Recent development of quantum spin liquids

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Fudan University
Opportunity for students and postdocs

• My research group is looking for graduate students and postdocs

• Our postdocs and visiting professors are generously funded.
Outline

• A general introduction to quantum spin liquids

• Spinon Fermi surface U(1) quantum spin liquid

• Rare earth triangular lattice quantum spin liquid and experiment prediction

• Control spinons in a quantum spin ice U(1) quantum spin liquid
Neel vs Landau (1930-40s)

\[ H = J \sum_{\langle ij \rangle} S_i \cdot S_j \]

Spin singlet = \( \frac{1}{\sqrt{2}} \left[ \begin{array}{c} \uparrow \downarrow \\ \downarrow \uparrow \end{array} \right] \)
The idea of quantum spin liquid (1973)

**P. W. Anderson**

*RESONATING VALENCE BONDS: A NEW KIND OF INSULATOR?*

The type would be insulating; it would represent an alternative state to the Neél antiferromagnetic state for $S = 1/2$. An estimate of

Pauling's RVB wavefunction for Benzene molecule

$$
\frac{1}{\sqrt{2}} [\uparrow \downarrow - \downarrow \uparrow]
$$
High temperature superconductivity (1986)

The idea is to view Mott insulator (QSL) as the parent state of high-temperature superconductor. In the QSL, there are preformed Cooper pairs. Doping it allows Cooper pairs to condense and lead to superconductivity.
Two milestones of 20th century condensed matter physics

Landau Fermi liquid theory

Landau symmetry breaking theory

These two paradigms break down after the discovery of fractional quantum Hall effect in 1980s.
Quantum spin liquid

- Quantum spin liquid is a new quantum phase of matter, and cannot be characterized by Landau symmetry breaking, instead by emergent gauge structure and deconfined fractionalized excitations.

- QSL, its existence, is very clear, at least at the level of theory.
  - Exactly solvable model with QSL ground state: e.g. Kitaev model and extension.
  - Classification of QSLs: many distinct symmetry enriched QSLs (XG Wen etc).
  - Numerical solutions: DMRG, QMC, exact diagonalization, etc.

QSL is robust against any local perturbation. So it should exist in Nature!
QSL: existing experiments

- 2D triangular and Kagome lattice
  - organics: kappa-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$, EtMe$_3$Sb[Pd(dmit)$_2$]$_2$, kappa–H$_3$(Cat-EDT-TTF)$_2$
  - herbertsmithite (ZnCu$_3$(OH)$_6$Cl$_2$), Ba$_3$NiSb$_2$O$_9$, Ba$_3$CuSb$_2$O$_9$, LiZn$_2$Mo$_3$O$_8$, ZnCu$_3$(OH)$_6$Cl$_2$
  - volborthite (Cu$_3$V$_2$O$_7$(OH)$_2$), BaCu$_3$V$_2$O$_3$(OH)$_2$, [NH$_4$]$_2$[C$_7$H$_{14}$N][V$_7$O$_6$F$_{18}$], Na$_2$IrO$_3$, CsCu$_2$Cl$_4$, CsCu$_2$Br$_4$, NiGa$_2$S$_4$, He-3 layers on graphite, etc

- 3D pyrochlore, hyperkagome, FCC lattice, diamond lattice, etc
  - Na$_4$Ir$_3$O$_8$, IrO$_2$, Ba$_2$YMoO$_6$, Yb$_2$Ti$_2$O$_7$, Pr$_2$Zr$_2$O$_7$, Pr$_2$Sn$_2$O$_7$, Tb$_2$Ti$_2$O$_7$, Nd$_2$Zr$_2$O$_7$, FeSc$_2$S$_4$, etc

- Ultracold atom and molecules on optical lattices: temperature is too high now.

Some candidate materials have already been ruled out.
Not being a QSL does not necessarily mean the physics is not interesting!
• Spinon Fermi surface U(1) quantum spin liquid
Any guiding rule to find QSL? Not really.

Frustrated lattice? Honeycomb Kitaev model.
Frustrated interaction? We do not really know unless we identify the interaction.
Low dimensionality? 3D lattice also has QSL.
Odd electrons per cell? Many QSLs have even electrons per cell.

• Hastings-Oshikawa-Lieb-Shultz-Mattis theorem.
• Recent extension to spin-orbit coupled insulators (Watanabe, Po, Vishwanath, Zaletel, PNAS 2016).
A rare-earth triangular lattice quantum spin liquid: $\text{YbMgGaO}_4$

This part is in collaboration with experimentalists

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Prof. Qingming Zhang (Renmin Univ, Beijing)
Wei Tong (High Magnetic field Lab, Hefei)
Pi Li (High Magnetic field Lab, Hefei)
Juanjuan Liu (Renmin Univ, Beijing)
Zhaorong Yang (Institute of Solid-State Physics, Hefei)
Xiaoqun Wang (Renmin, Shanghai Jiaotong)

YS Li, GC*,...., QM Zhang*
PhysRevLett 2015
A rare-earth triangular lattice quantum spin liquid: \( \text{YbMgGaO}_4 \)

- This is the \textbf{first} strong spin-orbit coupled QSL with odd number of electrons and effective spin-1/2.
- It is the \textbf{first} clear observation of \( T^{2/3} \) heat capacity.
- We understand the microscopic Hamiltonian and the physical mechanism.
YbMgGaO$_4$

- observation of $T^{2/3}$ heat capacity
- Entropy: effective spin-1/2 local moments

My proposal for ground state: spinon Fermi surface U(1) QSL.
Yb$^{3+}$ ion: $4f^{13}$ has $J=7/2$ due to SOC.

At $T \ll \Delta$, the only active DOF is the ground state doublet that gives rise to an effective spin-$1/2$. 
and a crystalline lattice or a magnetic field. Mott insulators are a particularly interesting class, with an odd number of electrons in each unit cell. Their low energy physics is captured by a spin model with an odd number of $S = 1/2$ moments in the unit cell. A powerful result due to Lieb, Schultz, and Mattis in 1D\(^1\), later extended to higher dimensions by Hastings and Oshikawa\(^2,3\), holds that if all symmetries remain unbroken, the ground state must be ‘exotic’ - such as a Luttinger liquid in 1D, or a quantum spin liquid in higher dimensions, with fractional ‘spinon’ excitations. These exotic states cannot be represented as simple product states, as a consequence of long ranged quantum entanglement. This general result is thought to be on the order of 10% of the Heisenberg exchange terms.

The quasi 2D Mott insulators are a particularly interesting class, with an odd number of electrons in each unit cell. A powerful result due to Luttinger, Schultz, and Mattis in 1D\(^1\), later extended to higher dimensions by Hastings and Oshikawa\(^2,3\), holds that if all symmetries remain unbroken, the ground state must be ‘exotic’ - such as a Luttinger liquid in 1D, or a quantum spin liquid in higher dimensions, with fractional ‘spinon’ excitations. These exotic states cannot be represented as simple product states, as a consequence of long ranged quantum entanglement. This general result is thought to be on the order of 10% of the Heisenberg exchange terms.

Filling constraints for spin-orbit coupled insulators in symmorphic and non-symmorphic crystals

Haruki Watanabe,\(^1\) Hoi Chun Po,\(^1\) Ashvin Vishwanath,\(^{1,2}\) and Michael P. Zaletel\(^3\)

May 2015

Can this kind of system support a QSL ground state? Yes.

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Can this kind of system support a QSL ground state? Yes.
What is the physical origin of the QSL?

4f electron is very localized, and dipolar interactions weak.

\[
\mathcal{H} = \sum_{\langle ij \rangle} \left[ J_{zz} S_i^z S_j^z + J_{\pm}(S_i^+ S_j^- + S_i^- S_j^+) \right. \\
+ J_{\pm\pm}(\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-) \\
- \frac{i J_{z\pm}}{2} (\gamma_{ij}^* S_i^z S_j^z - \gamma_{ij} S_i^- S_j^+ + \langle i \leftrightarrow j \rangle) \right],
\]

where \( S_i^{\pm} = S_i^x \pm i S_i^y \), and the phase factor \( \gamma_{ij} = 1, e^{i2\pi/3}, e^{-i2\pi/3} \) for the bond \( ij \) along the \( a_1, a_2, a_3 \) direction (see Fig. 1), respectively. This generic Hamil-

The spin-1/2 XXZ model supports conventional order. (Yamamoto, etc, PRL 2014)
Anisotropic spin interaction could potentially stabilize QSL.

![Phase diagram](image1.png)

Yao-Dong Li
Dept of Computer Sciences
Fudan University

![Parameter variation](image2.png)
Spinon Fermi surface U(1) QSL in organic magnets?

- kappa-(BEDT-TTF)$_2$Cu$_2$(CN)$_3$
- EtMe$_3$Sb[Pd(dmit)$_2$]$_2$
- kappa–H$_3$(Cat-EDT-TTF)$_2$

- No magnetic order down to 32mK
- Constant spin susceptibility at zero temperature

Other experiments: transport, heat capacity, optical absorption, etc.
Unfortunately, no neutron scattering so far.
• Theoretical understanding: expected phase diagram

\[ H = -t \sum_{\langle ij \rangle, \sigma} c_{i\sigma}^\dagger c_{j\sigma} + h.c. + U \sum_i n_{i\uparrow} n_{i\downarrow} \]

Senthil’s cartoon

- Fermi liquid
- Mott transition
- U(1) QSL with spinon Fermi surface
- supported by various different numerics

\[ H_{\text{pert}} = \sum_{ij} J_{ij} \mathbf{S}_i \cdot \mathbf{S}_j + K \sum_{1234} (P_{1234} + P_{1234}^{-1}) + \cdots \]

- 4-site ring exchange
- \((\mathbf{S}_1 \cdot \mathbf{S}_2)(\mathbf{S}_3 \cdot \mathbf{S}_4)\)
- \((\mathbf{S}_1 \cdot \mathbf{S}_4)(\mathbf{S}_2 \cdot \mathbf{S}_3)\)
- \(-(\mathbf{S}_1 \cdot \mathbf{S}_3)(\mathbf{S}_2 \cdot \mathbf{S}_4)\)

• Physical mechanism for weak Mott insulator spin liquids: perturbation in \(t/U\)

Gang Chen’s theory group

Sung-Sik Lee  T Senthil  Patrick Lee

Motrunich
Low energy property of spinon Fermi surface U(1) QSL: spinon non-Fermi liquid

Spinon Fermi surface coupled with dynamical U(1) gauge field: instanton event is suppressed.

dual to extremal/charged black hole?

\[ S = \int d^3x \left[ \Psi_j^* (\partial_0 - ia_0 - \mu_F) \Psi_j + \frac{1}{2m} \Psi_j^* (-i \nabla - a)^2 \Psi_j + \frac{1}{4g^2} f_{\mu \nu} f_{\mu \nu} \right]. \]

Re\(\Sigma\), Im\(\Sigma\) ~ \(\omega^{2/3}\)

Hermele et al., PRB 70, 214437 (04)
Sung-Sik Lee, PRB 78, 085129(08).
Spin wave vs (fractionalized) spinon continuum

Furthermore, and most importantly, our extracted exchange parameters correctly reproduce relative intensities as well as the shape of the spin wave dispersion for each of the five directions. Agreement is excellent for $H = 2T$, showing that these parameters produce a robust description of the field-induced ferromagnetic state. We note, however, that there is a significant quantitative disagreement with the exchange parameters quoted in Refs. [9, 10] (see Appendix H).

Implications.— The excellent agreement with spin wave theory for fields $H/C^2 \approx 2T$ clearly indicates that the high field state is accurately modeled semiclassically, and is smoothly connected to the fully polarized limit. Theoretically, the ground state in this regime breaks no symmetries, and supports a ferromagnetic polarization along the axis of the applied field (for the $h_{110}$ field used in the experiment).

However, the semiclassical analysis clearly and dramatically fails at small fields, where the measurements show no signs of spontaneous long-range order [18]. The classical zero-field ground state for our Hamiltonian parameters has a large spontaneous polarization along the $h_{100}$ axis. Extension of this analysis to a $T > 0$ mean-field theory wrongly predicts a spin wave continuum in Cs$_2$CuCl$_4$.

spin wave in Yb$_2$Ti$_2$O$_7$
L Savary, et al, PRX 2011

spinon continuum in Cs$_2$CuCl$_4$
Masanori, etc NatPhys 2009
but these are 1d spinons!
Huge spinon continuum at all energies

\( E = 1.5 \text{ meV} \)
\( T = 70 \text{ mK} \)

Yao Shen, …GC, Jun Zhao arxiv 2016
Kills the spinon quasi-particle weight, scatters the fermionic spinons strongly, gives a self-energy correction to the spinon Green's function, and the U(1) gauge photon is over-damped and becomes very soft. The soft gauge photon further the Fermi surface coupled with a noncompact U(1) gauge field. If the instanton event is suppressed here, the compactness of the U(1) gauge field is no longer an issue. Events is the cause of the confinement for a U(1) quantum spin liquid without gapless spinons. U(1) gauge field for a two-dimensional U(1) quantum spin liquid gapless fermionic spinons on the spinon Fermi surface help suppress the instanton events of the compact. Gang Chen's theory group.

Calculation of 0-flux Hamiltonian. a, b, c. Vertical dashed lines represent the high-symmetry points, and dotted lines of the 0-flux Hamiltonian. The grey plane marks the Fermi level at (r.l.u.)

<table>
<thead>
<tr>
<th>Energy (meV)</th>
<th>(1/2-K/2, K, 0) (r.l.u.)</th>
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<tbody>
<tr>
<td>0.2</td>
<td>(1, K, 0) (r.l.u.)</td>
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Intensity contour plot along high- 

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Intensity contour plot along high-
Summary

1. QSL is a field that bridges the fundamental ideas with the frontier experiments, it provides exciting opportunities for both theorists and experimentalists.

2. Rare-earth triangular lattice quantum spin liquid: $\text{YbMgGaO}_4$

   - To our best knowledge, this is the first strong spin-orbit coupled quantum spin liquid candidate with odd number of electrons per unit cell and effective spin-$1/2$ moment.
• Octupolar U(1) quantum spin liquid of quantum spin ice
Rare-earth pyrochlores

$\text{RE}_2\text{M}_2\text{O}_7$
Rare-earth local moments: a **crude** classification

Kramers’ doublet: $\text{R}^{3+}$ with **odd** number of electrons

Non-Kramers’ doublet / singlet: $\text{R}^{3+}$ with **even** number of electrons
Spin ice (Ising) limit

\[ H = J_{zz} \sum_{\langle i,j \rangle} S_{i}^{z} S_{j}^{z} + \ldots \]

2-in 2-out spin ice rule

2-in 2-out water ice rule

from wiki
Classical spin ice

- The “2-in 2-out” states are extensively degenerate.
- At temperature $T < J_{zz}$, the system thermally fluctuates within the ice manifold, leading to classical spin ice and interesting experimental discoveries.
Dipole-octupole doublet

The early classification of local moments is a bit crude!
One should carefully examine the wavefunction of the local doublet.
Local physics: start with $t_{2g}$ electrons

- Local moments on pyrochlore lattice: effective spin-1/2

\[
\begin{align*}
&d^3 \text{ configuration} \\
&\text{e.g. 5d transition metal} \\
&\text{octahedral CEF} \\
&\begin{array}{c}
\begin{array}{c}
\text{SOC} \\
j = \frac{1}{2}
\end{array}
\end{array}
\begin{array}{c}
\begin{array}{c}
H_{\text{tri}} \\
j = \frac{3}{2}
\end{array}
\end{array}
\end{align*}
\]

$\Gamma_5^+ \oplus \Gamma_6^+$

\[
\begin{array}{l}
\{ j^z = \pm 1/2 \}
\end{array}
\]

$d$ electrons under $D_{3d}$ point group crystal field
• Why is this Kramers doublet so special?

**ONE**-dimensional representations of the point group!

\[
R(2\pi/3)|J^z = \pm 3/2\rangle = -|J^z = \pm 3/2\rangle
\]

\[
R(2\pi/3) \equiv e^{-i \frac{2\pi}{3} J^z} = e^{-i \frac{2\pi}{3} \times (\pm \frac{3}{2})} = e^{\mp i\pi} = -1
\]

\[
|J^z = +3/2\rangle \xrightarrow{\text{time reversal}} |J^z = -3/2\rangle
\]
More generally, ... 

- Also applies to 4f electron moments on pyrochlore

\[ J = \frac{3}{2}, \frac{9}{2}, \frac{15}{2}, \ldots \]

with the local crystal field Hamiltonian

\[ H_{cf} = 3B_2^0(J^z)^2 + \cdots \text{ if } B_2^0 < 0. \]

E.g. local doublet wavefunction of Dy\(^{3+}\) (\(J = \frac{15}{2}\)) in Dy\(_2\)Ti\(_2\)O\(_7\)

\[ |\phi_0^\pm\rangle = 0.981|\pm\frac{15}{2}\rangle \pm 0.190|\pm\frac{9}{2}\rangle - 0.022|\pm\frac{3}{2}\rangle \mp 0.037|\mp\frac{3}{2}\rangle + 0.005|\mp\frac{9}{2}\rangle \pm 0.001|\mp\frac{15}{2}\rangle \]
Emphasis: what matters is the wavefunction, not the spin value!

- may generally apply to any Kramers’ doublets with \( J > 1/2 \)!
  
e.g., Ce: \( \text{Ce}_2\text{Sn}_2\text{O}_7 \)

Candidate Quantum Spin Liquid in the Ce\(^{3+}\) Pyrochlore Stannate Ce\(_2\)Sn\(_2\)O\(_7\)

Romain Sibille,\(^1\,^*\) Elsa Lhotel,\(^2\) Vladimir Pomjakushin,\(^3\) Chris Baines,\(^4\) Tom Fennell,\(^3,^+\) and Michel Kenzelmann\(^1\)

4\(f\)\(^1\) ion in \(D_{3d}\) local symmetry to the susceptibility was realized between \( T = 1.8 \) and 370 K, and the resulting calculation of the single ion magnetic moment is shown in Fig. 2(c). The wave functions of the ground state Kramers doublet correspond to a linear combination of \( m_J = \pm 3/2 \) states. The fitted coefficients result in energy levels at 50 ±
Realistic **XYZ model**

and

**Symmetry Enriched** U(1) topological order
Symmetry properties

- Effective spin-1/2 under lattice symmetry

\[ T_d \times \mathcal{I} \times \text{translations} \quad \text{and} \quad T_d = \{ C_3, M \} \]

Important: \( S^x \) and \( S^z \) transform identically (as a dipole), while \( S^y \) transforms as an octupole moment under mirror.

\[
\begin{align*}
S^+ &= \frac{1}{2} \left| \frac{3}{2} \right> \left< \frac{3}{2} \right| - \frac{1}{2} \left| \frac{3}{2} \right> \left< -\frac{3}{2} \right| \\
S^- &= -\frac{1}{2} \left| \frac{3}{2} \right> \left< -\frac{3}{2} \right|, \quad S^+ = \left| \frac{3}{2} \right> \left< \frac{3}{2} \right|
\end{align*}
\]
Generic model: XYZ model

$$H = \sum_{\langle ij \rangle} J_{ij} S_i^z S_j^z + J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_{\alpha z} (S_i^x S_j^z + S_i^z S_j^x)$$

VS

$$H = \sum \{ J_{zz} S_i^z S_j^z - J_\pm (S_i^z S_j^\mp + S_i^\mp S_j^z)$$

$$+ J_{\pm \pm} (\gamma_{ij} S_i^+ S_j^+ + \gamma_{ij}^* S_i^- S_j^-)$$

$$+ J_{zz} [S_i^z (\zeta_{ij} S_j^+ + \zeta_{ij}^* S_j^-) + i \leftrightarrow j]\}$$
A small transformation into XYZ model

\[ H = \sum_{\langle ij \rangle} J_z S^z_i S^z_j + J_x S^x_i S^x_j + J_y S^y_i S^y_j \]
\[ + J_{xz} (S^x_i S^z_j + S^z_i S^x_j) \]

Rotation around the y axis in the effective spin space

\[ H_{XYZ} = \sum_{\langle ij \rangle} \tilde{J}_z \tilde{S}^z_i \tilde{S}^z_j + \tilde{J}_x \tilde{S}^x_i \tilde{S}^x_j + \tilde{J}_y \tilde{S}^y_i \tilde{S}^y_j \]

XYZ model
XXZ model can lead to U(1) QSL

\[ H = J_{zz} \sum_{\langle i,j \rangle} S^z_i S^z_j - J_\pm \sum_{\langle i,j \rangle} (S^+_i S^-_j + S^-_i S^+_j) + \cdots \]

- Pretty much one can add any term to create **quantum** tunneling, as long as it is not too large to induce magnetic order, the **ground state** is a U(1) QSL !

Hermele, Fisher, Balents, Moessner, Isakov, ....
Emergent Quantum Electrodynamics

Spinon

Emergent electric field

Emergent vector potential

\[ S^z \sim E \]

\[ S^\pm \sim e^{\pm iA} \]
XYZ model is the generic model that describes the interaction between DO doublets.

\[ H_{\text{XYZ}} = \sum_{\langle ij \rangle} J_x \tau^x_i \tau^x_j + J_y \tau^y_i \tau^y_j + J_z \tau^z_i \tau^z_j \]

Study phase on a cube: \(-1 \leq \tilde{J}_{x,y,z} \leq 1\).
Gapped phases w/ symmetry → SET and SPT phases

- There are LRE symmetric states → Symm. Enriched Topo. phases
  - 100s symm. spin liquid through the PSG of topo. excit.  Wen 02
  - 8 trans. symm. enriched $\mathbb{Z}_2$ topo. order in 2D, 256 in 3D  Kou-Wen 09
  - 1000,000s symm. $\mathbb{Z}_2$ spin liquid through $[\mathcal{H}^2(SG, \mathbb{Z}_2)]^2 \times$  Hermele 12
- Classify SET phases through $\mathcal{H}^3[SG \times GG, U(1)]$  Ran 12

- There are SRE symmetric states → many different phases
  We may call them symmetry protected trivial (SPT) phase

\[ \begin{align*}
\text{topological order (tensor category)} & \quad \text{LRE 1} \\
\text{LRE 2} & \quad \text{SRE} \\
\text{SET orders (tensor category w/ symmetry)} & \quad \text{SY–LRE 1} \quad \text{SY–LRE 2} \\
\text{symmetry breaking (group theory)} & \quad \text{SB–LRE 1} \quad \text{SB–LRE 2} \\
\text{SPT phases (group cohomology theory)} & \quad \text{SY–SRE 1} \quad \text{SY–SRE 2}
\end{align*} \]
• Control spinons in a quantum spin ice U(1) quantum spin liquid

Yao-Dong Li (Fudan)

arxiv 1607
Field-driven Higgs transition for octupolar U(1) QSL

How to tell if Ce2Sn2O7 is an octupolar U(1) QSL or not?

The idea to use a little knob that could simply lead to some clear experimental consequence, very much like the isotope effect of BCS superconductors.

Here we apply external magnetic field, and expect a field-driven Higgs transition to magnetic ordering as the field only couples to the matter field (spinons).

\[
H_{\text{sim}} = \sum_{\langle i,j \rangle} J_y \tau_i^y \tau_j^y - J_\pm (\tau_i^+ \tau_j^- + \text{h.c.}) - \sum_i h (\hat{n}_i \cdot \hat{z}_i) \tau_i^z,
\]

\[\tau_i^\pm = \tau_i^z \pm i \tau_i^x\]
Inelastic neutron scattering and spinon continuum

In a quantum spin liquid, the elementary spin excitations are fractional, S=1/2 spinons.

Most of the information is in the continuum!

Neutron spinon S=1/2

$k, \omega$ magnon S=1

Spinon continuum in YbMgGaO$_4$ (today’s arXiv)

Calculation

$E = 1.2$ meV
$T = 70$ mK
Lower excitation edge

\[ q = k_1 + k_2, \]
\[ \Omega(q) = \omega_i(k_1) + \omega_j(k_2), \]
Neutron scattering and thermal transport

<table>
<thead>
<tr>
<th>Different U(1) QSLs</th>
<th>Heat capacity</th>
<th>Inelastic neutron scattering measurement</th>
</tr>
</thead>
<tbody>
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<td>Octupolar U(1) QSL for DO doublets</td>
<td>$C_v \sim T^3$</td>
<td>Gapped spinon continuum</td>
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</tr>
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<td>Dipolar U(1) QSL for usual Kramers’ doublets</td>
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</tr>
</tbody>
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Thermal transport

see both contribution, but there is a big separation of energy scales in spinon and gapless photons.
Material survey: other DO doublet systems

Our doublet can potentially be realized for any Kramers spin moment with $J > \frac{1}{2}$.

Two well-known systems:

- Pyrochlores $A_2B_2O_7$, e.g., $\text{Nd}_2\text{Ir}_2\text{O}_7$, $\text{Nd}_2\text{Sn}_2\text{O}_7$, $\text{Nd}_2\text{Zr}_2\text{O}_7$, etc $\text{Dy}_2\text{Ti}_2\text{O}_7$, $\text{Cd}_2\text{Os}_2\text{O}_7$, etc $\text{Ce}_2\text{Sn}_2\text{O}_7$,

- Spinels $A_2B_2X_4$, $B=$lanthanide? e.g. $\text{CdEr}_2\text{Se}_4$ $\text{CdYb}_2\text{S}_4$
Conclusion

• We propose a new doublet dubbed “dipole-octupole” doublet.

• We propose a generic XYZ model for our new doublet.

• This XYZ model supports both exotic (octupolar) order and symmetry enriched U(1) quantum spin liquid (quantum spin ice) ground states.

• There exist a large class of materials (not just pyrochlore, any other lattices with the same point group) that can support such doublets.

• The remarkable properties of the doublet allows a direct comparison between numerics and experiments. We propose a way to detect the consequence of symmetry enrichment.