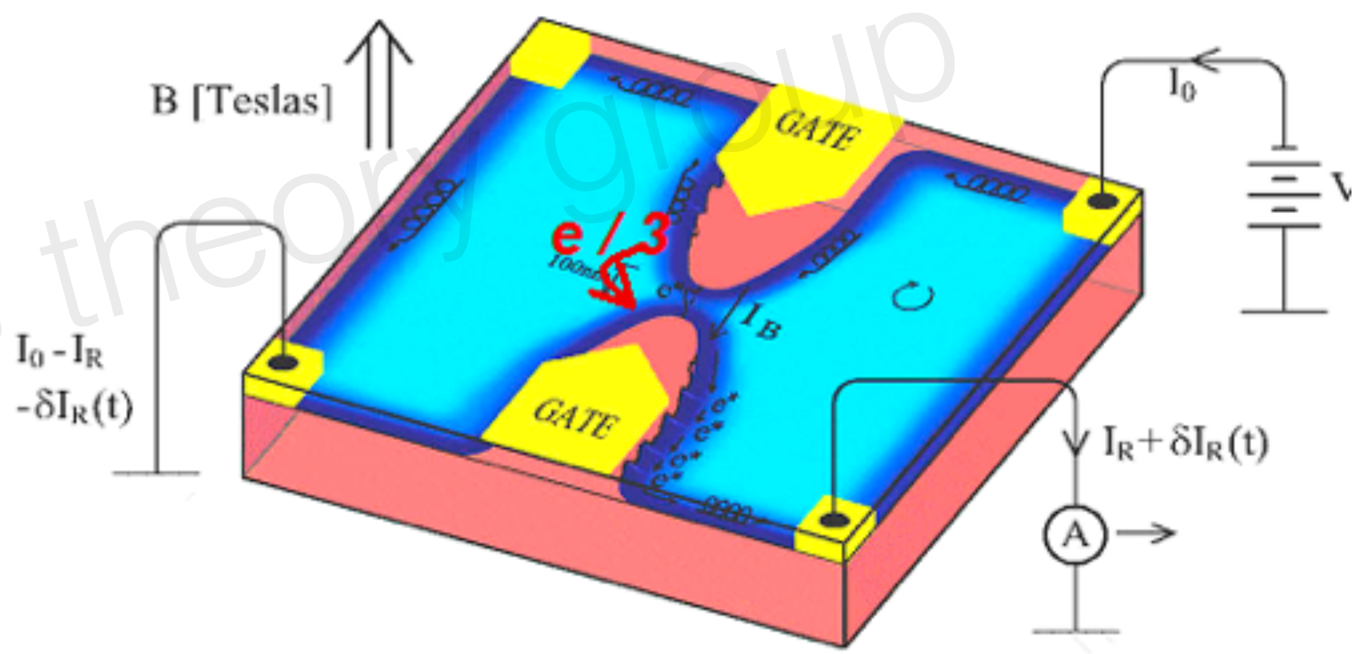
They are **fundamentally different** !
They are distinct symmetry enriched U(1) topological orders !

also string-net condensed phase!

Fractionalization in FQHE: shot-noise measurement







Etien et al, PRL 79, 2526 (1997)
also see Heiblum et al, Nature (1997)

FQHE is arguably the only existing topological order so far.

Chiral (Abelian) topological order

Fractionalization: fractionalized & deconfined exci
Chern-Simon gauge structure

with charge U(1) **symmetry**:
charge conservation

Fractionalized charge excitation

Symmetry makes topological order more visible in experiments.

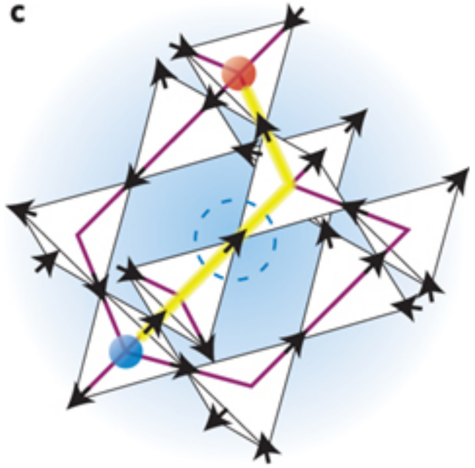
Answer to Prof Xiang Tao's previous question

Symmetry renders extra quantum number to the fractionalized excitation or particle, such that these fractionalized quantum number can be detected experimentally.

凝聚态物理历史上比较重要的实验之一

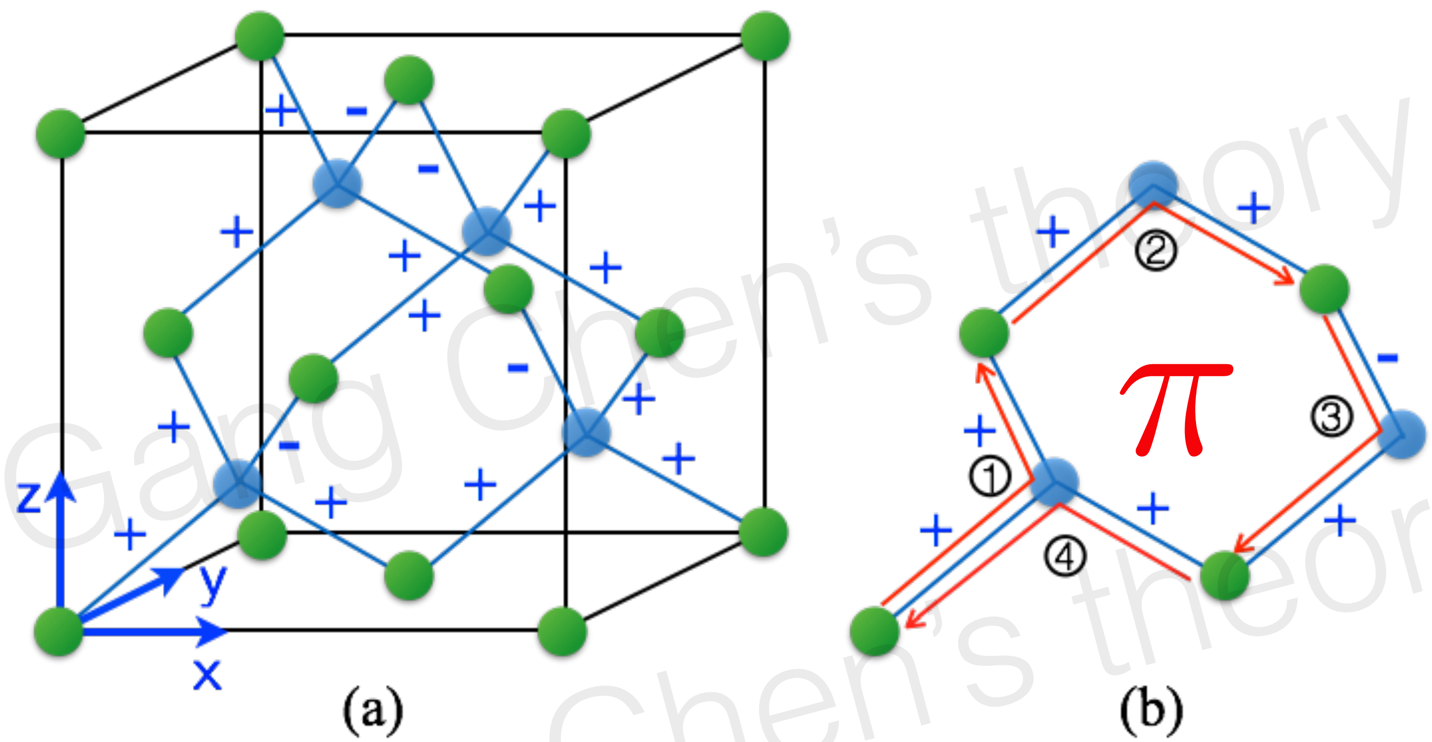
引入超导 就不会有shot noise的实验结果了。

观察！



Pi flux and the spinon trans

$$\mathcal{H}_{\rm LGT} = -K \sum_{\hexagon_{\rm d}} \cos(\mathrm{curl}\, A) + U \sum_{\boldsymbol{r}\boldsymbol{r}'} (E_{\boldsymbol{r}\boldsymbol{r}'} - \frac{\eta_{\boldsymbol{r}}}{2})^2$$



If $K < 0, \ curl A = \pi$

If $K > 0, \ curl A = 0$

$$T^s_\mu T^s_\nu (T^s_\mu)^{-1} (T^s_\nu)^{-1} = \pm 1$$

$$H = \sum_{\mathbf{r} \in \mathrm{I,II}} \frac{J_{zz}}{2} Q^2_{\mathbf{r}} - J_\pm \Big\{ \sum_{\mathbf{r} \in \mathrm{I}} \sum_{\mu, \nu \neq \mu} \Phi^\dagger_{\mathbf{r}+\mathbf{e}_\mu} \Phi_{\mathbf{r}+\mathbf{e}_\nu} \mathsf{s}^-_{\mathbf{r},\mathbf{r}+\mathbf{e}_\mu} \mathsf{s}^+_{\mathbf{r},\mathbf{r}+\mathbf{e}_\nu} + \sum_{\mathbf{r} \in \mathrm{II}} \sum_{\mu, \nu \neq \mu} \Phi^\dagger_{\mathbf{r}-\mathbf{e}_\mu} \Phi_{\mathbf{r}-\mathbf{e}_\nu} \mathsf{s}^+_{\mathbf{r},\mathbf{r}-\mathbf{e}_\mu} \mathsf{s}^-_{\mathbf{r},\mathbf{r}-\mathbf{e}_\nu} \Big\}$$

Aharonov-Bohm flux experienced by spinon via the 4 translation is identical to the flux in the hexagon.

Gang Chen, Phys. Rev. B 96, 085136 (2017)

Pi flux means crystal symmetry fractionalization

with definitive momentum
quantum number

It is like symmetry breaki

$$T^s_\mu T^s_\nu = -T^s_\nu T^s_\mu$$

2-spinon scattering state in an inelastic neutron scattering measurement

$$|a\rangle \equiv |\boldsymbol{q}_a; z_a\rangle,$$

construct another 3 equal-energy states by translating one spinon by 3 lattice vector

$$|b\rangle = T^s_1(1)|a\rangle, \quad |c\rangle = T^s_2(1)|a\rangle, \quad |d\rangle = T^s_3(1)|a\rangle$$

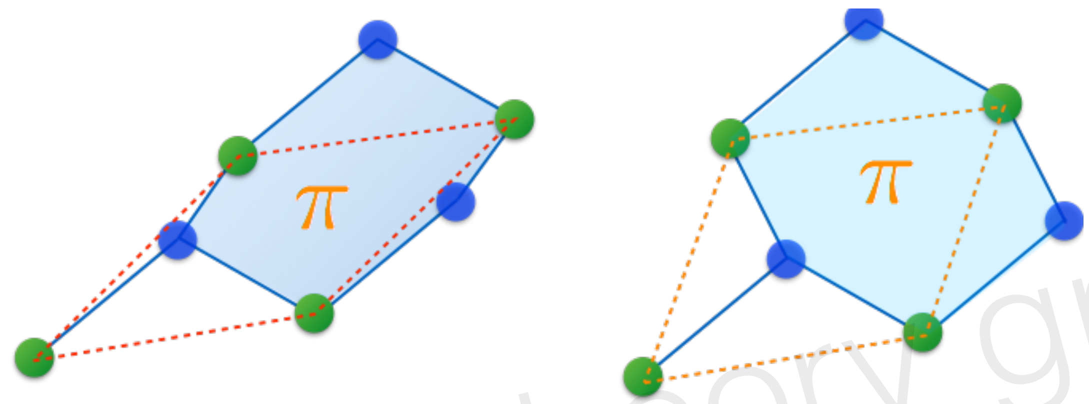
$$T_1|b\rangle = T^s_1(1)T^s_1(2)T^s_1(1)|a\rangle = +T^s_1(1)[T_1|a\rangle],$$
$$T_2|b\rangle = T^s_2(1)T^s_2(2)T^s_1(1)|a\rangle = -T^s_1(1)[T_2|a\rangle],$$
$$T_3|b\rangle = T^s_3(1)T^s_3(2)T^s_1(1)|a\rangle = -T^s_1(1)[T_3|a\rangle],$$

$\Rightarrow$

$$\mathbf{q}_b - \mathbf{q}_a = 2\pi(100)$$

X-G Wen, 2001, 2002, Essin, Hermele, 2014
Gang Chen, Phys. Rev. B 96, 085136 (2017)

Spectral periodicity of the spinon continuum



Lower edge of 2-spinon continuum $\mathcal{L}(\boldsymbol{q}) = \mathcal{L}(\boldsymbol{q} + 2\pi(100)) = \mathcal{L}(\boldsymbol{q} + 2\pi(010)) = \mathcal{L}(\boldsymbol{q} + 2\pi(001))$,

Upper edge of 2-spinon continuum $\mathcal{U}(\boldsymbol{q}) = \mathcal{U}(\boldsymbol{q} + 2\pi(100)) = \mathcal{U}(\boldsymbol{q} + 2\pi(010)) = \mathcal{U}(\boldsymbol{q} + 2\pi(001))$.

But elastic neutron scattering will NOT see extra Bragg peak.

Gang Chen, Phys. Rev. B 96, 085136 (2017)

Calculate spinon continuum to demonstrate the above prediction

$$\mathcal{H}_{\text{XXZ}} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_{\perp}(S_i^+ S_j^- + S_i^- S_j^+),$$

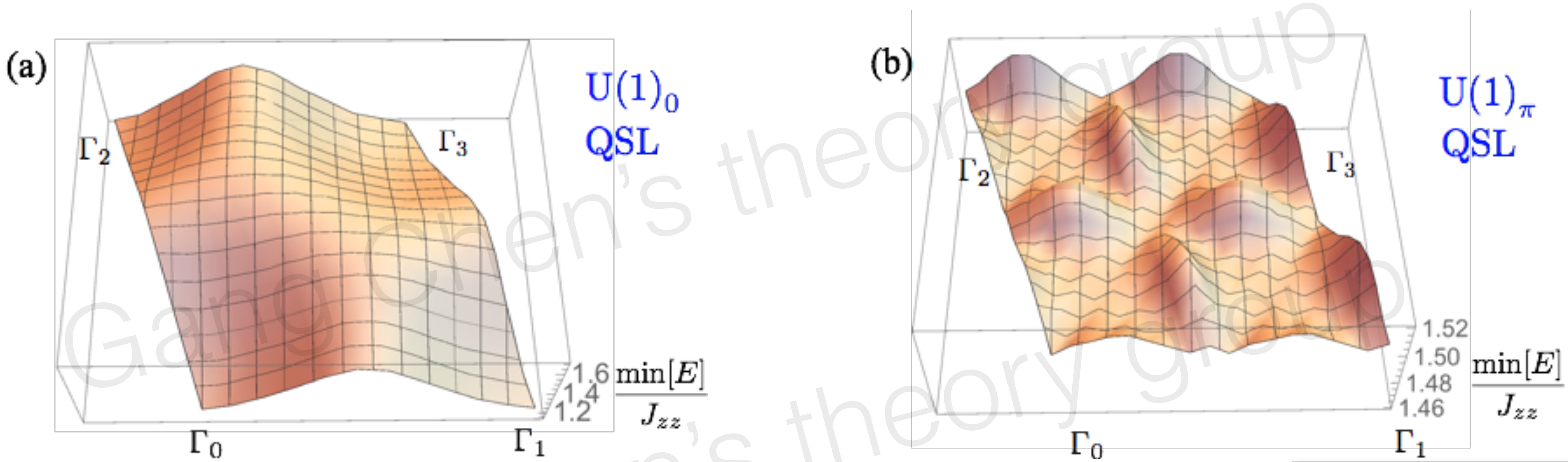


FIG. 3. (Color online.) The lower excitation edge of the spinon continuum in U(1)$_0$ and U(1)$_\pi$ QSLs. Here, $\Gamma_0\Gamma_1 = 2\pi(\bar{1}11), \Gamma_0\Gamma_2 = 2\pi(1\bar{1}1)$. We set $J_\perp = 0.12 J_{zz}$ for U(1)$_0$ QSL in (a) and $J_\perp = -J_{zz}/3$ for U(1)$_\pi$ QSL in (b).

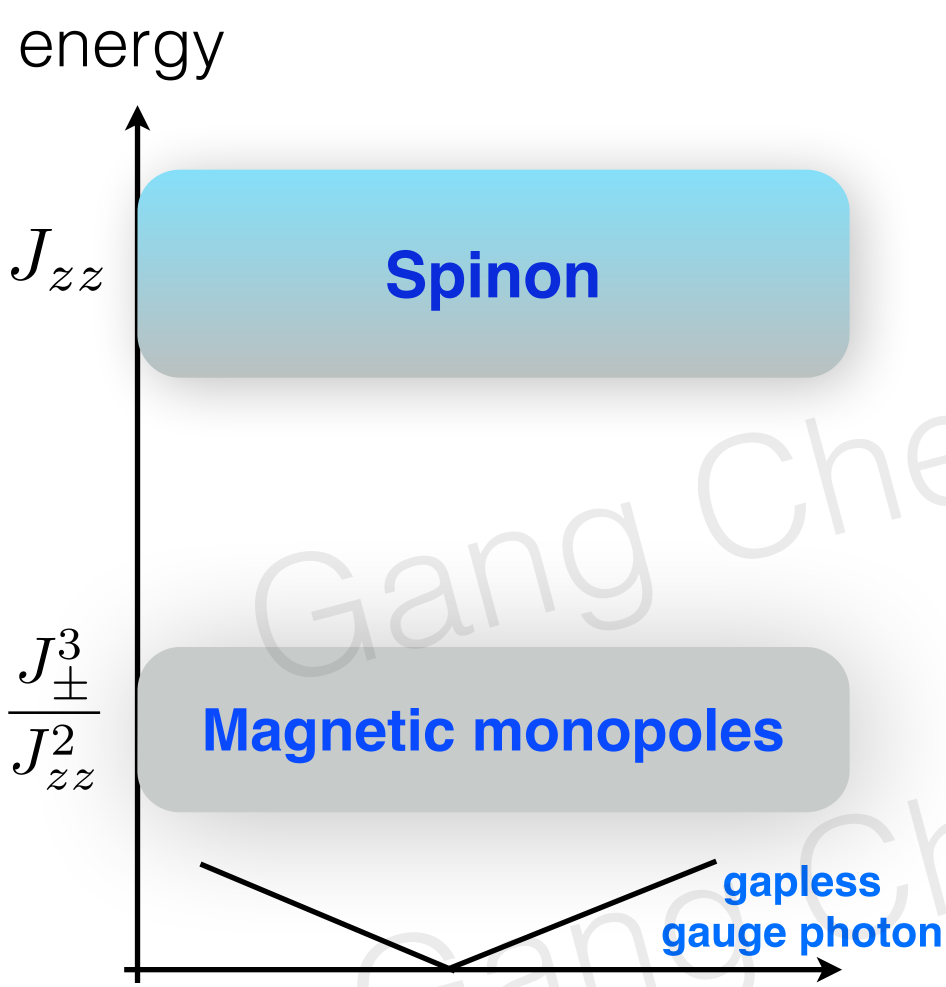
Gang Chen, Phys. Rev. B 96, 085136 (2017)

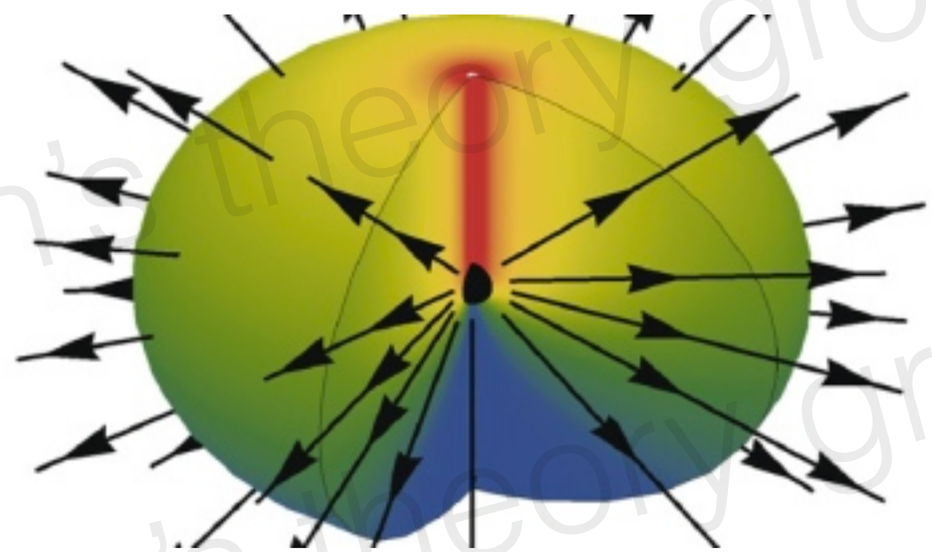
these wiggles are observable,

xxxx like Dirac fermion

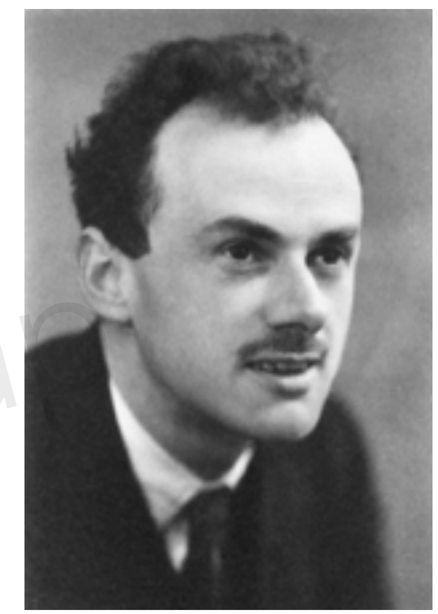
和电荷分数化的联系， 晶格动量的分数化

How to observe Dirac's "magnetic monopole"?



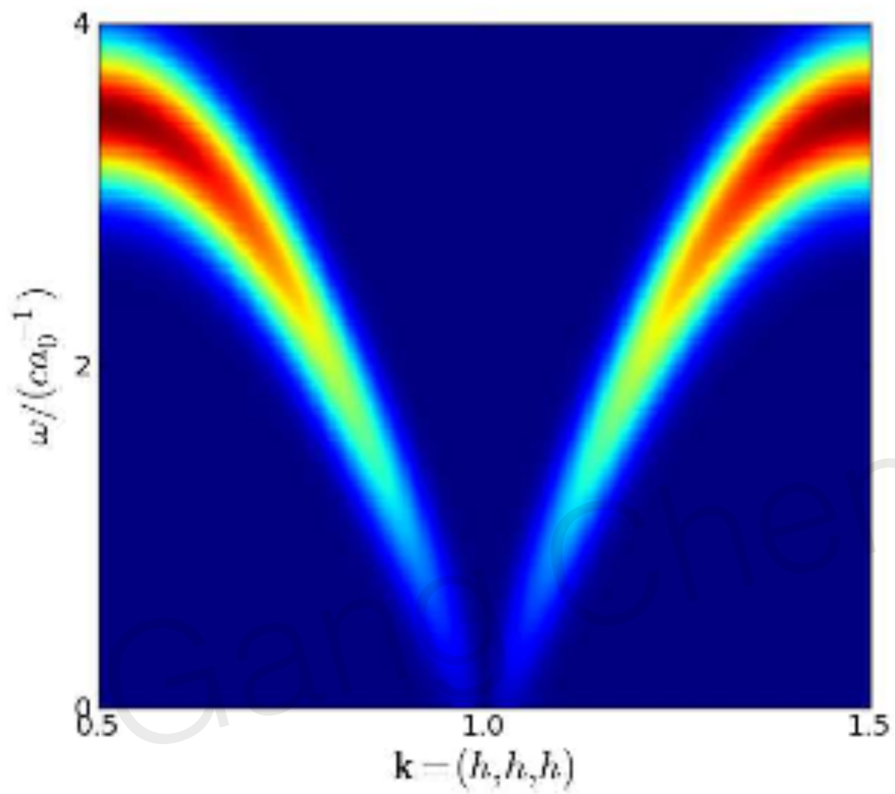


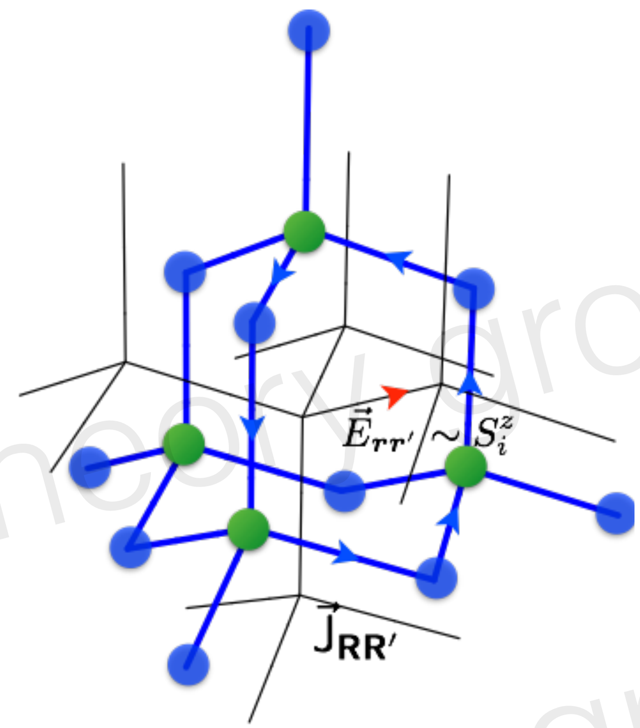
Dirac magnetic monopole

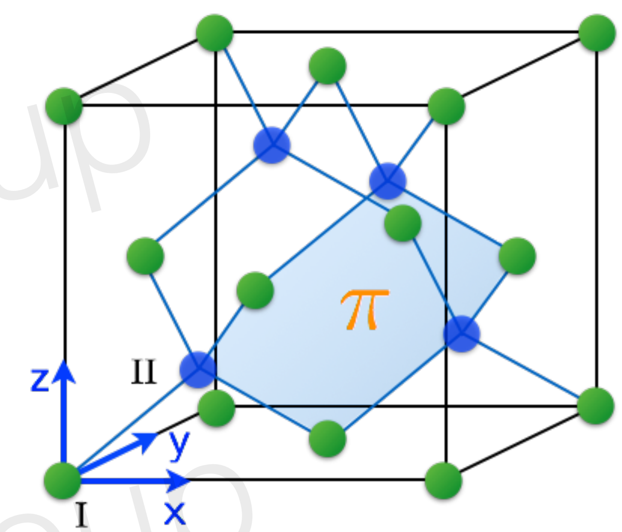


Dirac

How to observe the "magnetic monopole"?







Electromagnetic duality

$S_z \sim E$ (emergent electric field)

$$\mathrm{Im}[E^{\alpha}_{-\mathbf{k},-\omega} E^{\beta}_{\mathbf{k},\omega}] \propto [\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{\mathbf{k}^2}]\, \omega\, \delta(\omega - v|\mathbf{k}|),$$

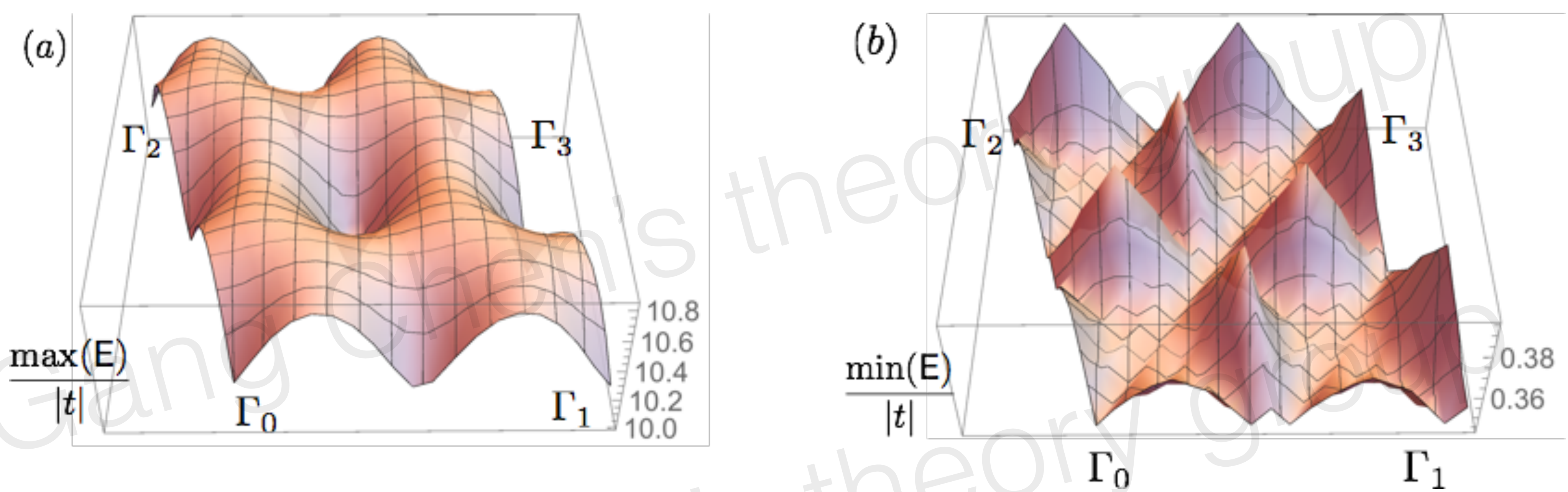
Low energy theory

Electric loop current -> Magnetic field
Magnetic loop current -> Electric field

at higher energy, detect monopole continuum

Gang Chen, arXiv 1706.04333 (2017)

Enhanced spectral periodicity of the monopole continuum



Upper edge

Lower edge

Gang Chen, arXiv 1706.04333 (2017)

Two distinct symmetry enriched U(1) QSLs

Properties	$U(1)_{0,\pi}$ QSL	$U(1)_{\pi,\pi}$ QSL
spinon flux	0	π
"monopole" flux	π	π
spinon continuum	not enhanced	enhanced
"monopole" continuum	enhanced	enhanced

TABLE II. A classification of distinct U(1) QSLs from the symmetry classification patterns of the spinons and the "magnetic monopoles". The first subindex refers to the flux that is

One can think about the symmetry fractionalization pattern of "**fermionic dyons**".

Gang Chen, arXiv 1706.04333 (2017)

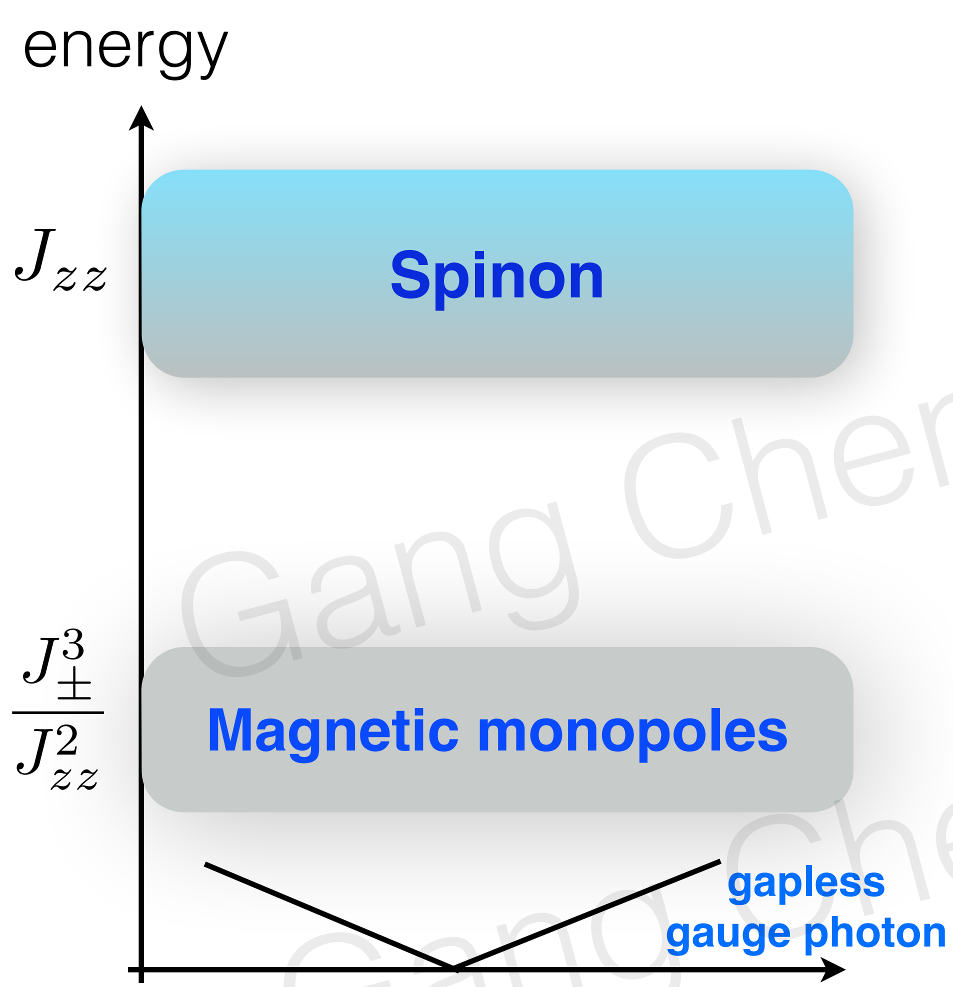
3D U(1) spin liquid / topological order

Fractionalization: fractionalized & deconfined excitation
Maxwell field theory with compact U(1) gauge structure

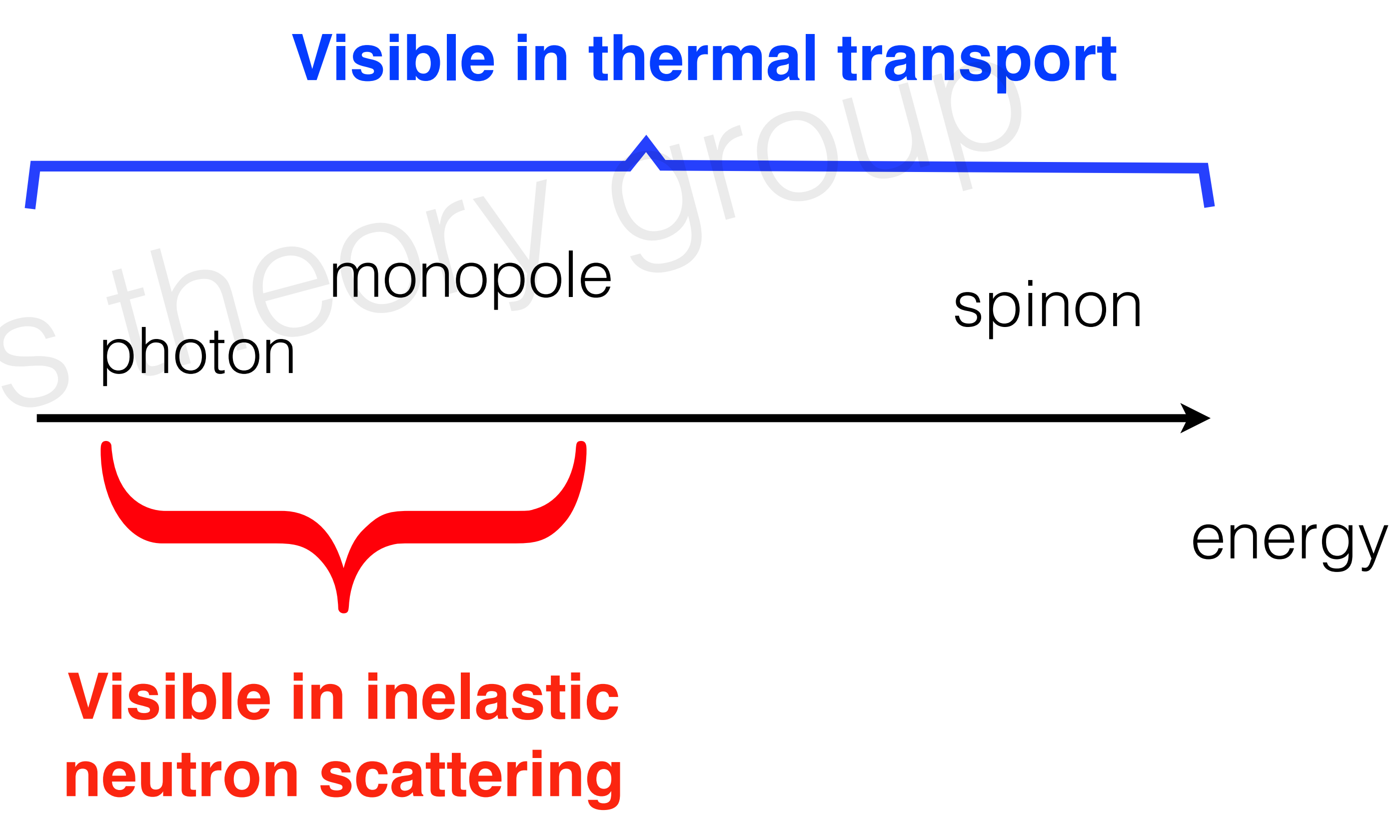
with lattice translations

Fractionalized crystal momentum for
the spinons and magnetic monopoles

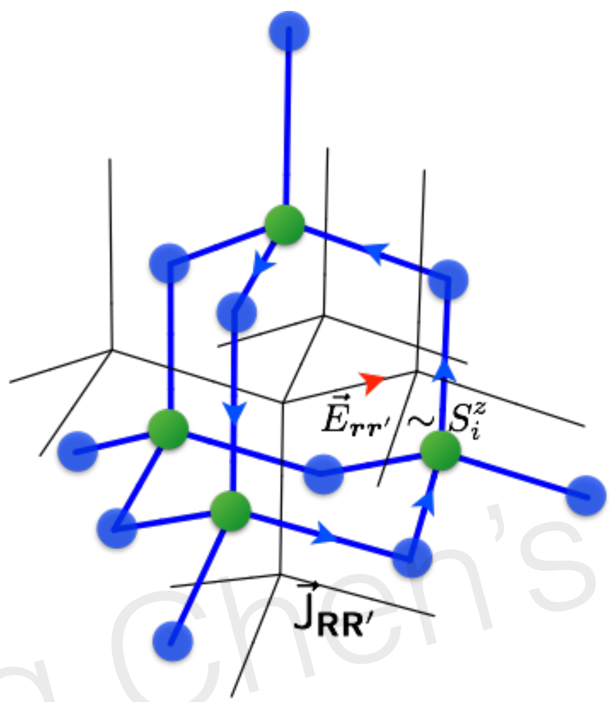
Suggestion 1: combine thermal transport with inelastic neutron



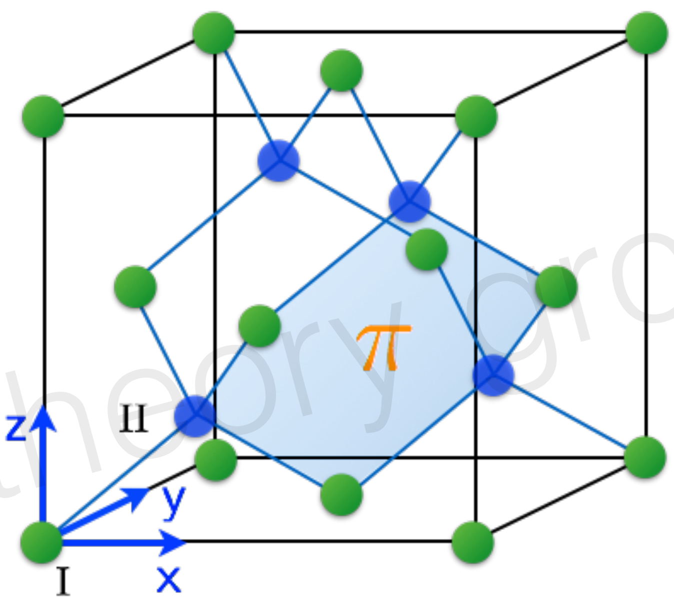
For **non-Kramers doublets** such as Pr ion in $Pr_2Zr_2O_7$ and Tb ion in $Tb_2Ti_2O_7$



Suggestion 2: effect of the external magnetic field



(a)



(b)

$$H_{Zeeman} = \vec{B} \cdot \sum_i S_i^z \hat{z}_i$$

The weak magnetic field polarizes Sz slightly, and thus modifies the background electric field distribution. This further modulates monopole band structure, creating "**Hofstadter**" monopole band, which may be detectable in inelastic neutron.

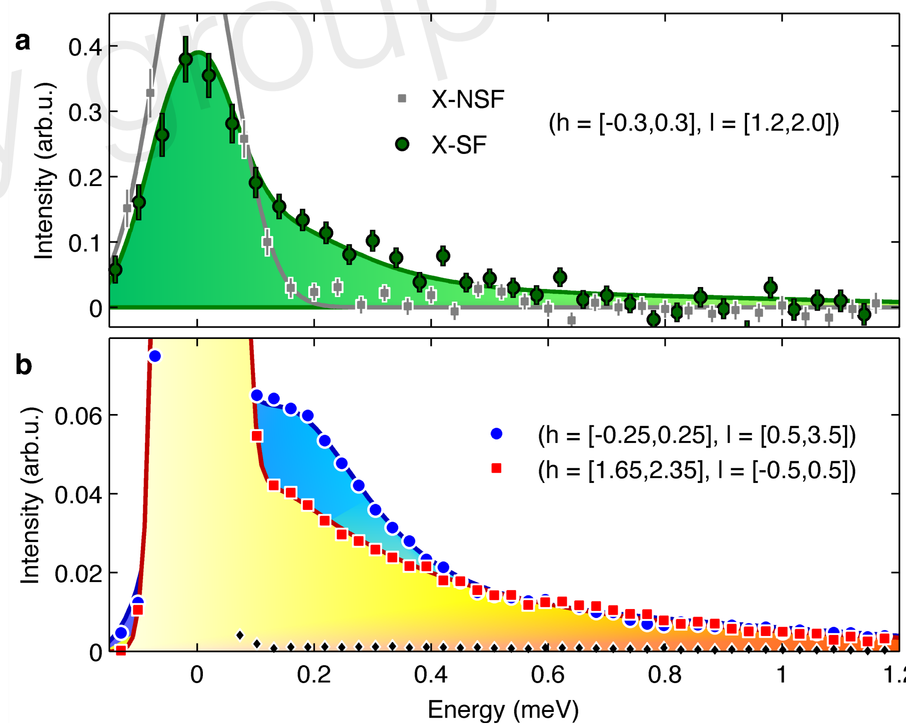
THz can also do the job, but only at Gamma point.

Summary

1. We point out the existence of "magnetic monopole continuum" in the U(1) quantum spin liquid, and monopole is purely **quantum origin**.

2. We point out that the "magnetic monopole" always experiences a Pi flux, and thus supports enhanced spectral periodicity with **folded Brillouin zone, while** spinons most of the time experience Pi flux**.**

In fact, continuum has been observed in $Pr_2Hf_2O_7$ (R. Sibille, et al, arXiv 1706.03604).



Gang Chen, Phys. Rev. B 96, 085136 (2017)

Gang Chen, arXiv 1706.04333 (2017)