

Wilson Ratio enhancement in a quantum spin liquid candidate: $\text{Na}_4\text{Ir}_3\text{O}_8$ (hyperkagome)

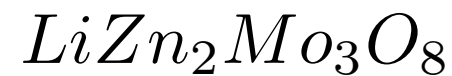
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Collaborator: Prof. Yong-Baek Kim
(University of Toronto)

Acknowledge Dr. Perry and Prof. Takagi
for sharing their experimental results

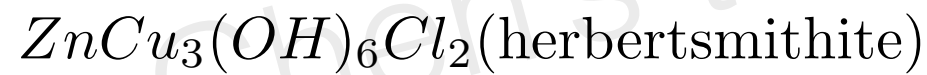
Quantum spin liquid candidates

Triangle $\kappa - (ET)_2Cu_2(CN)_3$, $EtMe_3Sb[Pd(dmit)_2]_2$,



$He - 3$ on graphite layer

Kagome $Cu_3Zn(OH)_6Cl_2$ (kapellasite),



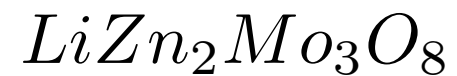
FCC Ba_2YMoO_6

Pyrochlore $some R_2TM_2O_7$ R=rare earth,
TM=transition metal Quantum spin ice

Hyperkagome $Na_4Ir_3O_8$

Quantum spin liquid candidates

Triangle $\kappa - (ET)_2Cu_2(CN)_3, EtMe_3Sb[Pd(dmit)_2]_2,$



He – 3 on graphite layer

Kagome $Cu_3Zn(OH)_6Cl_2$ (kapellasite),
 $BaCu_3V_2O_8(OH)_2$ (vesignieite),
 $ZnCu_3(OH)_6Cl_2$ (herbertsmithite)

FCC



Pyrochlore

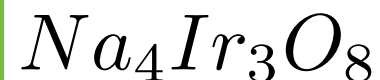
some



R=rare earth,
TM=transition metal

Quantum spin ice

Hyperkagome



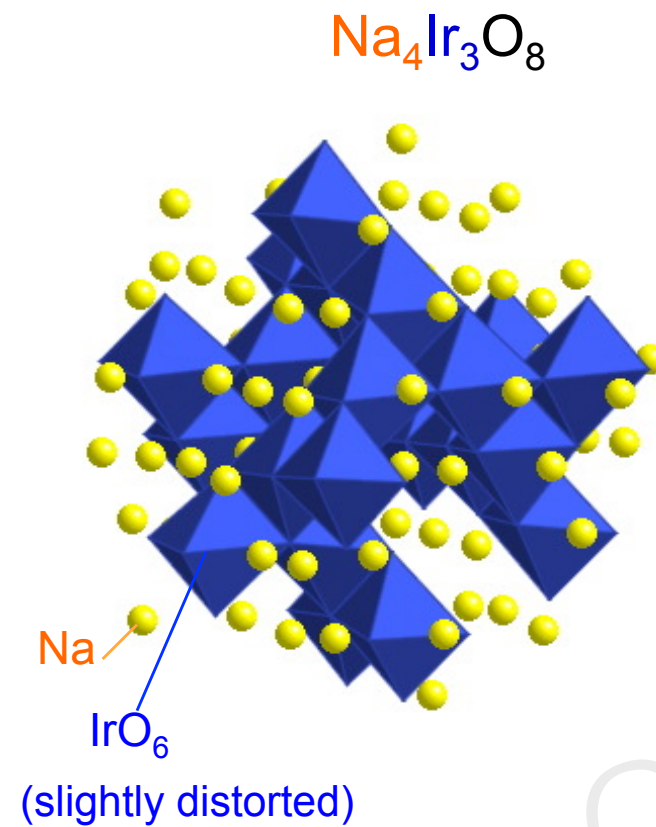
4d, 5d and f electrons: Spin-orbit coupling is expected to be important, and may lead to some new physics.

Outline

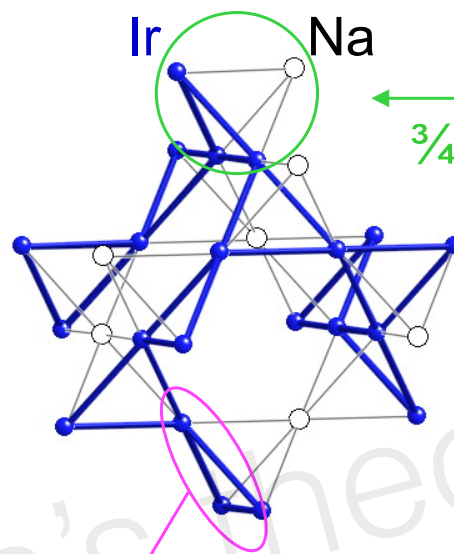
Part 1. Review experiments and current theories

Part 2. Present a possible explanation for the experiments

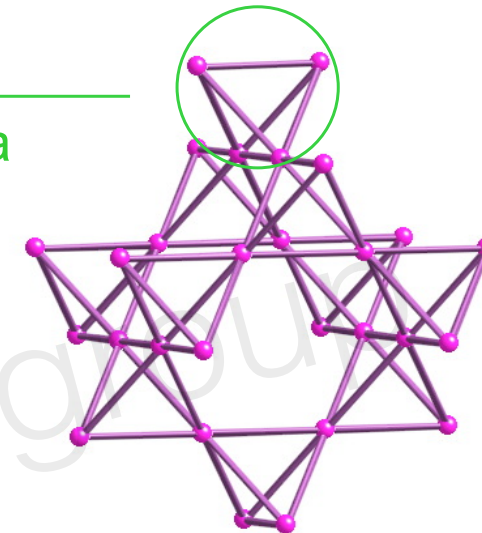
$\text{Na}_4\text{Ir}_3\text{O}_8$: a hyperkagome Ir sublattice



hyperkagome

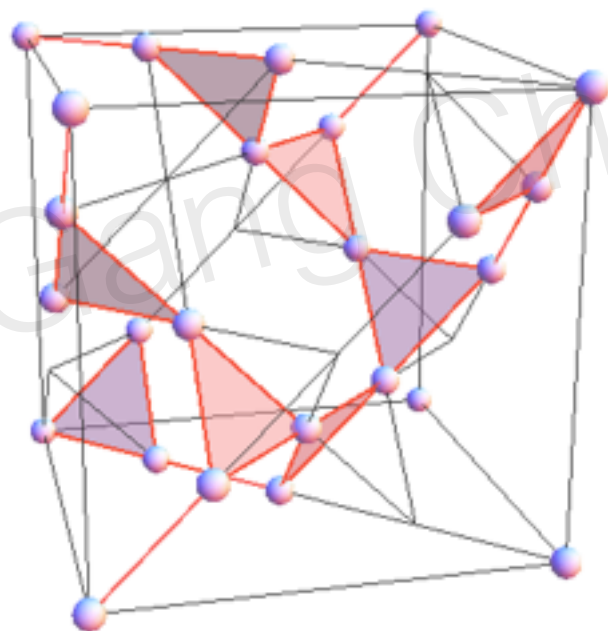


pyrochlore



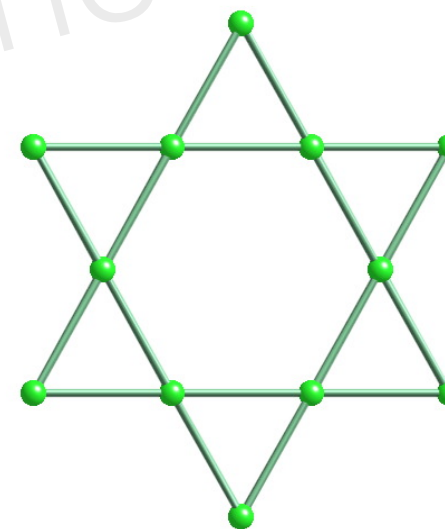
all Ir-Ir bonds: equivalent

hyperkagome



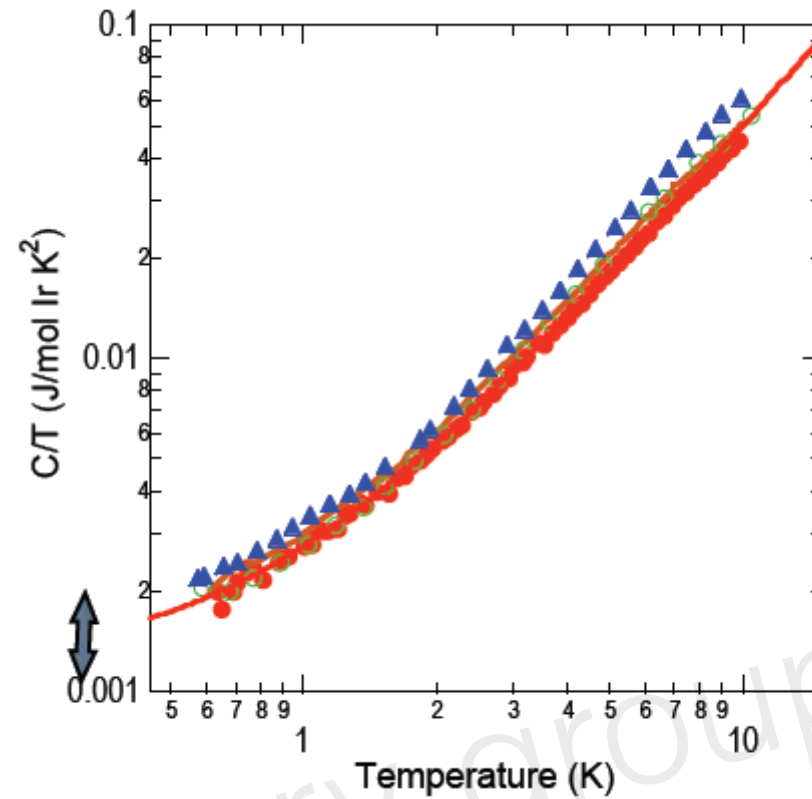
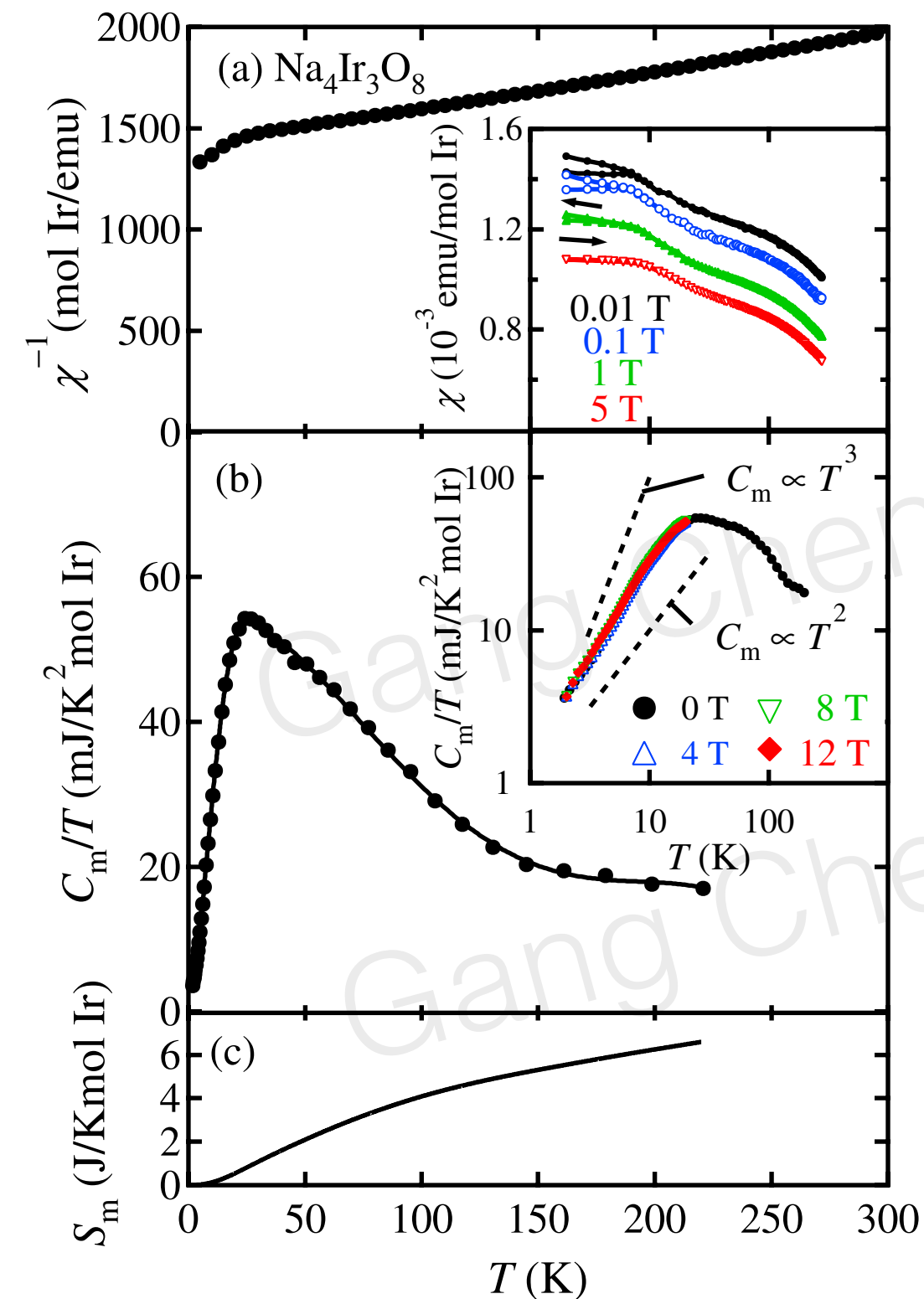
12 sites per unit cell

kagome



3 sites per unit cell

Polycrystal sample



From
Prof. Takagi's talk

Curie-Weiss fit:

$$\mu_{eff} = 1.96\mu_B \quad \text{close to spin-1/2}$$

$$\Theta_{CW} = -650\text{K}$$

No indication of ordering down to 2K from C_v and χ
NMR measurement confirms the absence of magnetic
ordering. (See Prof. Takagi's talk on Monday)

$$\chi|_{T \rightarrow 0} = \text{constant}$$

$$C_v/T|_{T \rightarrow 0} = \text{constant}$$

Very large Wilson Ratio, 35!

Y. Okamoto, et al, H. Takagi

Phys. Rev. Lett. **99**, 137207 (2007)

Wilson Ratios of some QSL candidates

Table 1 Some experimental materials studied in the search for QSLs					
Material	Lattice	S	Θ_{CW} (K)	R^*	Status or explanation
κ -(BEDT-TTF) $_2$ Cu $_2$ (CN) $_3$	Triangular †	$\frac{1}{2}$	-375 ‡	1.8	Possible QSL
EtMe $_3$ Sb[Pd(dmit) $_2$] $_2$	Triangular †	$\frac{1}{2}$	-(375-325) ‡	? ~1.0-3.0	Possible QSL
Cu $_3$ V $_2$ O $_7$ (OH) $_2$ •2H $_2$ O (volborthite)	Kagomé †	$\frac{1}{2}$	-115	6	Magnetic
ZnCu $_3$ (OH) $_6$ Cl $_2$ (herbertsmithite)	Kagomé	$\frac{1}{2}$	-241	?	Possible QSL
BaCu $_3$ V $_2$ O $_8$ (OH) $_2$ (vesignieite)	Kagomé †	$\frac{1}{2}$	-77	4	Possible QSL
Na $_4$ Ir $_3$ O $_8$	Hyperkagomé	$\frac{1}{2}$	-650	70 30-40	Possible QSL
Cs $_2$ CuCl $_4$	Triangular †	$\frac{1}{2}$	-4	0	Dimensional reduction
FeSc $_2$ S $_4$	Diamond	2	-45	230	Quantum criticality

BEDT-TTF, bis(ethylenedithio)-tetrathiafulvalene; dmit, 1,3-dithiole-2-thione-4,5-dithiolate; Et, ethyl; Me, methyl. * R is the Wilson ratio, which is defined in equation (1) in the main text. For EtMe $_3$ Sb[Pd(dmit) $_2$] $_2$ and ZnCu $_3$ (OH) $_6$ Cl $_2$, experimental data for the intrinsic low-temperature specific heat are not available, hence R is not determined. † Some degree of spatial anisotropy is present, implying that $J' \neq J$ in Fig. 1a. ‡ A theoretical Curie-Weiss temperature (Θ_{CW}) calculated from the high-temperature expansion for an $S = \frac{1}{2}$ triangular lattice; $\Theta_{\text{CW}} = 3J/2k_B$, using the J fitted to experiment.

Wilson Ratio quantifies spin fluctuations that enhance the susceptibility.

Basic physics in Na $_4$ Ir $_3$ O $_8$

- Fermi gas $W = 1$
- He-3 (almost localized fermi liquid) $W = 4$
- Fe-Superconductor (Fe $_{1.04}$ Te $_{0.67}$ Se $_{0.33}$) $W = 5.7$

- Strong spin-orbit coupling (Z=77)
- Multi-orbital bands, 3 t $_2$ g orbitals
- Close to metal-insulator transition (true for almost all iridates under current investigation)

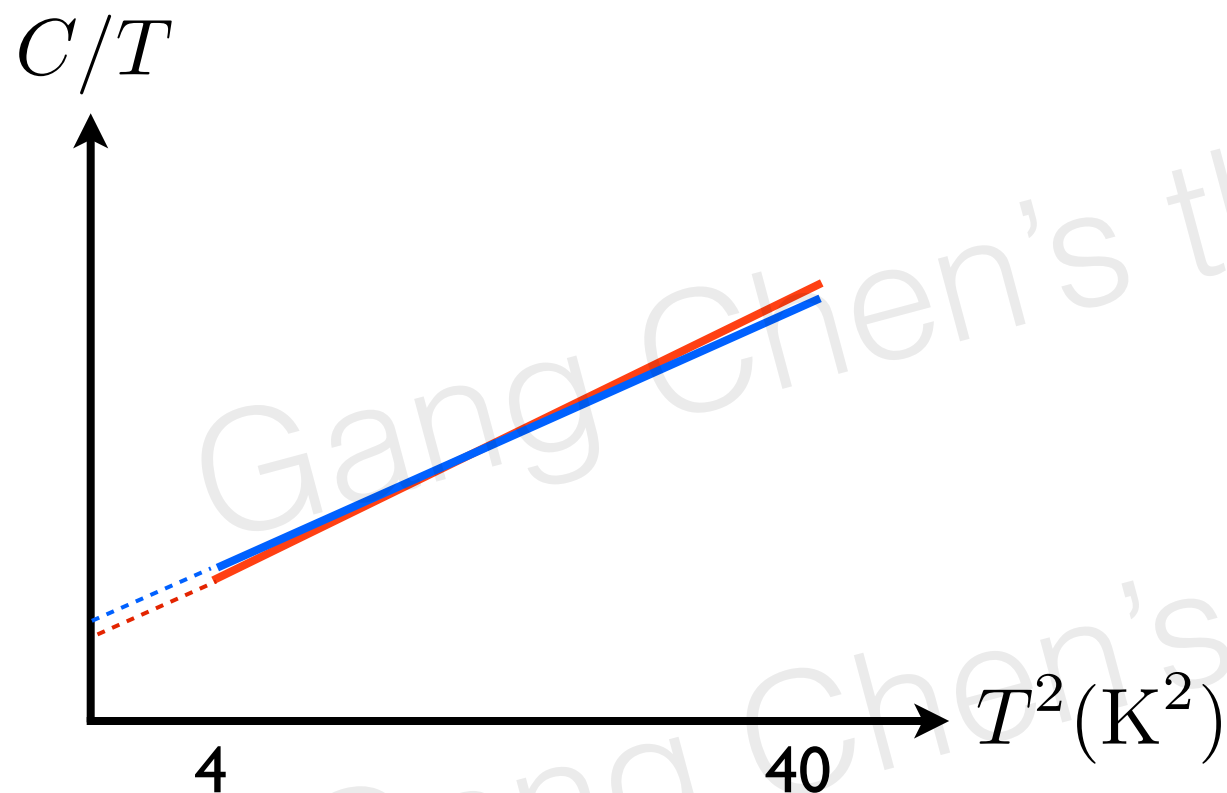
With SOC, spin-rotational symmetry is broken, large Wilson Ratio is certainly possible, e.g. ordered AFMagnet with gapped spin-wave excitations

$W = \infty$

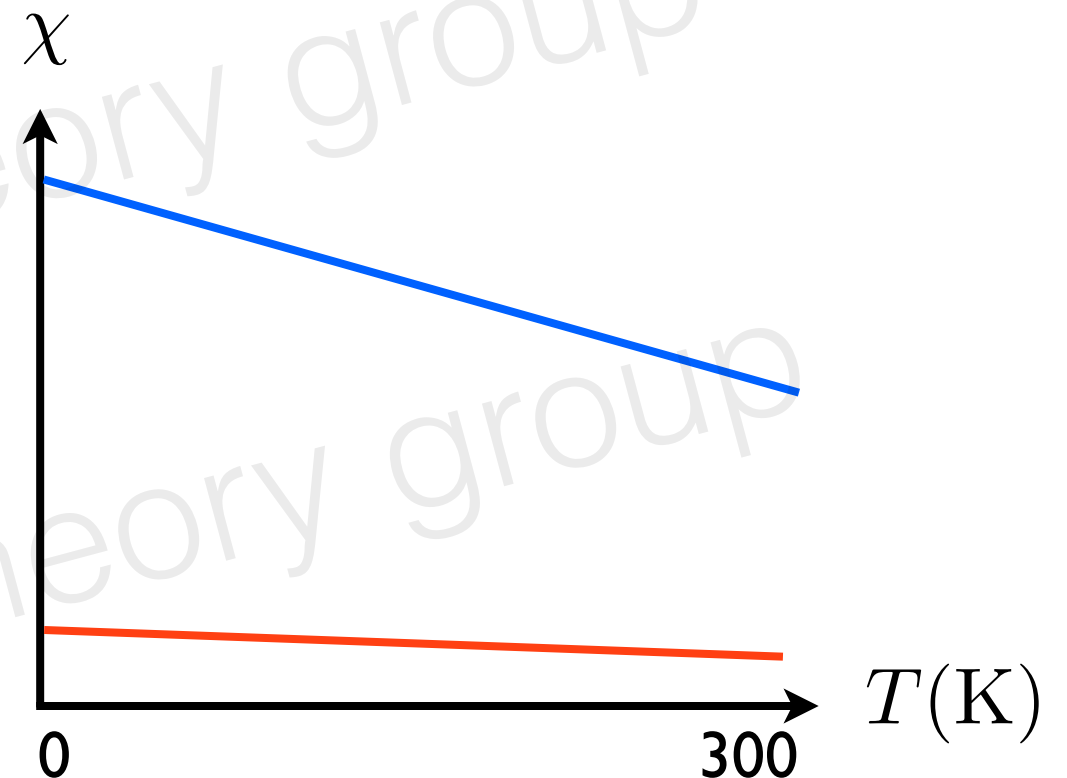
L. Balents, Nature **464**, 199 (2010)
G. Chen, et al, Phys. Rev. Lett. **102**, 096406 (2009)
D. Vollhardt, Rev. Mod. Phys. **56**, 99 (1984)
J. Yang, et al, JPSJ, **79**, 074704, (2010)

New data: **schematic** plots

- Single-crystal metallic sample (R. Perry, et al, unpublished, Prof. Takagi's group)
- Polycrystal insulating sample (Okamoto, et al, PRL 2007)



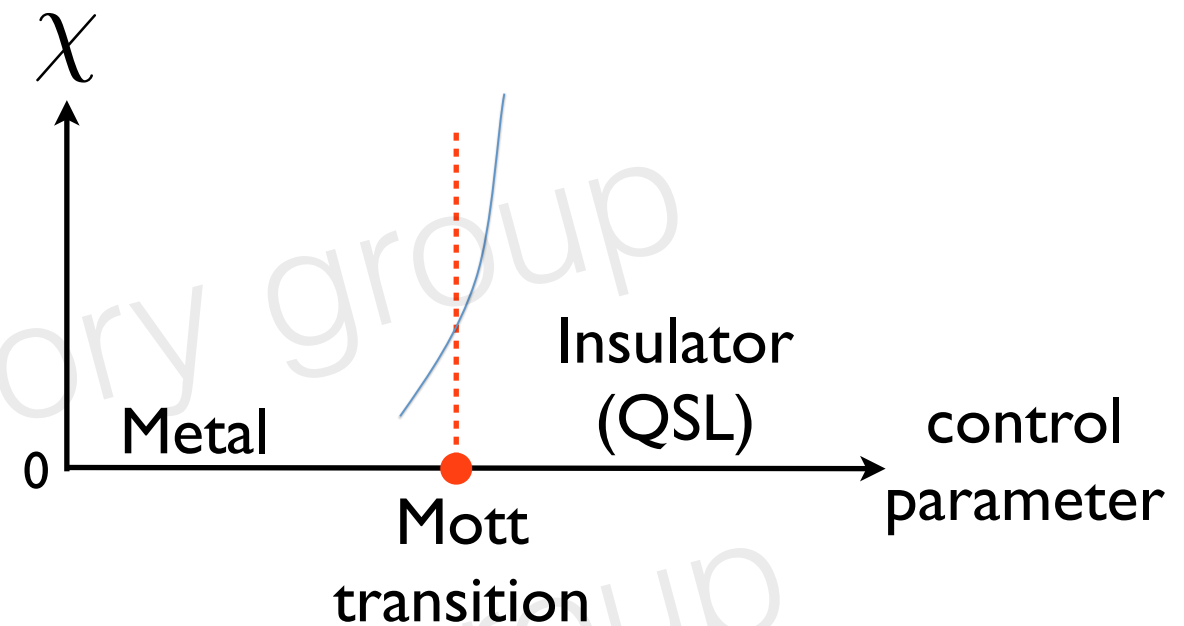
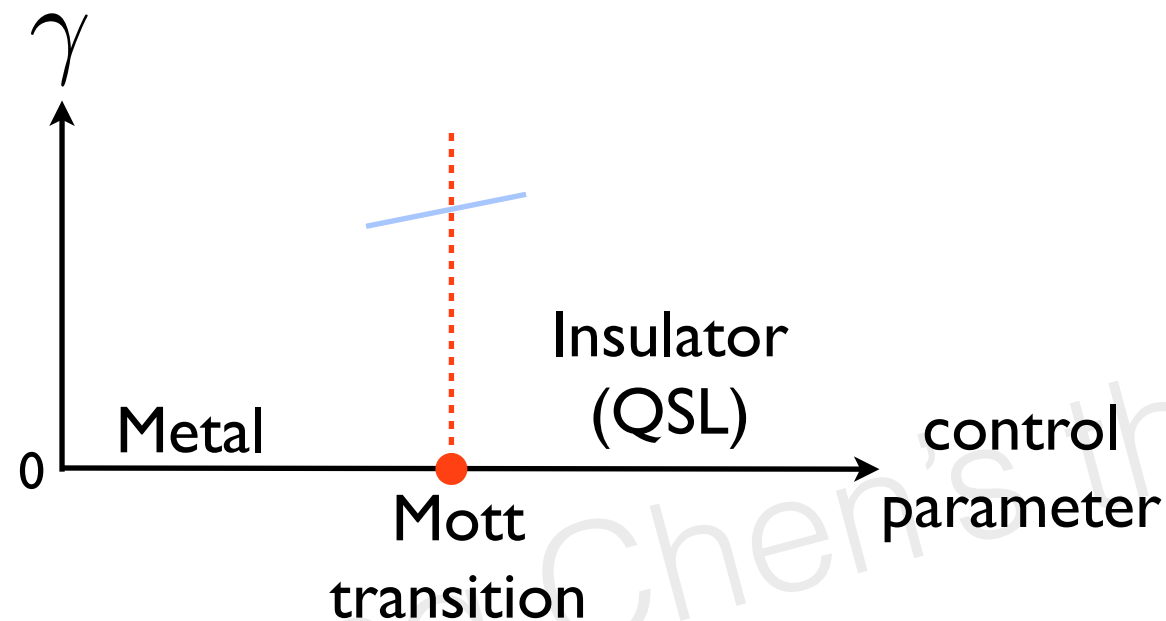
Small change ($\sim 18\%$) in linear-T heat capacity



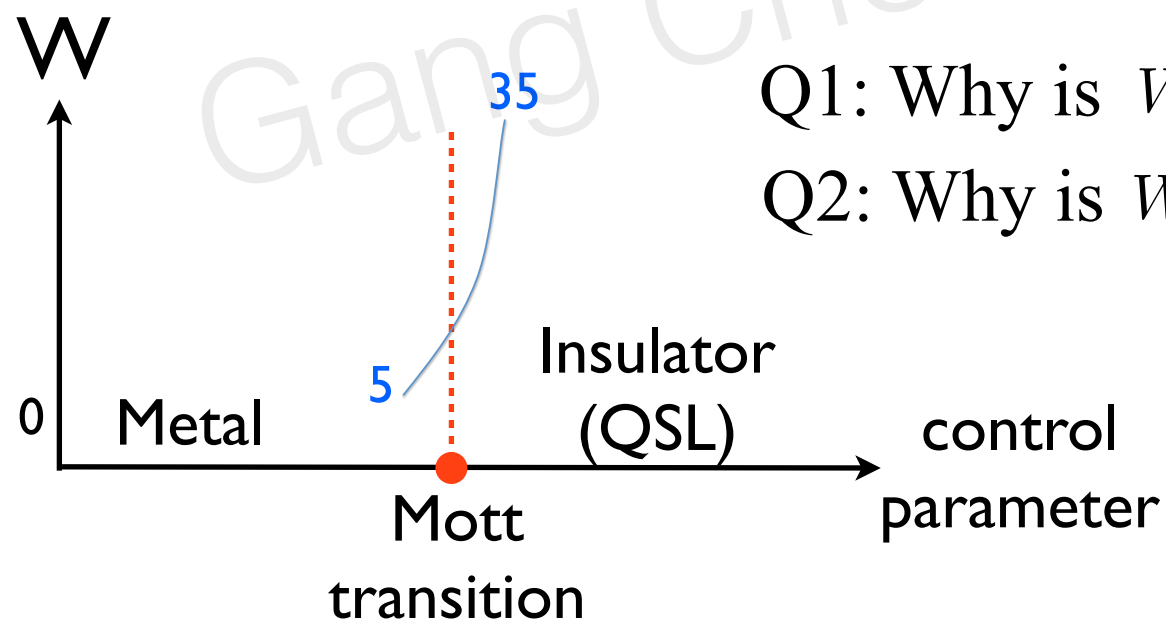
Large enhancement of magnetic susceptibility.
Susceptibility increases with resistivity
(several other single-crystal samples)

Summary of the experiments: **schematic** plots

$$\gamma \equiv \frac{C_v}{T} \Big|_{T \rightarrow 0} \quad \text{Control parameter: carrier concentration? chemical pressure? etc?}$$



Wilson Ratio $W \equiv \frac{\pi^2}{3} \frac{\chi / \mu_B^2}{\gamma / k_B^2}$



Q1: Why is W (or χ) enhanced in the insulating phase?

Q2: Why is W (or χ) so sensitive to Mott transition?

Current theoretical work on $\text{Na}_4\text{Ir}_3\text{O}_8$

U(1) QSL M. Lawler, et al Phys. Rev. Lett. **101**, 197202 (2008)

Z_2 QSL M. Lawler, et al Phys. Rev. Lett. **100**, 227201 (2008)

Y. Zhou, et al Phys. Rev. Lett. **101**, 197201 (2008)

VBS E. J. Bergholtz, et al Phys. Rev. Lett. **105**, 237202 (2010)

Other works focus on various other things

John M. Hopkinson, et al Phys. Rev. Lett. **99**, 037201, (2007)

G. Chen, et al Phys. Rev. B **78**, 094403, (2008)

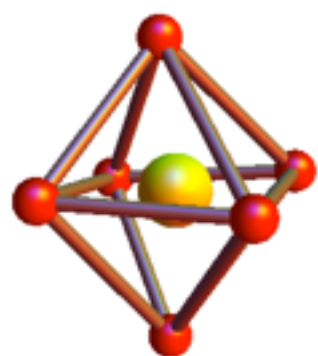
D. Podolsky, et al Phys. Rev. Lett. **102**, 186401, (2009)

T. Micklitz, et al Phys. Rev. B **81**, 174417, (2010)

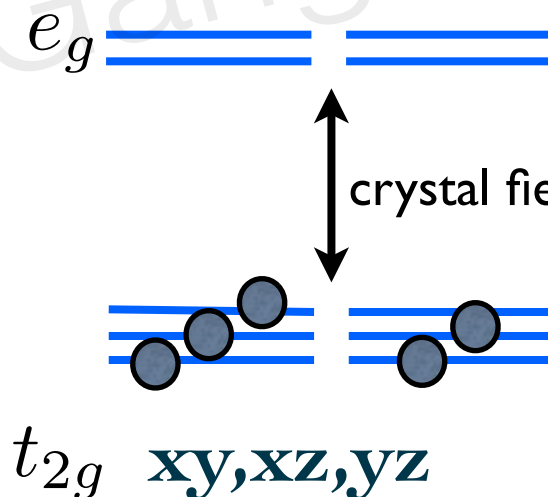
M. R. Norman, et al Phys. Rev. B **81**, 024428, (2010)

D. Podolsky, et al Phys. Rev. B **83**, 054401, (2011)

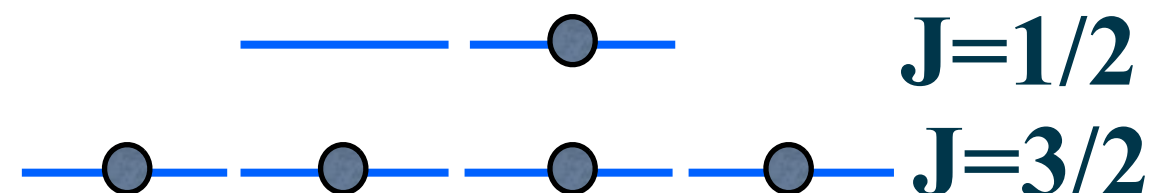
Formation of local moment in the strong Mott regime



IrO_6



SOC



Theoretical proposals

U(1) QSL? M. Lawler, et al Phys. Rev. Lett. **101**, 197202 (2008)

Spinon fermi surface: (nearly) linear-T C_v , constant χ (Heisenberg model)

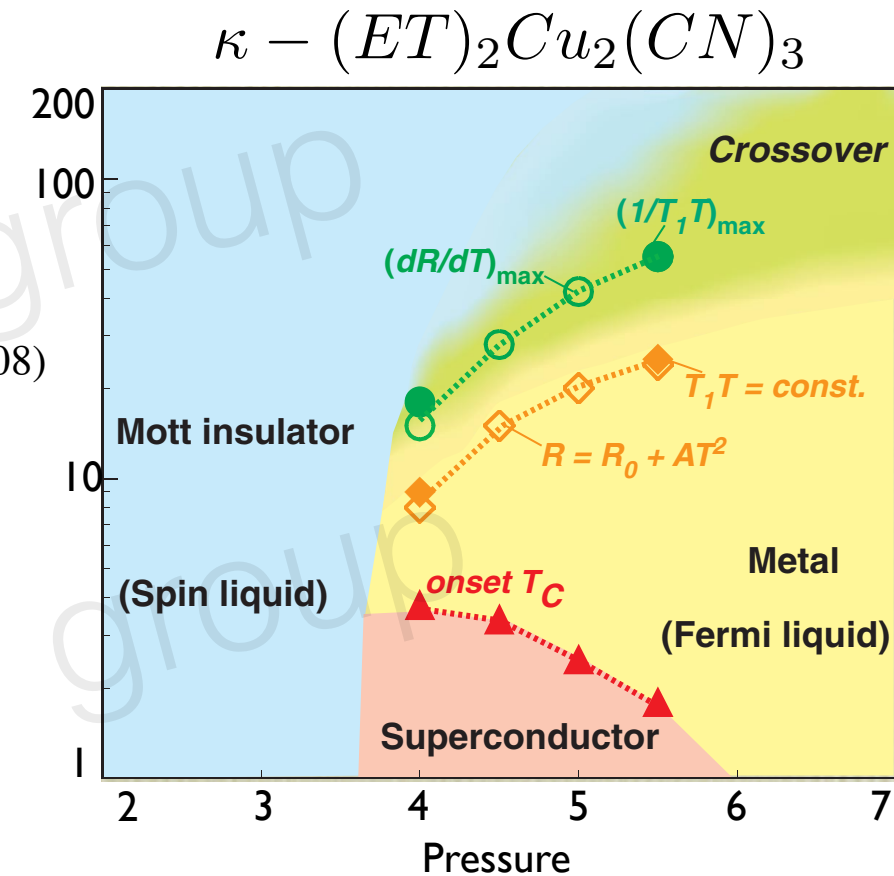
If other interactions are included to break spin-rotational symmetry, large W might be obtained for this state.

Z₂ QSL (less likely) Y. Zhou, et al Phys. Rev. Lett. **101**, 197201 (2008)

Suppress C_v by spinon pairing to enhance W (interesting)

Explain the susceptibility remaining constant by large SOC $\lambda \gg \Delta$

Expect suppressed C_v from metal to QSL, and also superconductivity in metallic side just like kappa-ET organics



K. Kanoda's group 2003-

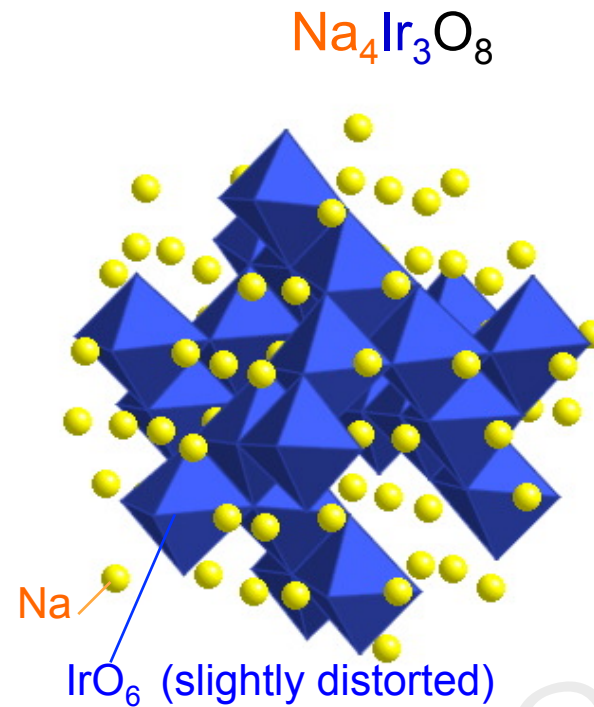
VBS (less likely) E. J. Bergholtz, et al Phys. Rev. Lett. **105**, 237202 (2010)

Similar series expansion like Huse+Singh's work on kagome

Complicated ground state: 72 sites in one cell

a bit hard to explain power-law C_v and constant χ over a large temperature range

Extended Hubbard Model



$$\mathcal{H} = \mathcal{H}_{hop} + \mathcal{H}_{soc} + \mathcal{H}_{ion} + \mathcal{H}_{int}$$

\mathcal{H}_{hop} - Tight-binding model

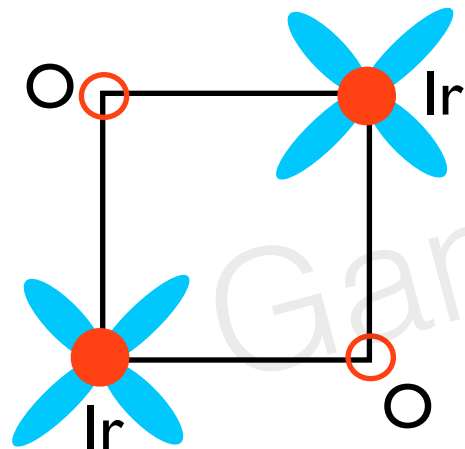
\mathcal{H}_{soc} - Atomic spin-orbit coupling

\mathcal{H}_{ion} - single-ion (crystal field) term due to IrO₆ distortion
(drive transition from TBI to metal in 227 iridates)

\mathcal{H}_{int} - Multiorbital interactions

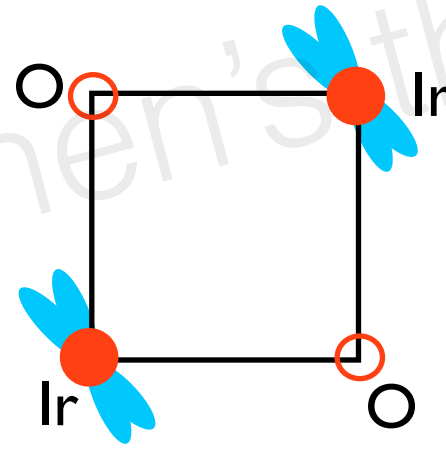
Tight-binding model

σ - bonding



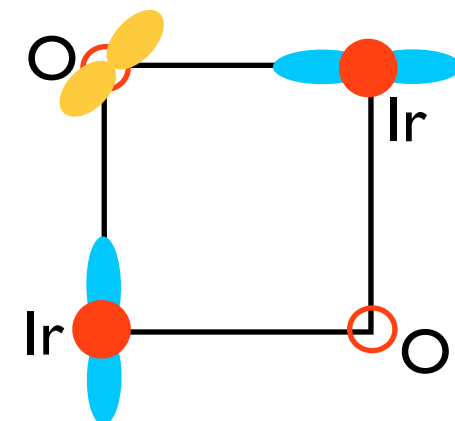
t_σ

π - bonding



t_π

indirect hop through oxygen



t_2

$$t_\sigma = 1, t_\pi = 0.2, t_2 = 0.5$$

No band insulator

T. Micklitz, et al

Phys. Rev. B **81**, 174417, (2010)

M. R. Norman, et al

Phys. Rev. B **81**, 024428, (2010)

Wilson ratio for the non-interacting case

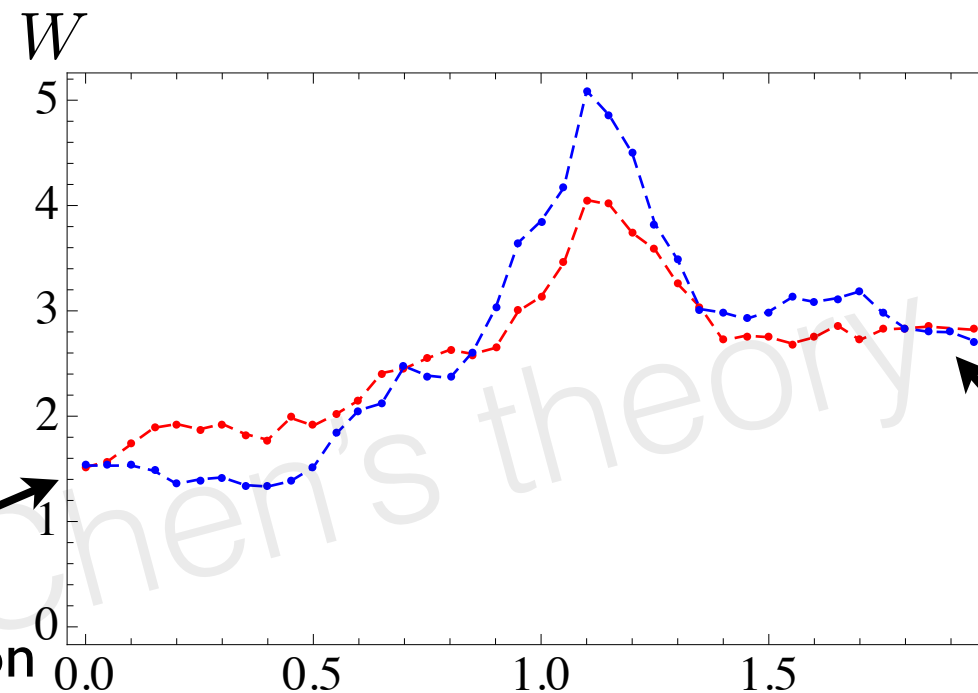
$$\mathcal{H}_0 = \mathcal{H}_{hop} + \mathcal{H}_{soc} + \mathcal{H}_{ion}$$

$$t_\sigma = 1, t_\pi = 0.2, t_2 = 0.5$$

$$\mathbf{M}_i \equiv \mu_B (\mathbf{L}_i + 2\mathbf{S}_i)$$

$$W \neq 1$$

is because of the hybridization of different orbitals



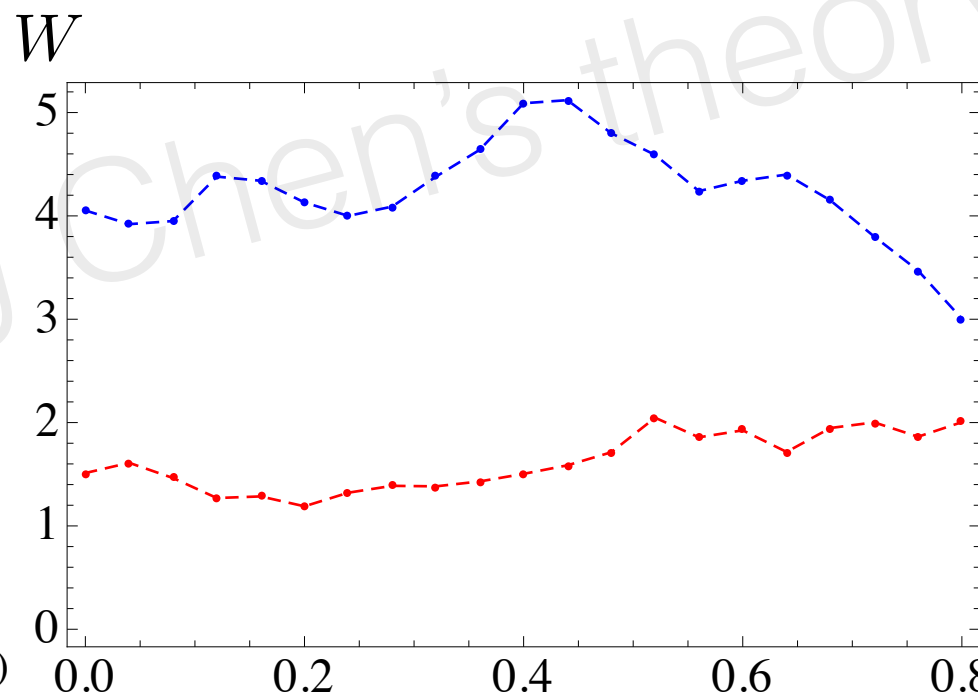
two anisotropic parameters

$$D = 0$$

$$D = 0.4$$

Same reason why Heisenberg model is relevant for Sr_2IrO_4

$$\lambda/t_\sigma$$



$$\lambda = 0$$

$$\lambda = 1.1$$

$$D/t_\sigma$$

G. Chen, et al Phys. Rev. B **78**, 094403, (2008)

G. Jackeli, et al Phys. Rev. Lett. **102**, 017205, (2009)

F. Wang, et al Phys. Rev. Lett. **106**, 136402, (2011)

Multi-orbital interactions

$$H_{int} = U \sum_{i,m} \hat{n}_{i,m,\uparrow} \hat{n}_{i,m,\downarrow} + \frac{U'}{2} \sum_{i,m \neq m'} \hat{n}_{i,m} \hat{n}_{i,m'}$$

i is a position index.
 m is an orbital index.

$$+ \frac{J}{2} \sum_{i,m \neq m'} d_{im\sigma}^\dagger d_{im'\sigma}^\dagger d_{im\sigma'} d_{im'\sigma} + \frac{J'}{2} \sum_{i,m \neq m'} d_{im\uparrow}^\dagger d_{im\downarrow}^\dagger d_{im'\downarrow} d_{im'\uparrow}$$

In atomic limit,

$$U = U' + J + J'$$

$$J = J'$$

Rewrite interaction, $\mathcal{H}_{int} = \mathcal{H}_{c-int} + \mathcal{H}_{ex-int}$

$$\mathcal{H}_{c-int} = \frac{U}{2} \sum_i (\hat{n}_i - 5)^2$$

$$\mathcal{H}_{ex-int} = -J \sum_{i,m \neq m'} \hat{n}_{i,m} \hat{n}_{i,m'} + \frac{J}{2} \sum_{i,m \neq m'} d_{im\sigma}^\dagger d_{im'\sigma}^\dagger d_{im\sigma'} d_{im'\sigma}$$

$$+ \frac{J}{2} \sum_{i,m \neq m'} d_{im\uparrow}^\dagger d_{im\downarrow}^\dagger d_{im'\downarrow} d_{im'\uparrow}$$

U is the energy scale for excessive electron/charge occupation.

J is the energy scale for electron distribution among different spin and orbital states.

\mathcal{H}_{ex-int} is like an onsite exchange interaction in the *Kugel-Khomskii* picture.

Strong coupling mean field: slave-rotor theory

$$\mathcal{H} = \mathcal{H}_{hop} + \mathcal{H}_{soc} + \mathcal{H}_{ion} + \mathcal{H}_{c-int}$$

Original electron Hamiltonian

$$H_{hop} = \sum_{Rim, R'i'm'} t_{mm'}^{ii'} d_{im\sigma}^\dagger(R) d_{im'\sigma}(R') + h.c.$$

$$H_{c-int} = \frac{U}{2} \sum_{Ri} \left(\sum_{m,\alpha} d_{im\alpha}^\dagger(R) d_{im\alpha}(R) - 5 \right)^2$$

$$H_{ion} = D \sum_{Ri\alpha} (L_i^\mu)_{mn}^2 d_{im\alpha}^\dagger(R) d_{in\alpha}(R)$$

$$H_{soc} = \frac{\lambda}{2} \sum_{Ri} \mathbf{L}_{mn} \cdot \boldsymbol{\sigma}_{\alpha\beta} d_{im\alpha}^\dagger(R) d_{in\beta}(R)$$

Slave-rotor approach to obtain fermionic spinons
(see Prof. Senthil's talk)

$$d_{im\alpha} = e^{-i\theta_i} f_{im\alpha}$$

$$L_i(R) = \sum_{m\sigma} f_{im\sigma}^\dagger(R) f_{im\sigma}(R) - 5$$

$$[\theta_i, L_i] = i$$

Slave-rotor mean field Hamiltonian

$$H_f = Q_f \sum_{Rim, R'i'm'} (t_{mm'}^{ii'} f_{im\sigma}^\dagger(R) f_{im'\sigma}(R') + h.c.)$$

$$+ \frac{\lambda}{2} \sum_{Ri} \mathbf{L}_{mn} \cdot \boldsymbol{\sigma}_{\alpha\beta} f_{im\alpha}^\dagger(R) f_{in\beta}(R) + D \sum_{Ri\alpha} (L_i^\mu)_{mn}^2 f_{im\alpha}^\dagger(R) f_{in\alpha}(R)$$

$$H_L = \frac{U}{2} \sum_{Ri} L_i^2(R) + \sum_{Ri} (hL_i(R) + 5h) + Q_r \sum_{Ri, R'i'} e^{i\theta_i(R) - i\theta_{i'}(R')} + h.c.$$

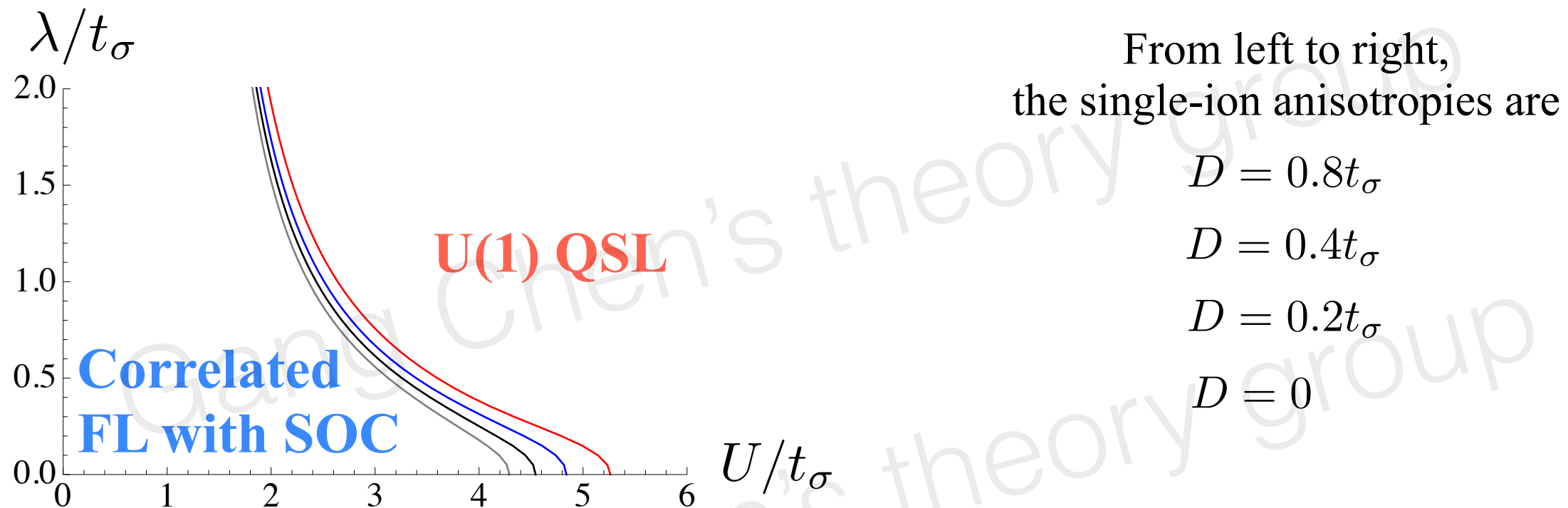
R is unit cell index
 i is sublattice index

$$Q_f \equiv \langle e^{i\theta_i(R) - i\theta_{i'}(R')} \rangle_\theta \quad Q_r \equiv \sum_{mm'\sigma} t_{mm'} \langle f_{im\sigma}^\dagger f_{i'm'\sigma}(R) \rangle_f$$

Slave-rotor phase diagram

$\langle e^{-i\theta_i} \rangle \neq 0$, $Z \neq 0$, spin and charge are confined, we have a “correlated FL”.

$\langle e^{-i\theta_i} \rangle = 0$, $Z = 0$, we have a “U(1) QSL”.



Three energy scales: SOC, correlation, bandwidth

Two observations (also see Prof. Balents' talk):

1. SOC enhances correlation effects. \longrightarrow Strong correlation physics may be seen in 4d/5d electron system
2. Correlation effects enhance SOC. \longrightarrow SOC may be also important even in 3d electron system in certain cases: FeSc_2S_4 , ZnV_2O_4 , etc

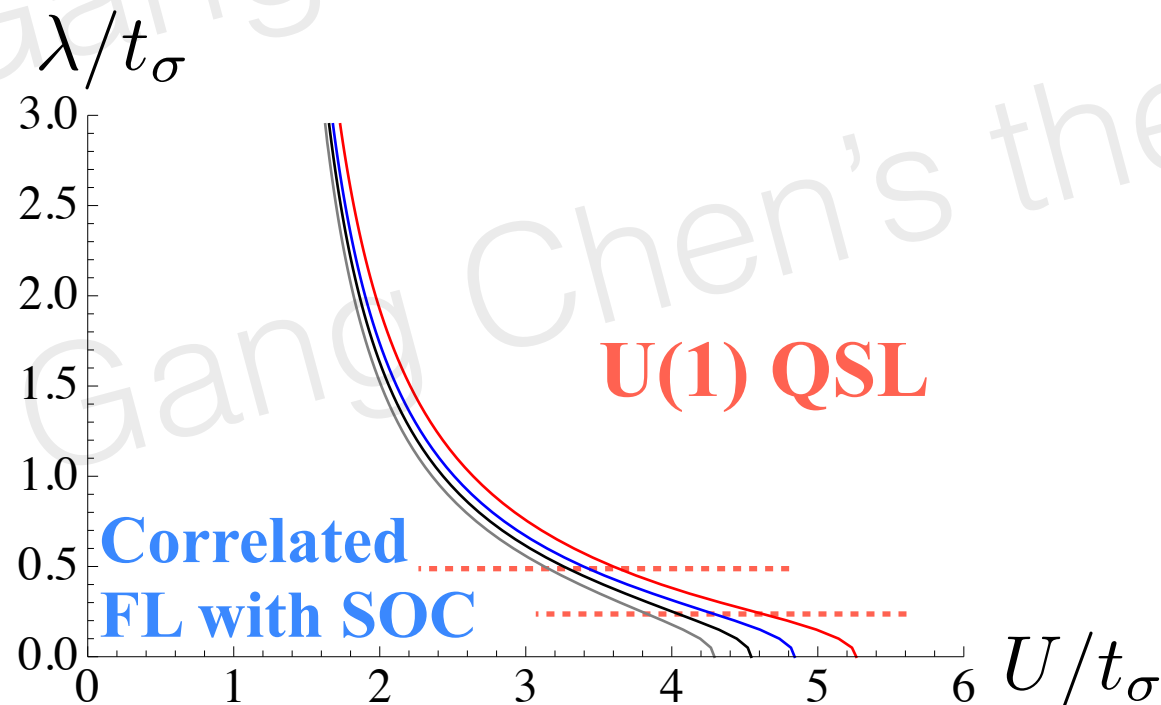
Onsite exchange interaction

We put the onsite exchange interaction in the spinon mean field hamiltonian.

W. Ko, P.A. Lee, Phys. Rev. B. **83**, 134515 (2011)

$$H_{ex-int} = \sum_i \left[-J \sum_{m \neq m'} f_{im\sigma}^\dagger f_{im\sigma} f_{im'\sigma'}^\dagger f_{im'\sigma'} + \frac{J}{2} \sum_{m \neq m'} f_{im\sigma}^\dagger f_{im'\sigma'}^\dagger f_{im\sigma'} f_{im'\sigma} \right. \\ \left. + \frac{J}{2} \sum_{m \neq m'} f_{im\uparrow}^\dagger f_{im\downarrow}^\dagger f_{im'\downarrow} f_{im'\uparrow} \right]$$

$$H_f \rightarrow H_f + H_{ex-int}$$

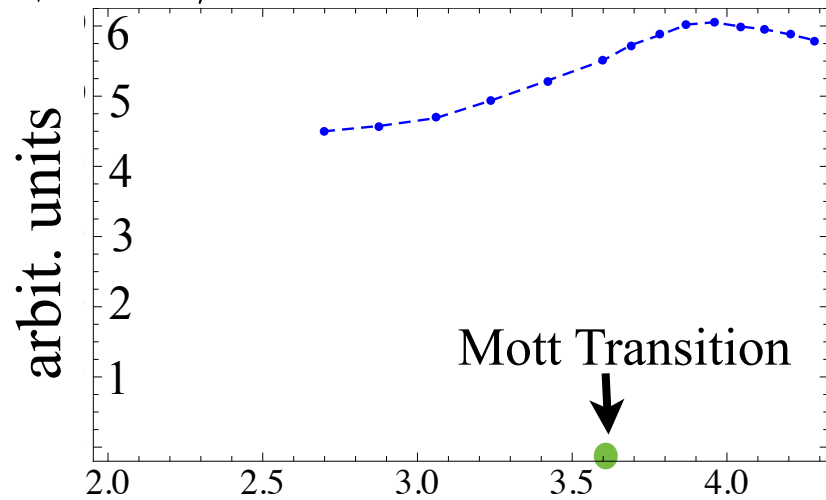


$$\mathbf{M}_i \equiv \mu_B (\mathbf{L}_i + 2\mathbf{S}_i)$$

Study Wilson ratio
along the dashed line

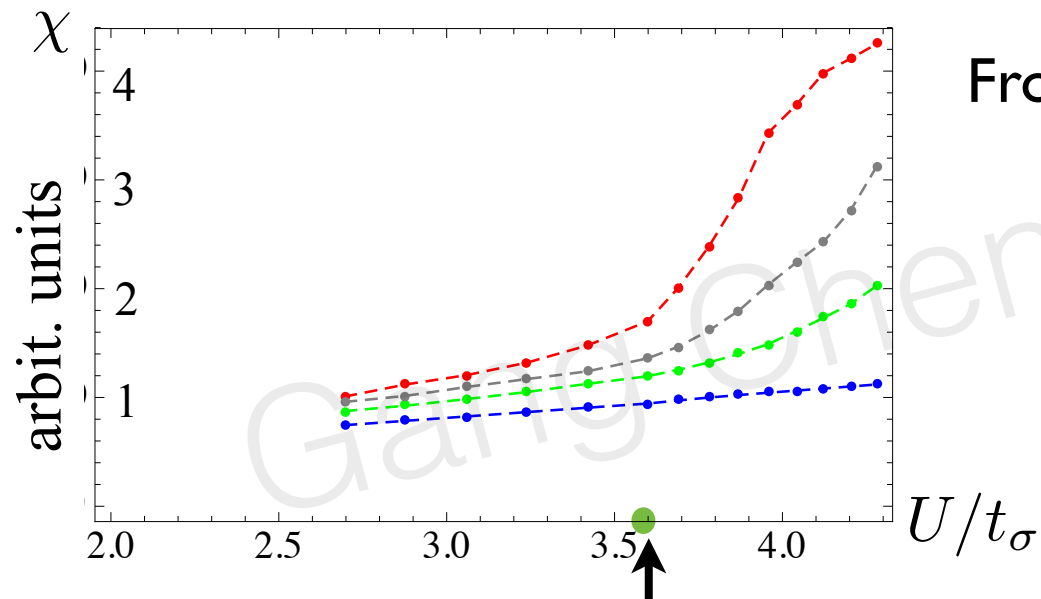
$$\lambda = 0.5t_\sigma$$

$$\gamma \equiv C_v/T$$



Both “effective mass”
and fermi surfaces are
changed due to SOC

$$J \lesssim 0.1U_c$$



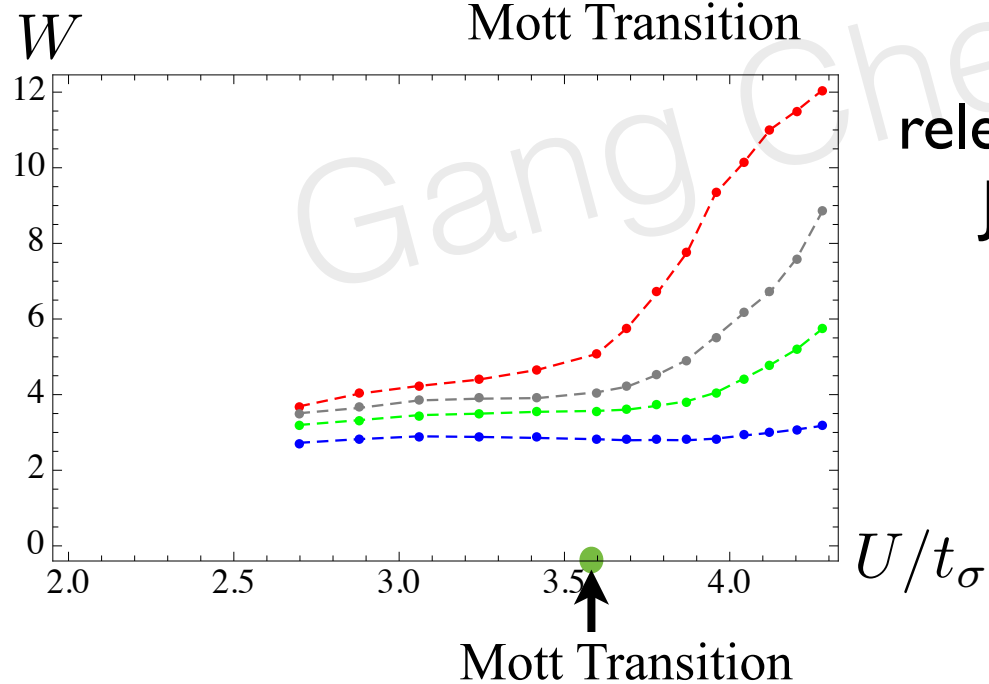
From bottom to top,

$$J = 0$$

$$J = 0.2t_\sigma$$

$$J = 0.3t_\sigma$$

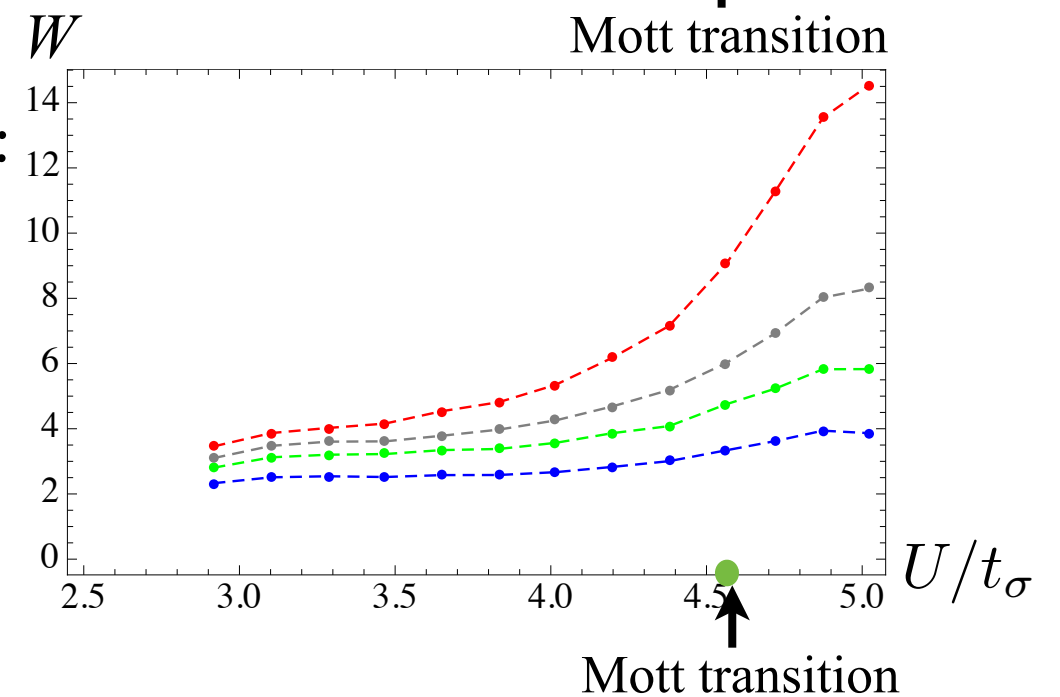
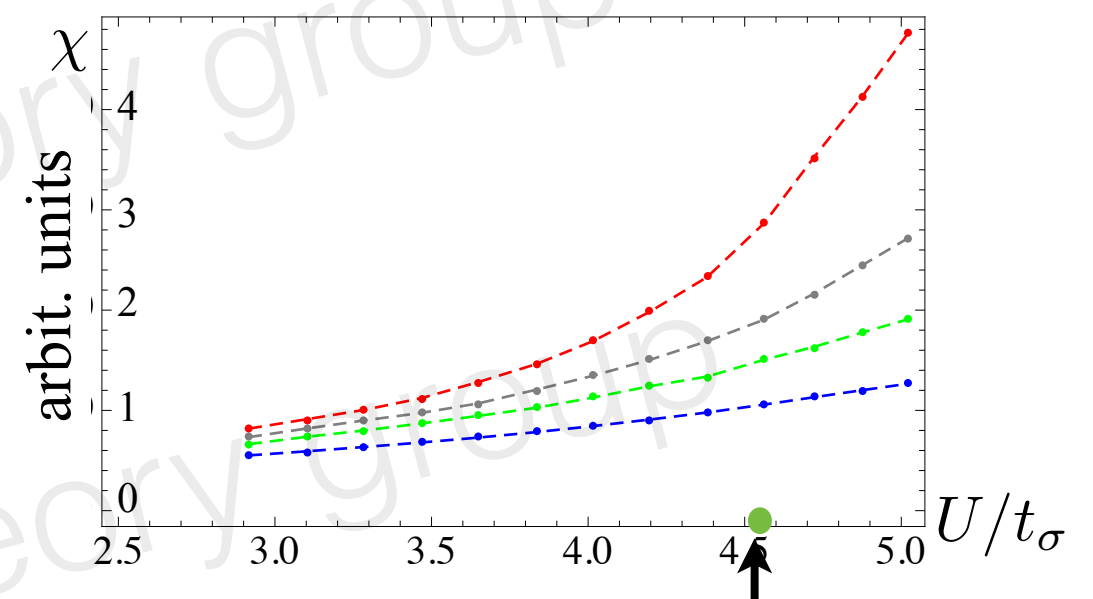
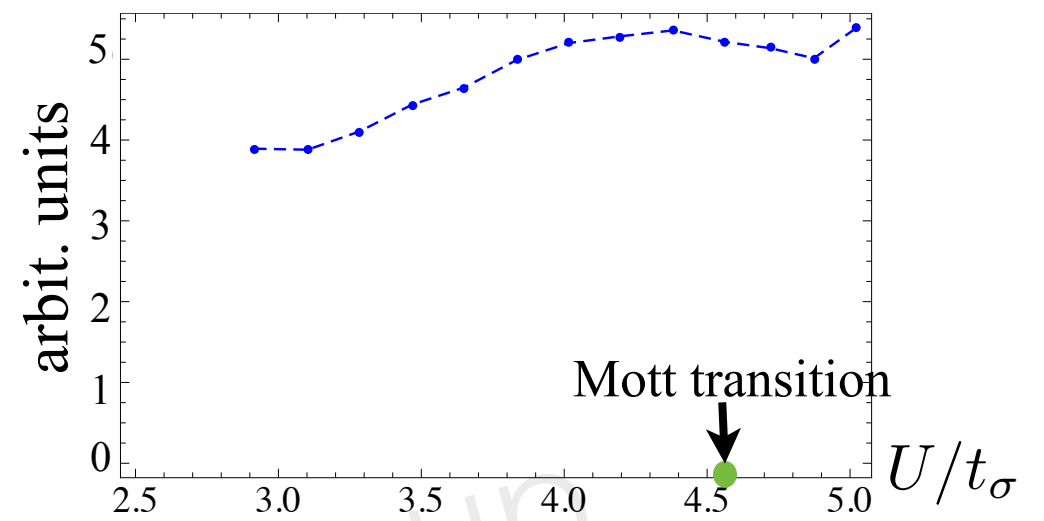
$$J = 0.4t_\sigma$$



relevant energy scales:
J and bandwidth

$$\lambda = 0.25t_\sigma$$

$$\gamma \equiv C_v/T$$



Summary

$\text{Na}_4\text{Ir}_3\text{O}_8$ is likely to be a U(1) quantum spin liquid with spinon fermi surfaces.

The large Wilson ratio might arise from the combined effect of spin-orbit coupling, correlation and onsite spin-orbital exchange.

(other possible explanation, gauge fluctuations?)

For experiments,

Other experiments: resonant inelastic x-ray scattering (planned), thermal conductivity (seems like **a metal**), quantum oscillations (too soft gauge field? O. Motrunich, PRB 2005)

Can similar physics be observed in related materials?
e.g. nonmagnetic $\text{R}_2\text{Ir}_2\text{O}_7$, Os-compounds, etc