The spectral periodicity of spinon continuum in quantum spin ice

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I thank Jeff and Michel for discussion and comments
Our recent works on quantum spin ice

Quantum spin ice with dipole-octupole doublet
Yi-Ping Huang, Gang Chen, M Hermele, PRL 112, 167203, 2014

Ce2Sn2O7, symmetry enriched U(1) QSL and field-driven Anderson-Higgs transition
Yao-Dong Li, Gang Chen, PRB Rapid Comm, 95, 041106, 2017

Quantum spin ice on the breathing pyrochlore lattice.
L Savary, XQ Wang, HY Kee, YB Kim, Y Yu, Gang Chen PRB 94, 205107, 2016

“Magnetic Monopole” condensation transition out of spin ice U(1) QSL: Pr2Ir2O7
Gang Chen PRB 94, 205107, 2016

The spectral periodicity of spinon continuum in quantum spin ice
Gang Chen, arXiv 1704.02734, 2017
Fractionalization in FQHE: shot-noise measurement

Etien et al, PRL 79, 2526 (1997)
also see Heiblum et al, Nature (1997)
FQHE is arguably the only existing topological order so far.

Chiral (Abelian) topological order

Fractionalization: fractionalized & deconfined excitation

Chern-Simon gauge structure

with charge U(1) symmetry:
charge conservation

Fractionalized charge excitation

Symmetry makes topological order more visible in experiments.
What is the sharp physical observable for the U(1) QSL in quantum spin ice?

$J_{zz}$

Spinon

$J^3_{\pm} / J^2_{zz}$

“Magnetic” monopoles

gapless gauge photon

$\omega f(\alpha u) \sim I(\omega) \sim \omega$

low energy scale suppressed intensity

Hermele, Fisher, Balents 2004

Nic Shannon, etc 2012, Savary, Balents, 2012

heat capacity (Savary&Balents: 1000 times larger than phonon!) and spinon continuum
One answer:

the spectral periodicity of the spinon continuum

![Graph of the spinon continuum](image)

(a) U(1)$_0$ QSL
(b) U(1)$_\pi$ QSL

regular periodicity
enlarged periodicity

**Enlarged periodicity is like the fractional charge in FQHE.**

Gang Chen, arXiv 1704.02734, 2017
Realistic models

• Usual Kramers’ doublet and non-Kramers’ doublet

\[ H = \sum_{\langle ij \rangle} \{ J_{zz} S^z_i S^z_j - J_{\pm} (S^+_i S^-_j + S^-_i S^+_j) \]

\[ + J_{\pm\pm} (\gamma_{ij} S^+_i S^+_j + \gamma^*_{ij} S^-_i S^-_j) \]

\[ + J_{z\pm} [S^z_i (\zeta_{ij} S^+_j + \zeta^*_{ij} S^-_j) + i \leftrightarrow j] \],

Savary, Balents, PRL 2012
S. Onoda, etc, 2009, 2010
SB Lee, Onoda, Balents, 2012

• Dipole-octupole doublet

\[ H = \sum_{\langle ij \rangle} J_x S^x_i S^x_j + J_y S^y_i S^y_j + J_z S^z_i S^z_j \]

\[ + J_{xz} (S^x_i S^z_j + S^z_i S^x_j) \].

Y-P Huang, Gang Chen, M Hermele, PRL 2014
Yao-Dong Li, Gang Chen, PRB 2017

Yi-Ping Huang
(Fudan -> UCSB)

Yao-Dong Li
Use the XXZ model to illustrate the universal physics

\[ \mathcal{H}_{XXZ} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_\perp (S_i^+ S_j^- + S_i^- S_j^+) \]

Transverse spin order
\[ \langle S_i^{\pm} \rangle \neq 0 \]

Hermele, Fisher, Balents, 2004,
Banerjee, Isakov, Demle, YongBaek Kim 2008
Savary, Balents, 2012
Kato, Onoda, 2015
Frustrated regime: early theoretical study

one. We also consider the case of frustrated $XY$ exchange, and find that it favors a $\pi$-flux QSL, with an emergent line degeneracy of low-energy spinon excitations. This feature greatly enhances the stability of the QSL with respect to classical ordering.
Nic Shannon’s interesting work of semiclassic phases

complementary study in the classical regime:
three classical spin liquids
Besides the quantitative differences, are there sharp distinctions between the $U(1)_{\pi}$ QSL on the left and the $U(1)_0$ QSL on the right? 

Related by unitary transformation (Hermele, Fisher, Balents 2004)
Lattice gauge theory

\[ \mathcal{H}_{XXZ} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_\perp (S_i^+ S_j^- + S_i^- S_j^+), \]

3rd order degenerate perturbation
(Hermele, Fisher, Balents 2004)

\[ \mathcal{H}_{\text{eff}} = -\frac{12J_3^3}{J_{zz}^2} \sum_{\mathcal{O}_p} (S_i^+ S_j^- S_k^+ S_l^- S_m^+ S_n^- + h.c.), \]

\[ E_{rr'} \simeq S_{rr'}^z, \]
\[ e^{iA_{rr'}} \simeq S_{rr'}^\pm, \]

\[ \mathcal{H}_{\text{LGT}} = -K \sum_{\mathcal{O}_d} \cos(\text{curl} A) + U \sum_{rr'} (E_{rr'} - \frac{\eta_r}{2})^2 \]

\[ K = 24J_3^3 / J_{zz}^2 \]
Pi flux and the spinon translation

\[ \mathcal{H}_{LGT} = -K \sum_{\bigcirc_d} \cos(\text{curl} A) + U \sum_{rr'} (E_{rr'} - \frac{\eta r}{2})^2 \]

If \( K < 0 \), \( \text{curl} A = \pi \)

If \( K > 0 \), \( \text{curl} A = 0 \)

\[ T_\mu^s T_\nu^s (T_\mu^s)^{-1} (T_\nu^s)^{-1} = \pm 1 \]

Aharonov-Bohm flux experienced by spinon via the 4 translation is identical to the flux in the hexagon.
Pi flux means crystal symmetry fractionalization

\[ T^s_\mu T^s_\nu = -T^s_\nu T^s_\mu \]

2-spinon scattering state in an inelastic neutron scattering measurement

\[ |a\rangle \equiv |q_a; z_a\rangle, \]

construct another 3 equal-energy states by translating one spinon by 3 lattice vector

\[ |b\rangle = T^s_1(1)|a\rangle, \quad |c\rangle = T^s_2(1)|a\rangle, \quad |d\rangle = T^s_3(1)|a\rangle \]

\[ T_1|b\rangle = T^s_1(1)T^s_1(2)T^s_1(1)|a\rangle = +T^s_1(1)[T_1|a\rangle], \]
\[ T_2|b\rangle = T^s_2(1)T^s_2(2)T^s_1(1)|a\rangle = -T^s_1(1)[T_2|a\rangle], \]
\[ T_3|b\rangle = T^s_3(1)T^s_3(2)T^s_1(1)|a\rangle = -T^s_1(1)[T_3|a\rangle], \]

\[ \mathbf{q}_b - \mathbf{q}_a = 2\pi(100) \]

Xiao-Gang Wen, 2001, 2002,
Andrew Essin, Michael Hermele, 2014
Gang Chen, 1704.02734
Spectral periodicity of the spinon continuum

spectral periodicity for the spinon continuum. The spectral periodicity can be reflected by the spectral intensity \( \mathcal{I}(q, E) \), the lower \( \mathcal{L}(q) \) and upper excitation edge \( \mathcal{U}(q) \) of the spinon continuum. For \( U(1)_\pi \) QSL, we have

\[
\mathcal{I}(q, E) = \mathcal{I}(q + 2\pi(100), E) = \mathcal{I}(q + 2\pi(010), E) \\
= \mathcal{I}(q + 2\pi(001), E),
\]

\[
\mathcal{L}(q) = \mathcal{L}(q + 2\pi(100)) = \mathcal{L}(q + 2\pi(010)) \\
= \mathcal{L}(q + 2\pi(001)),
\]

\[
\mathcal{U}(q) = \mathcal{U}(q + 2\pi(100)) = \mathcal{U}(q + 2\pi(010)) \\
= \mathcal{U}(q + 2\pi(001)).
\]

But elastic neutron scattering will NOT see extra Bragg peak.

Xiao-Gang Wen, 2001, 2002,  
Andrew Essin, Michael Hermele, 2014  
Gang Chen, 1704.02734
Calculation to demonstrate the above prediction

$$\mathcal{H}_{\text{XXXZ}} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_\perp (S_i^+ S_j^- + S_i^- S_j^+)$$

FIG. 3. (Color online.) The lower excitation edge of the spinon continuum in U(1)$_0$ and U(1)$_\pi$ QSLs. Here, $\Gamma_0 \Gamma_1 = 2\pi(\bar{1}11)$, $\Gamma_0 \Gamma_2 = 2\pi(1\bar{1}1)$. We set $J_\perp = 0.12J_{zz}$ for U(1)$_0$ QSL in (a) and $J_\perp = -J_{zz}/3$ for U(1)$_\pi$ QSL in (b).

Lower excitation edge of spinon continuum within the gauge MFT calculation
Conclusion

<table>
<thead>
<tr>
<th>U(1) QSLs</th>
<th>U(1) (_0) QSL</th>
<th>U(1) (_\pi) QSL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Background U(1) Flux</td>
<td>0 Flux</td>
<td>(\pi) Flux</td>
</tr>
<tr>
<td>Heat Capacity</td>
<td>(C_v \sim T^3)</td>
<td>(C_v \sim T^3)</td>
</tr>
<tr>
<td>Proximate XY Order</td>
<td>Keep Translation</td>
<td>Enlarged Cell</td>
</tr>
<tr>
<td>Spectral Periodicity</td>
<td>Not Enhanced</td>
<td>Enhanced</td>
</tr>
</tbody>
</table>

For usual Kramers’ doublet, spinon continuum is detectable by INS.

Lucile & Kate: Yb2Ti2O7 is either in or proximate to 0-flux.

So Yb2Ti2O7 does not have enhanced spectral periodicity.

For non-Kramers’ doublet, spinon continuum cannot be detected by INS. But the proximate quadrupolar order would break translation symmetry, and can however be detectable.

Lee, Onoda, Balents: Pi flux state is more robust. This is great!

This means it is more likely for a candidate material to have spectral periodicity enhancement.