

# Topological Engineering in $\text{Pr}_2\text{Ir}_2\text{O}_7$

Gang Chen  
Fudan University



# Outline

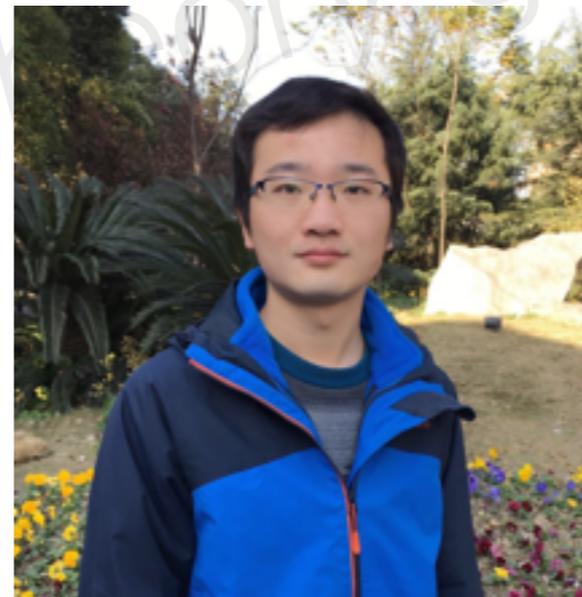
1. Microscopics of  $\text{Pr}_2\text{Ir}_2\text{O}_7$ : conduction electrons and local moments
2. Pr-magnetism induced Weyl nodes and symmetry protected Dirac band touching
3. Proximate phase transition out of U(1) quantum spin liquid

Refs:

GC, PRB 94, 205107, (2016)

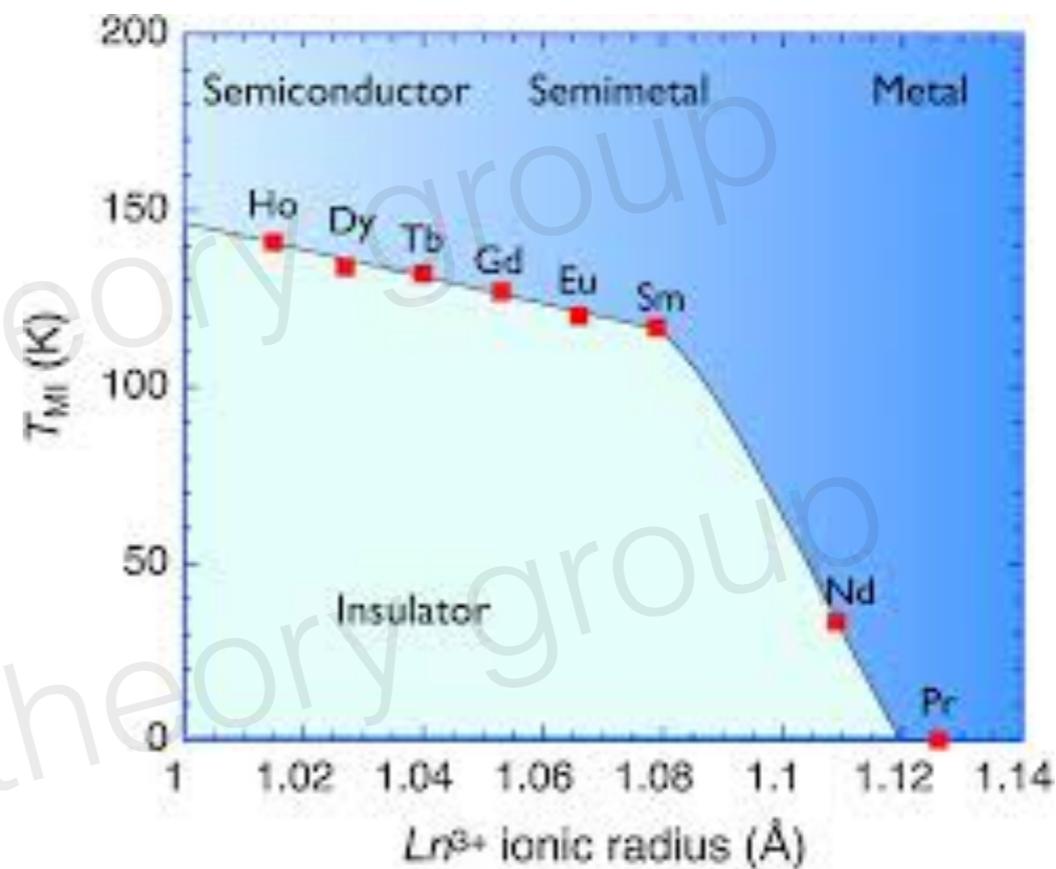
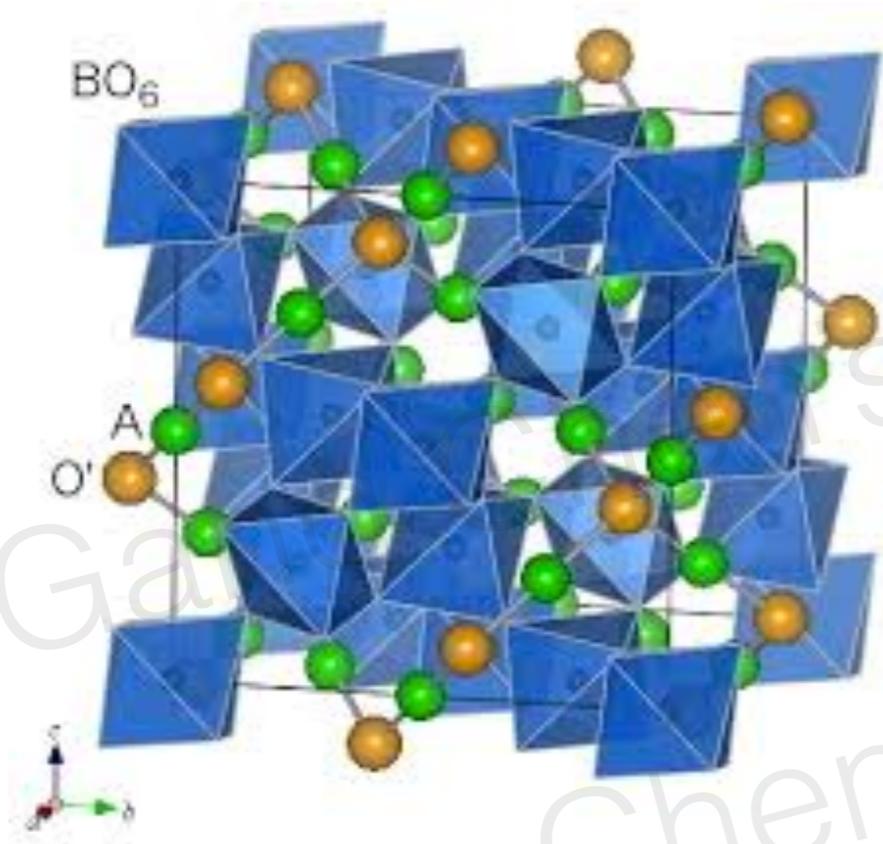
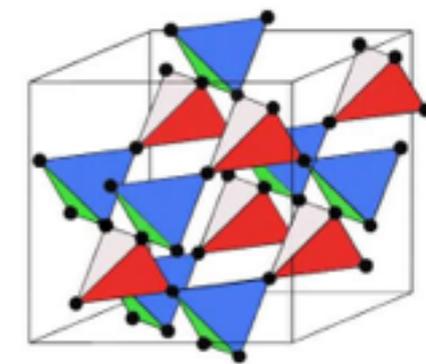
GC, PRB 96, 195127, (2017)

X-P Yao, GC, arXiv:1712.06534



**Xu-Ping Yao**  
**(Fudan)**

# Pyrochlore iridates



K Matsuhira, M Wakeshima, Y Hinatsu, S. Takagi  
JPSJ, 2011

$Pr_2Ir_2O_7$  remains metallic and disordered !

# Early motivation: correlation physics in spin-orbit (to

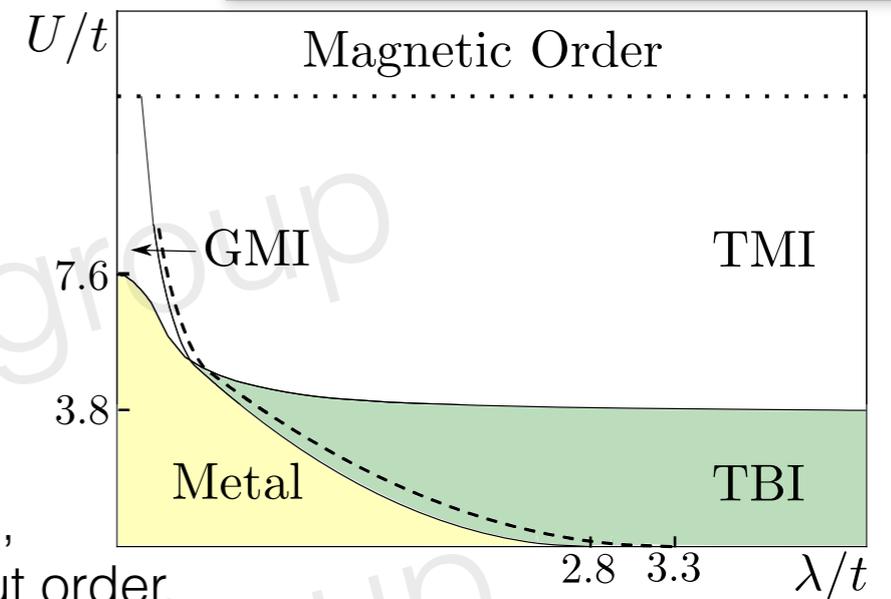
iridate provides such a setting, has interaction and strong spin-orbit coupling

Leon knows a lot of solid state physics, Mott transition, density of state is suppressed, the longer range part of interaction becomes important and could induce exciton magnetism.

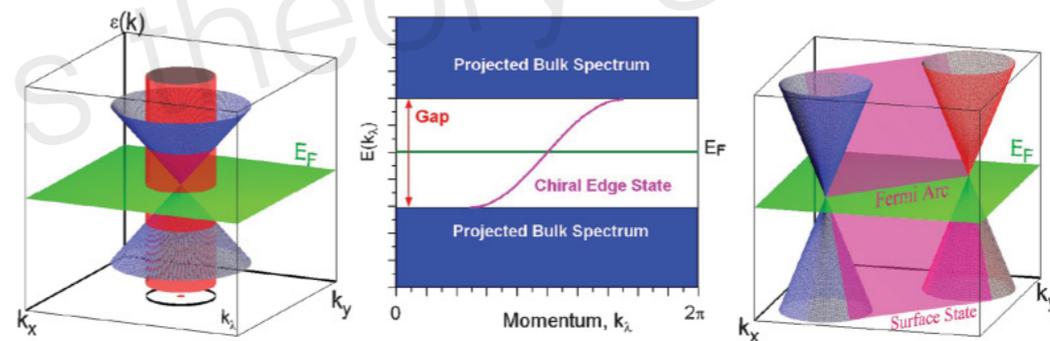
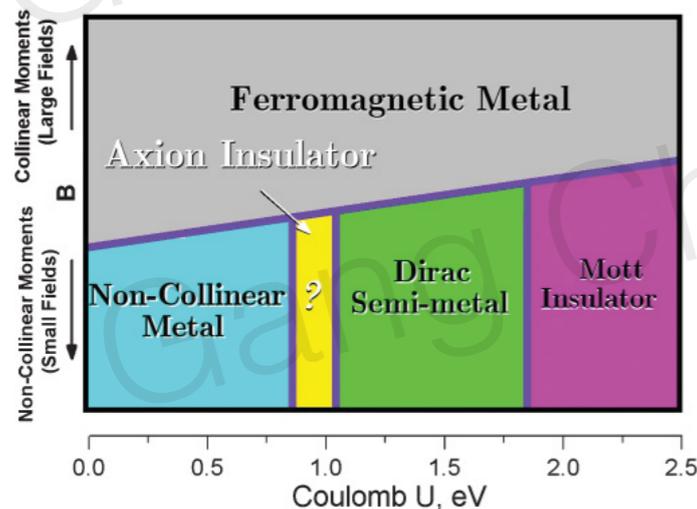
Halperin 1960s.

D Pesin, L Balents, NatPhys 2010, Topological Mott insulator (or A 3D U(1) quantum spin liquid)

$$H = \sum_{Ri\alpha} (\varepsilon_\alpha - \mu) d_{Ri\alpha}^\dagger d_{Ri\alpha} + t \sum_{\langle Ri, R'i' \rangle} T_{\alpha\alpha'}^{ii'} d_{Ri\alpha}^\dagger d_{R'i'\alpha'} + \frac{U}{2} \sum_{Ri} \left( \sum_{\alpha} d_{Ri\alpha}^\dagger d_{Ri\alpha} - n_d \right)^2$$



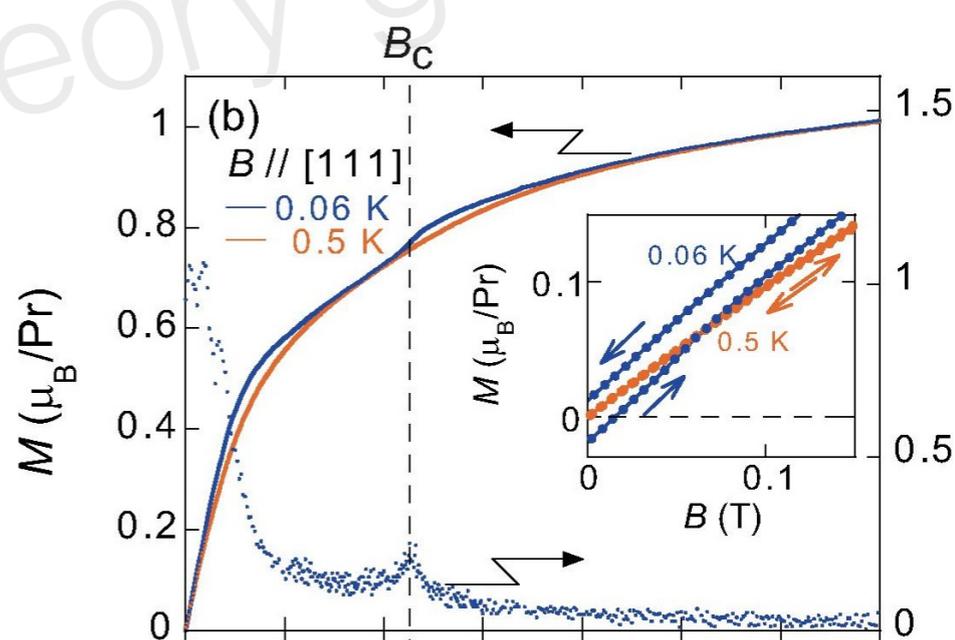
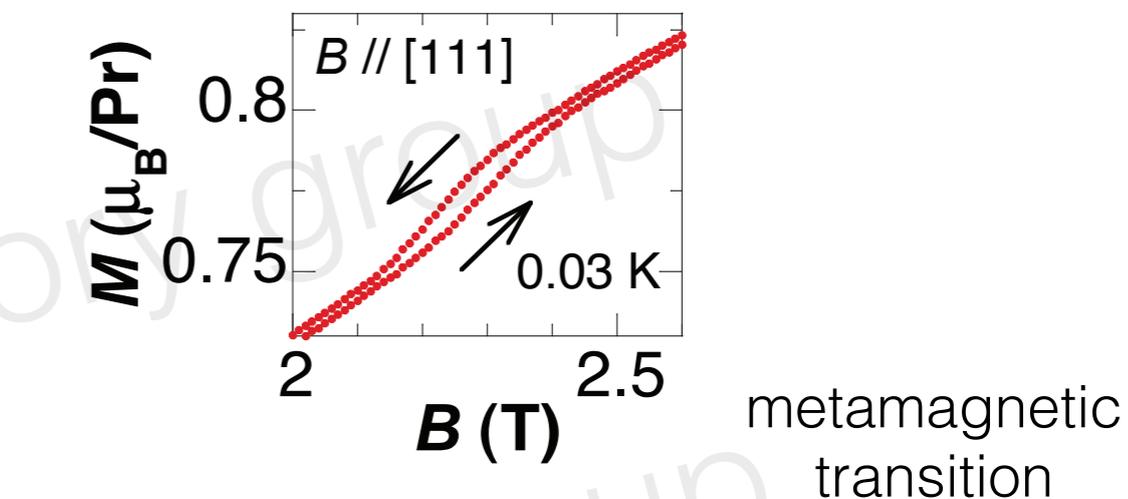
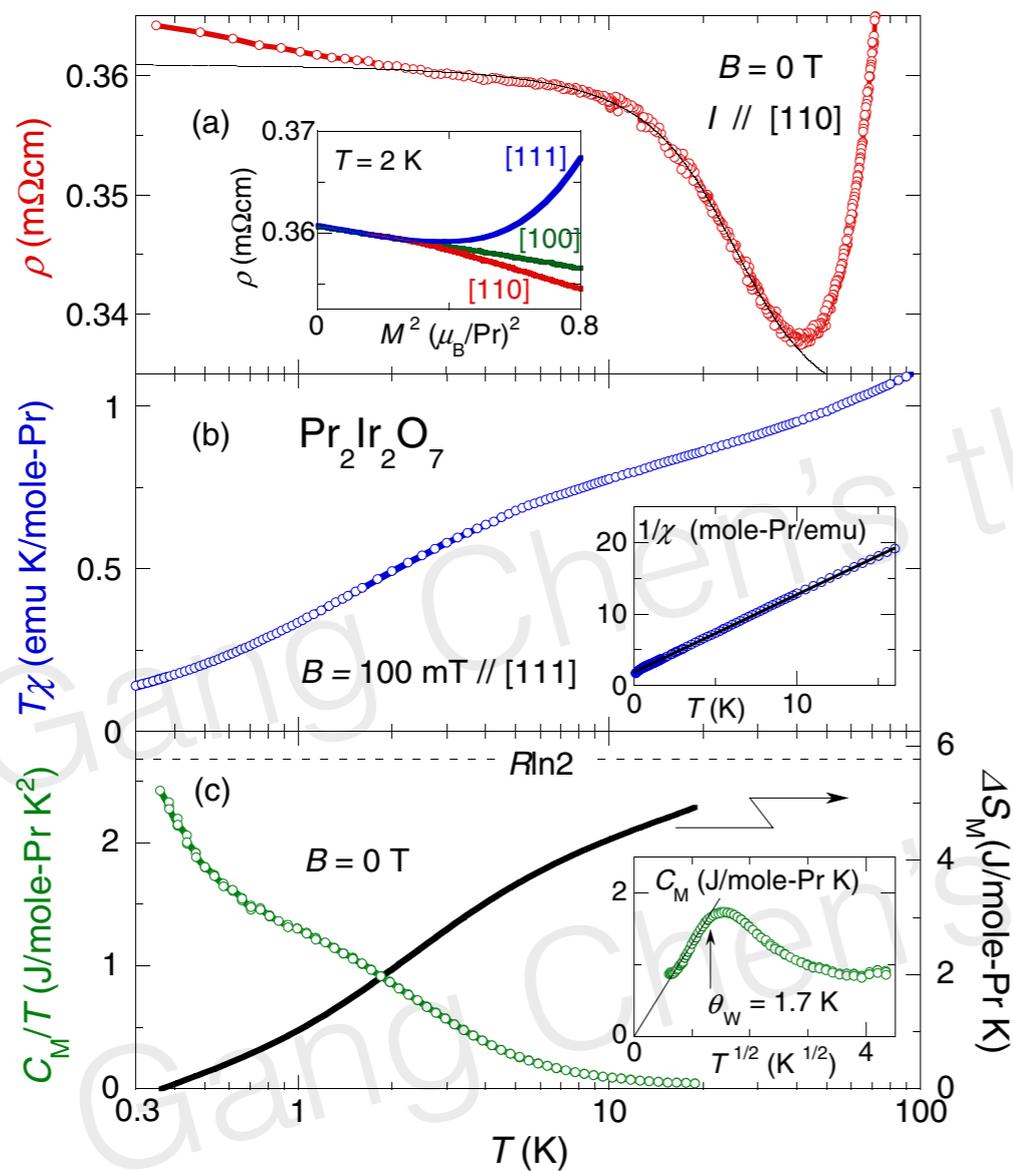
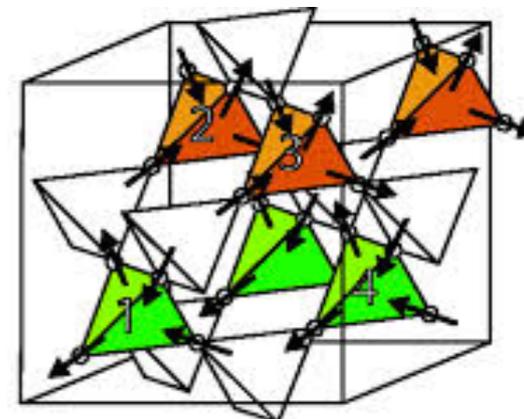
Xiangang Wan, Turner, Vishwanath, Savrasov, PhysRevB 2011, **Magnetic Weyl semimetal** from the Ir correlation driven all-in all-out order.



Weyl semimetal and surface Fermi arcs

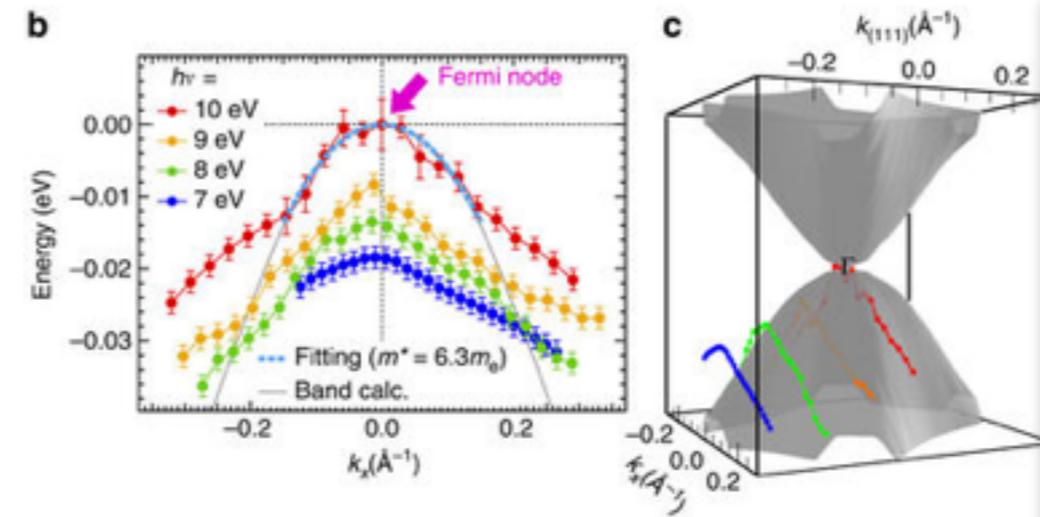
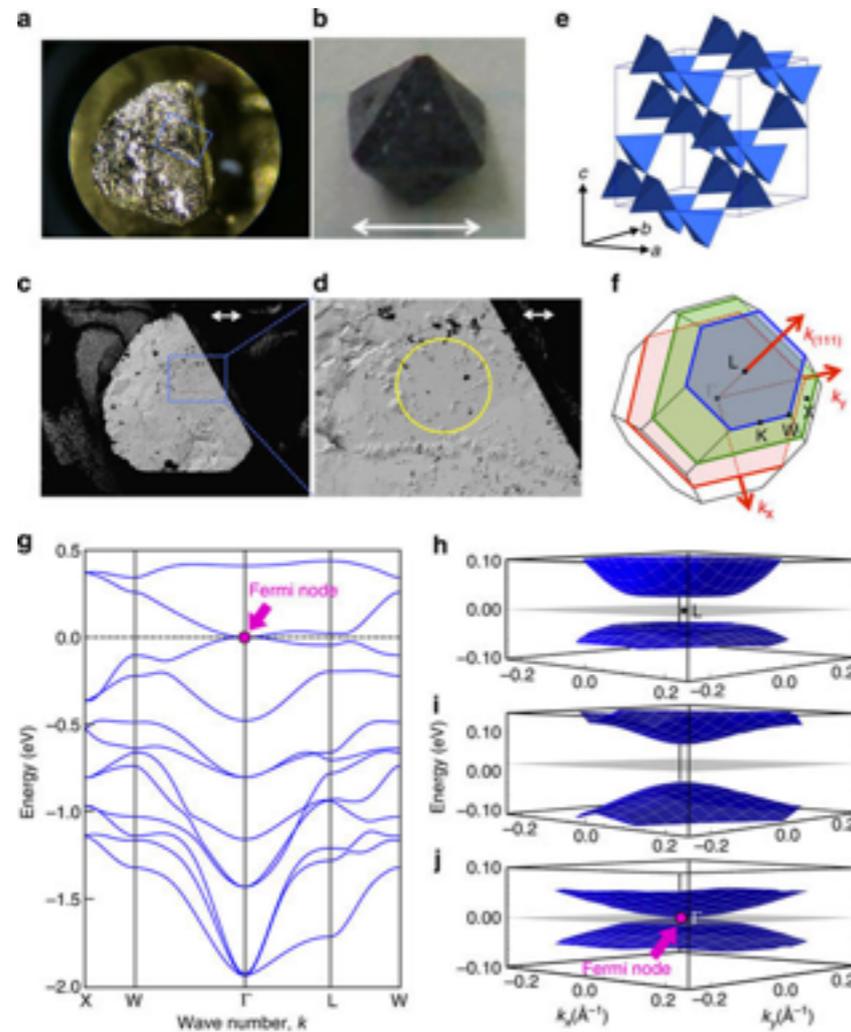
Later on, many interesting works about Iridium physics (YB Kim, L Savary, L Fu, Xi Dai, Imada, BJ Yang, EG Moon, Nagaosa, etc)

# Peculiar one: Pr<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub>



Nakatsuji, etc  
PRL **96**, 087204 (2006)

# Ir conduction electron: Luttinger semimetal



ARPES: Quadratic band

vanishing DOS, p  
the coulomb inte

T Kondo, ...**Ru Chen**, ..., Nakatsuji, Balents, Shin  
Nature Comm, 2015

P Amitage's optical measurement 2017

Correlation effect: EG Moon, L Savary, YB Kim,  
Cenke Xu, L Balents

Partial screening of long  
range Coulomb interaction

# Some $\text{Pr}_2\text{Ir}_2\text{O}_7$ sample does order magnetically

## Unstable Spin-Ice Order in the Stuffed Metallic Pyrochlore $\text{Pr}_{2+x}\text{Ir}_{2-x}\text{O}_{7-\delta}$

D. E. MacLaughlin,<sup>1,2,\*</sup> O. O. Bernal,<sup>3</sup> Lei Shu,<sup>1,4,5</sup> Jun Ishikawa,<sup>2</sup> Yosuke Matsumoto,<sup>2</sup>  
J.-J. Wen,<sup>6,†</sup> M. Mourigal,<sup>6,‡</sup> C. Stock,<sup>6,7,§</sup> G. Ehlers,<sup>8</sup> C. L. Broholm,<sup>6,7,8,9</sup> Yo Machida,<sup>2,¶</sup>  
Kenta Kimura,<sup>2</sup> Satoru Nakatsuji,<sup>2,10,\*\*</sup> Yasuyuki Shimura,<sup>2</sup> and Toshiro Sakakibara<sup>2</sup>

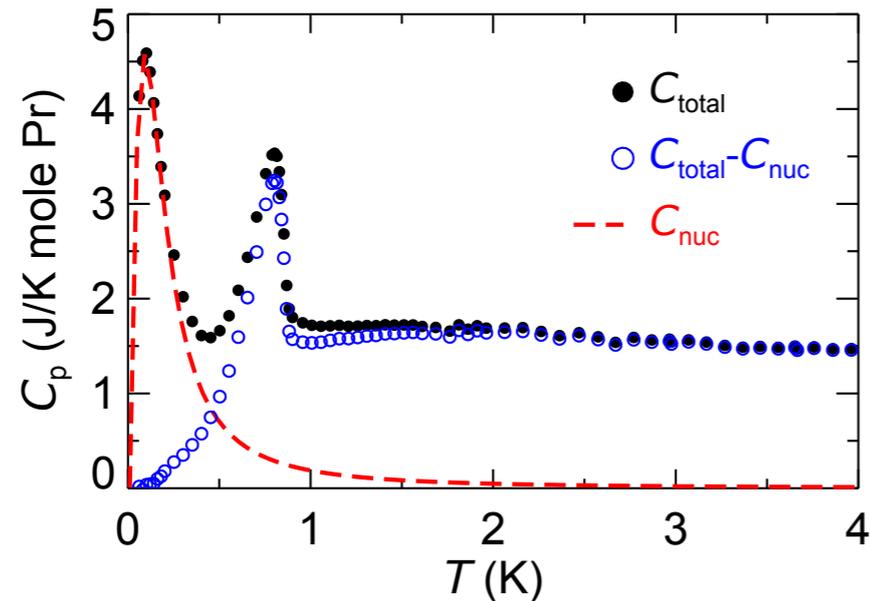


FIG. 1. (color online) Temperature dependence of the specific heat of  $\text{Pr}_{2+x}\text{Ir}_{2-x}\text{O}_{7-\delta}$  in zero field. Filled circles: experimental total specific heat. Dashed curve: calculated specific

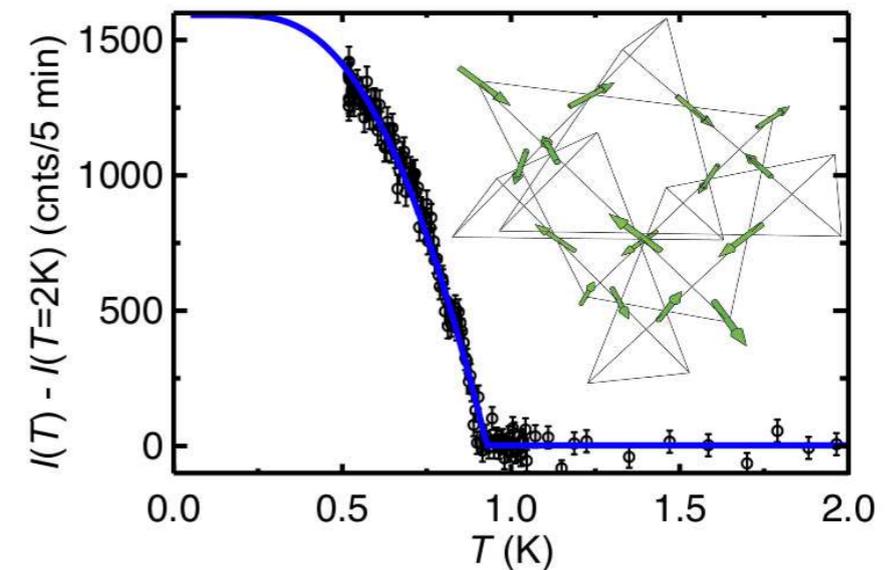
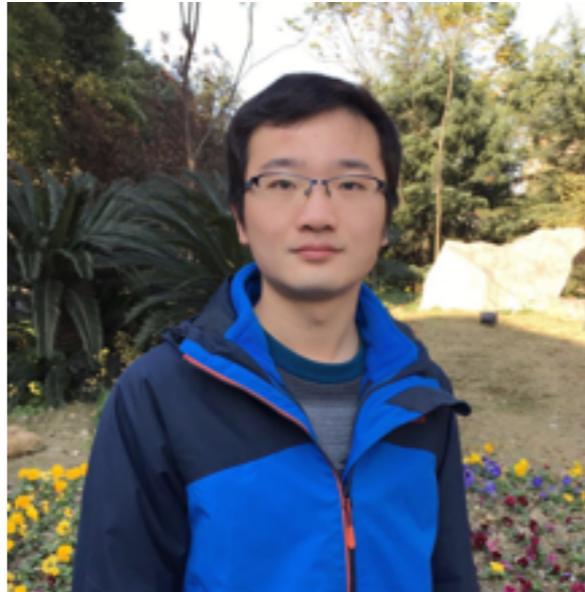


FIG. 2. (color online) Temperature dependence of elastic neutron scattering intensity of  $\text{Pr}_{2+x}\text{Ir}_{2-x}\text{O}_{7-\delta}$  at the position of the  $\mathbf{q}_m = (100)$  reflection. The intensity measured at  $T = 2$  K

actually “Melko-Hertog-Gingras” spin state  
(obtained numerically for a **different and classical** system)

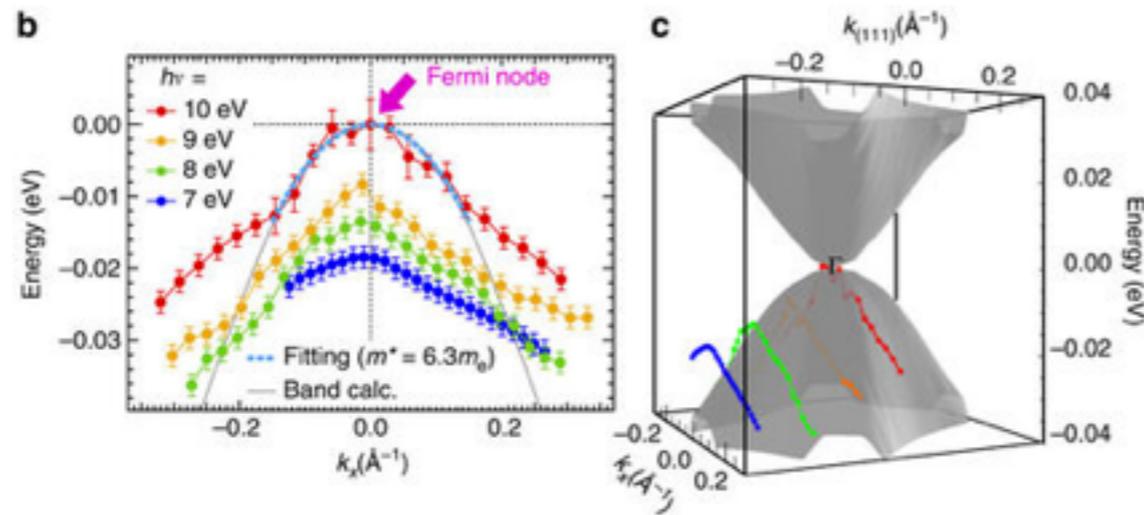
## 2. Pr-magnetism induced Weyl nodes and symmetry protected Dirac band touching

**Here we focus on the ordered side/sample.**



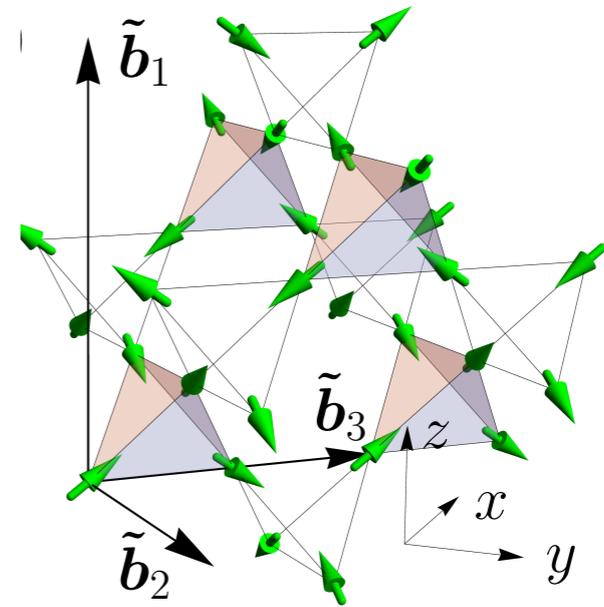
Xu-Ping Yao, GC, 1712.06534

What is the impact of the Pr magnetism on Ir conduction electrons?



Ir Luttinger semimetal

+



Pr magnetism

= ?????

**When electron behaves as electron,  
when spin behaves as spin !**

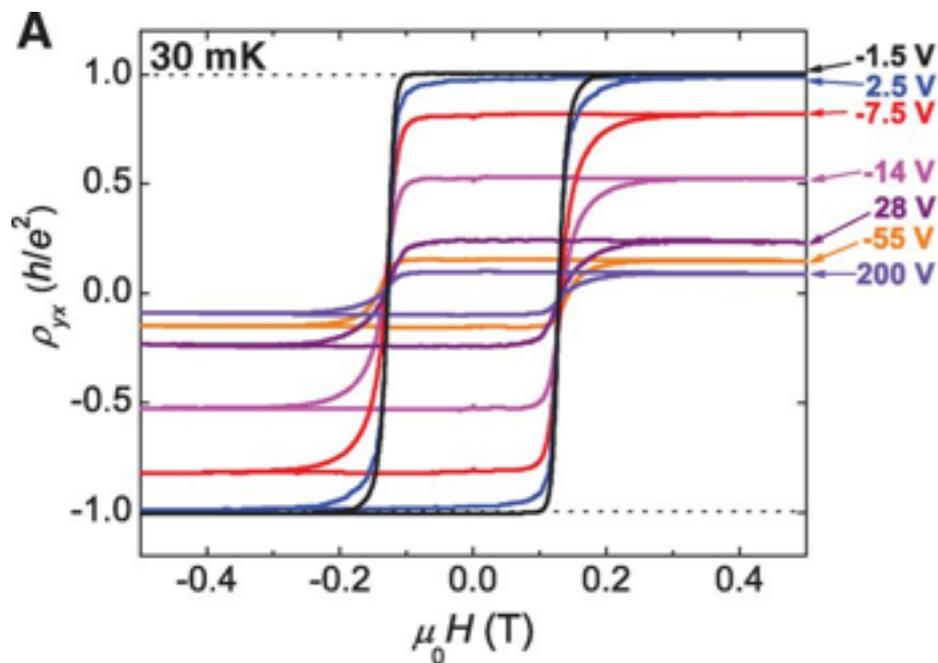
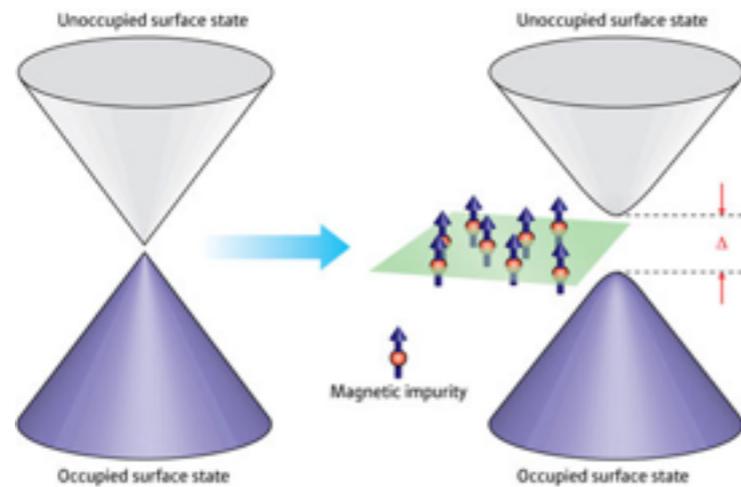


Xue's group

# Quantum Anomalous Hall Effect

1. One understanding: TI  $\rightarrow$  Dirac cone  
ferromagnetism  $\rightarrow$  gapped Dirac fermion  $\rightarrow$   
QAHE

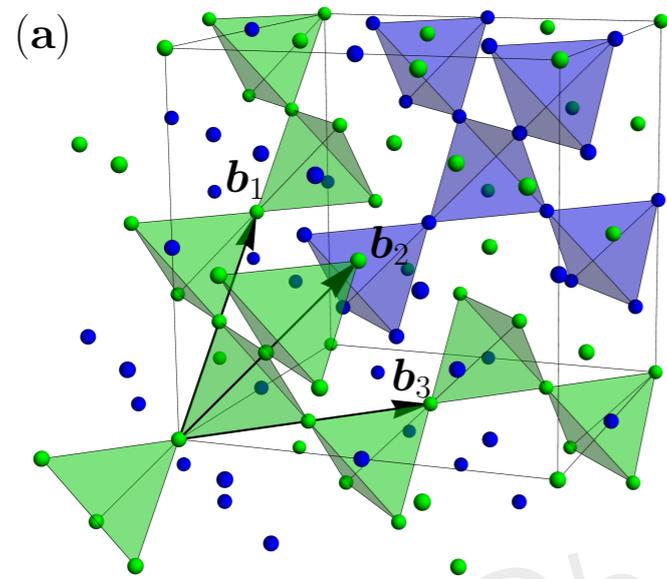
2. Our understand: QAHE is an example of interplay  
between conduction electron and local moments.  
Here in QAHE, itinerant electron band topology is  
modulated by magnetism, and magnetism is rather  
simple.



Here, we study the system with both local moments  
and itinerant electrons, trying to understand their interplay  
and interactions. How local moments influence conduction  
electrons, and vice versa.

# Microscopics: Ir conduction electron + Pr local moment

interact with  
interact with  
very complex  
system



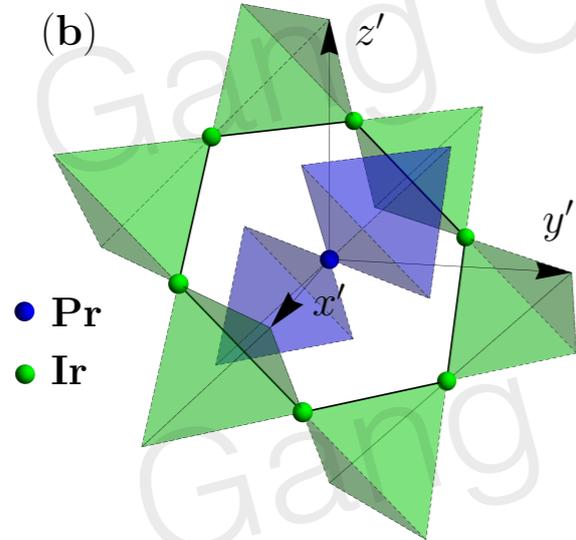
● Pr  
● Ir

$\sim < 1\text{eV}$

Ir 5d electron: hopping, SOC, interaction

$\sim$

**Pr-Ir interaction: f-d exchange**



● Pr  
● Ir

$\sim 10\text{K}$

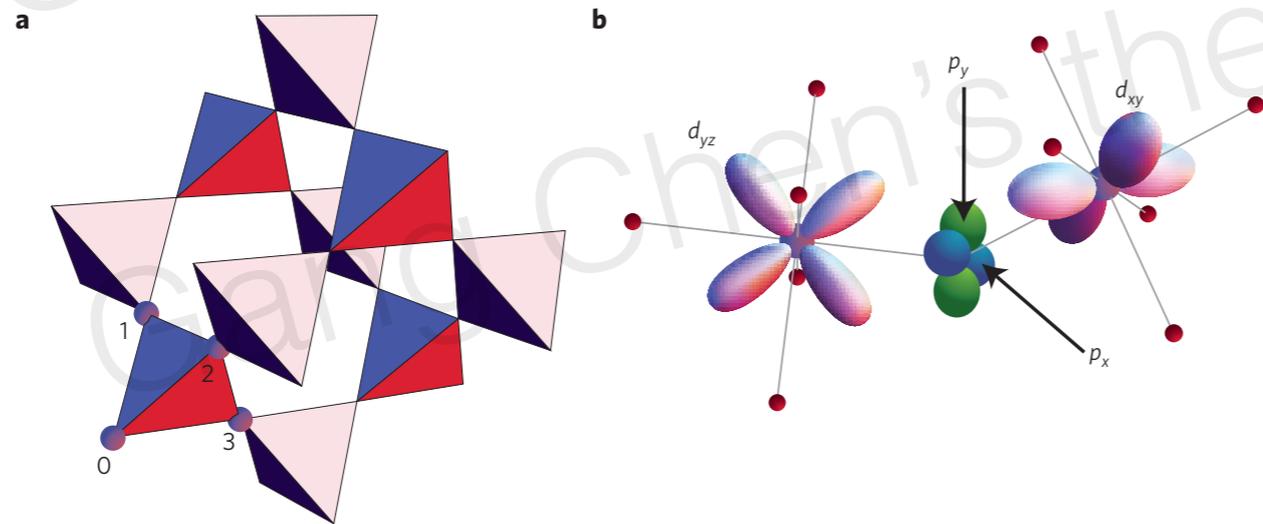
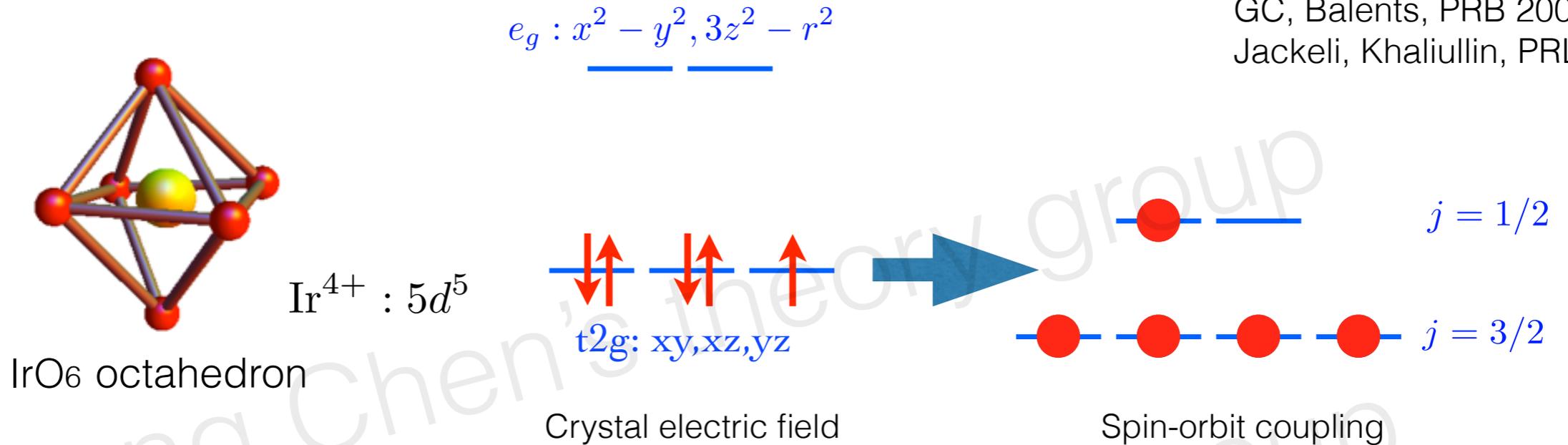
Pr 4f electron: exchange interaction

Energy  
Scale

GC, Hermele, PRB 2012  
X-P Yao, GC, 1712.06534

# Ir 5d electron: SOC, hopping and correlation

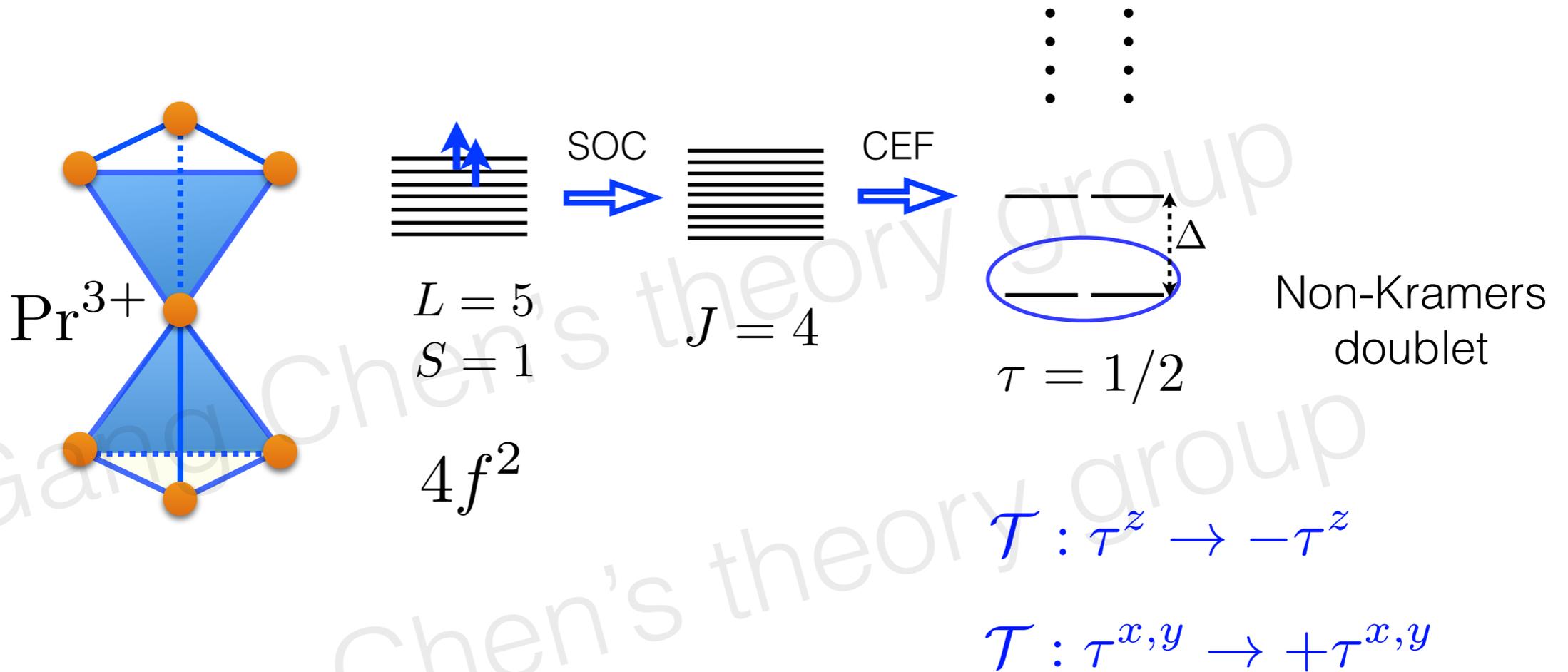
BJ Kim, etc 2008,  
GC, Balents, PRB 2008  
Jackeli, Khaliullin, PRL 2009



Besides Ir electron hopping via intermediate oxygens, there is also direct electron hopping

For Pr<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub>, correlation renormalizes the band width.

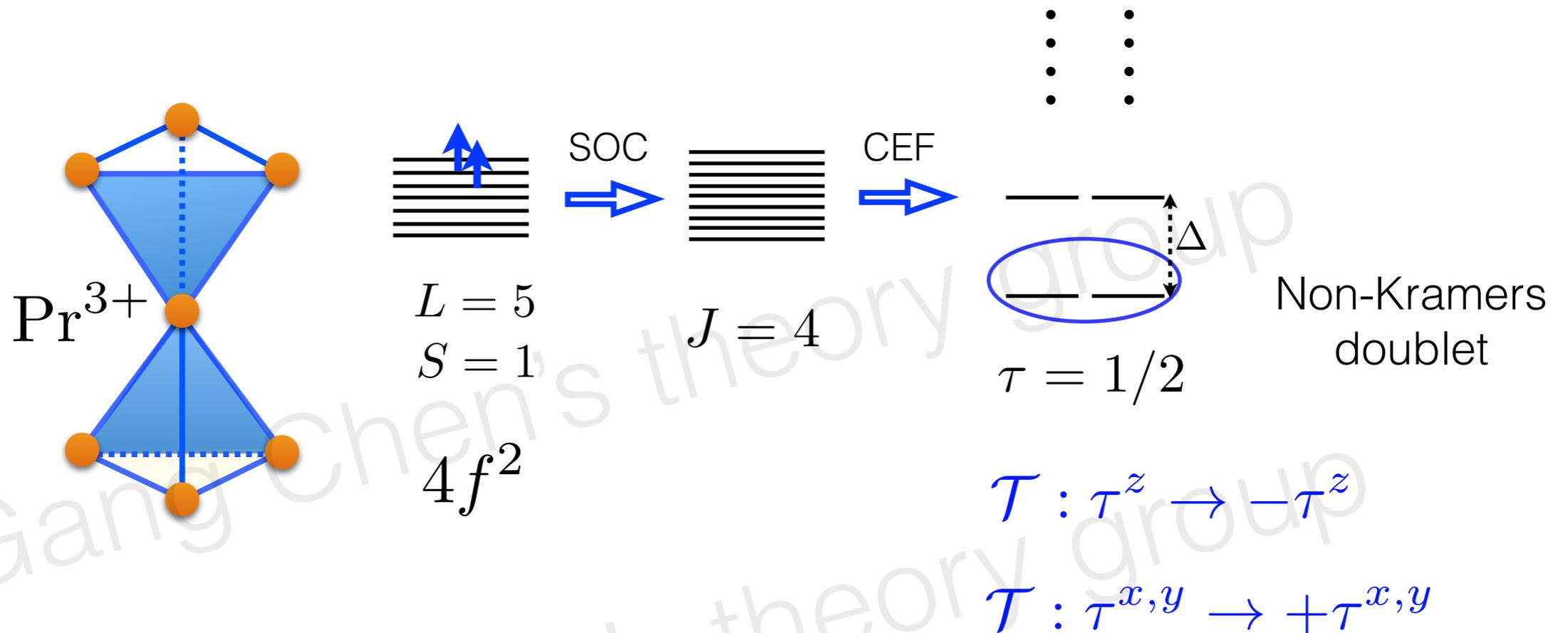
# Pr local moments: non-Kramers doublets



Indication:

1. Only z (or Ising) component couples to external magnetic field.
2. Magnetic order necessarily implies z (or Ising) component ordering.
3. Only z (or Ising) component couples to the Ir electron spin density.

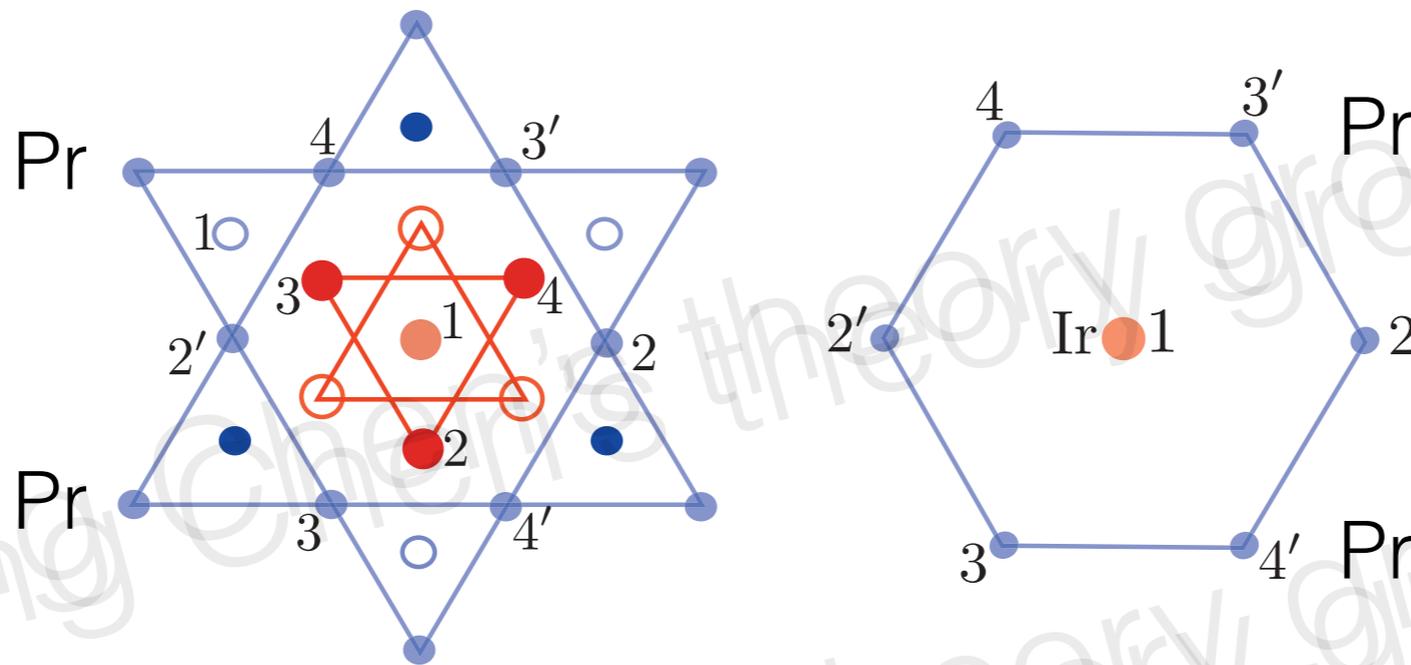
# Pr's 4f local moments: exchange interaction



$$\tilde{H}_{ex} = \sum_{ij} J_{z,ij} \tau_i^z \tau_j^z + \sum_{ij} J_{\perp,ij} \sum_{\mu,\nu=x,y} \tau_i^\mu \tau_j^\nu,$$

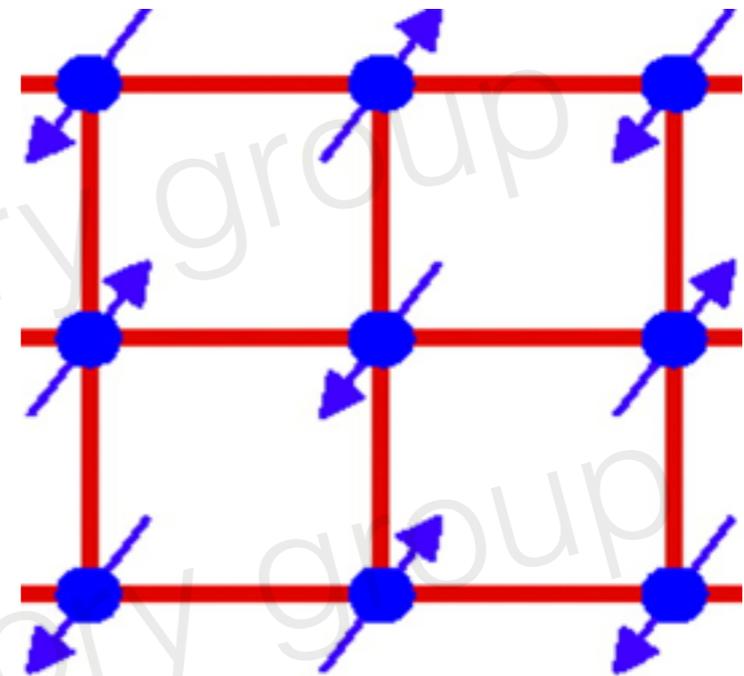
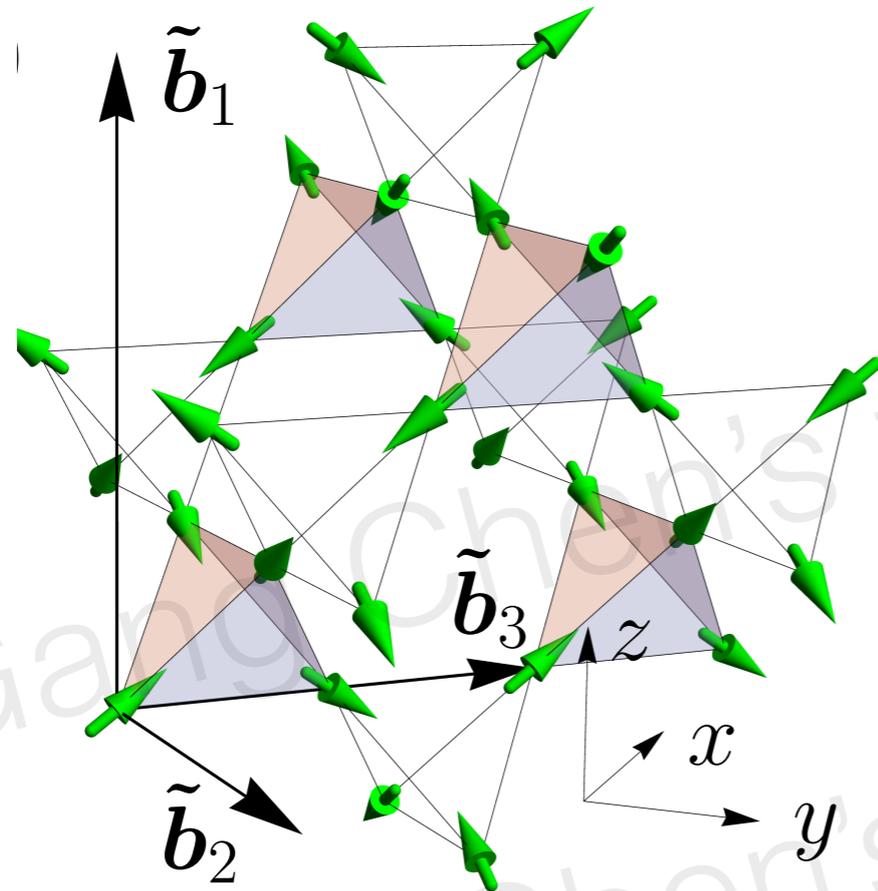
$$\tilde{H}_{ex} \simeq \sum_{\langle ij \rangle} J_{1z} \tau_i^z \tau_j^z + \sum_{\langle\langle\langle ij \rangle\rangle\rangle} J_{3z} \tau_i^z \tau_j^z,$$

# Pr-Ir interaction: 4f-5d exchange



$$\mathcal{H}_{fd} = [c_1 \tau_4^z - c_2 (\tau_2^z + \tau_3^z)] j_1^x + [c_1 \tau_3^z - c_2 (\tau_2^z + \tau_4^z)] j_1^y + [c_1 \tau_2^z - c_2 (\tau_3^z + \tau_4^z)] j_1^z + [2 \leftrightarrow 2', 3 \leftrightarrow 3', 4 \leftrightarrow 4'],$$

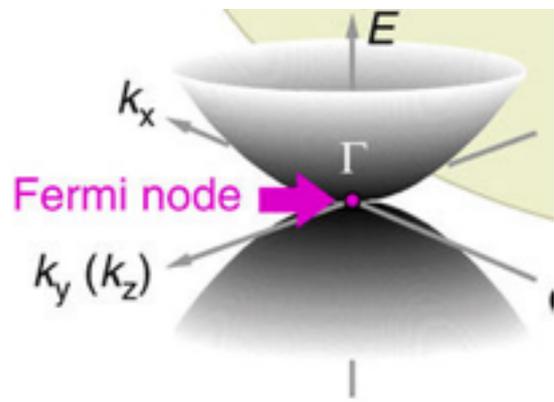
# Magnetic translation of Pr magnetic state



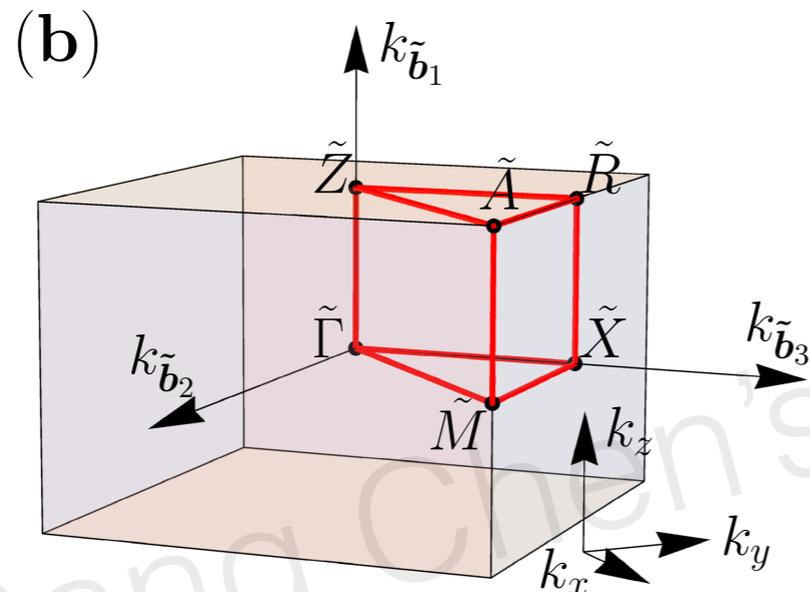
Neel state on square lattice

$$\tilde{T} \equiv T \circ t$$

3D analogue of the magnetic translation for Neel state

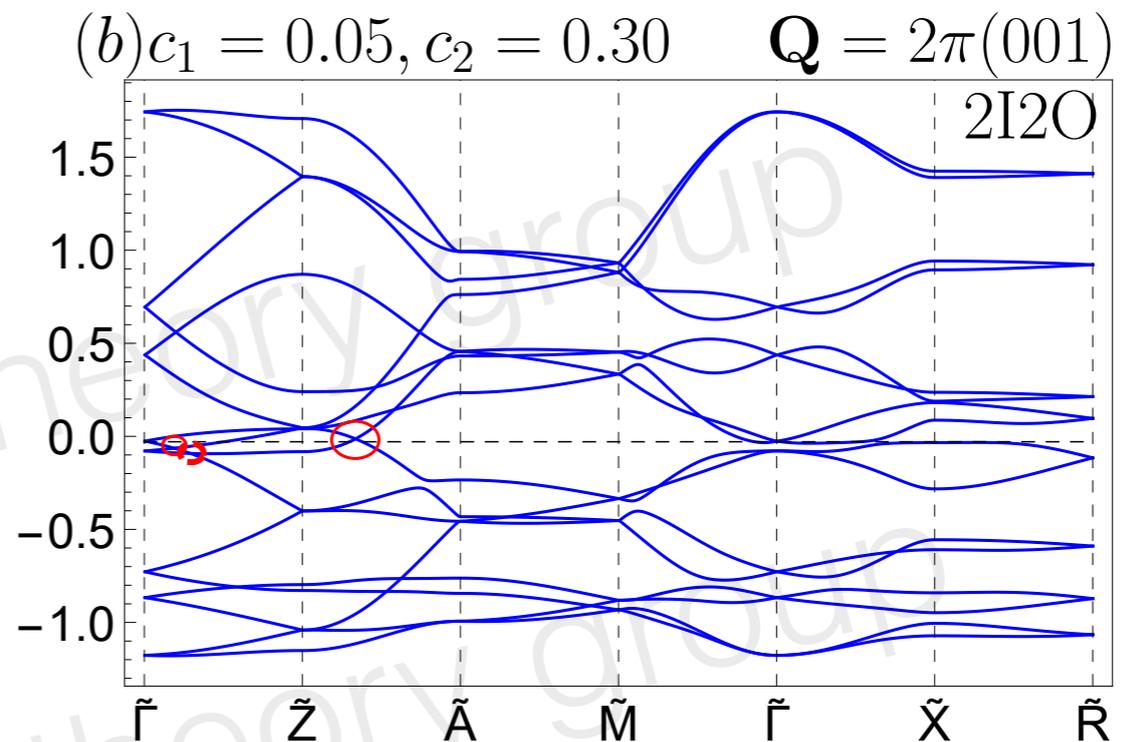


# Symmetry protected Dirac band touching



magnetic Brillouin zone

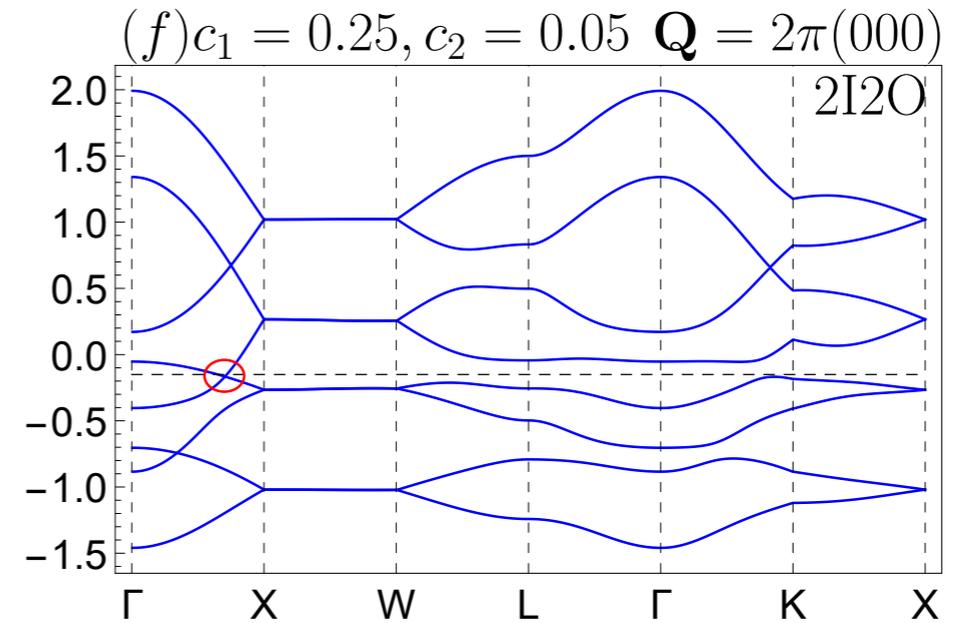
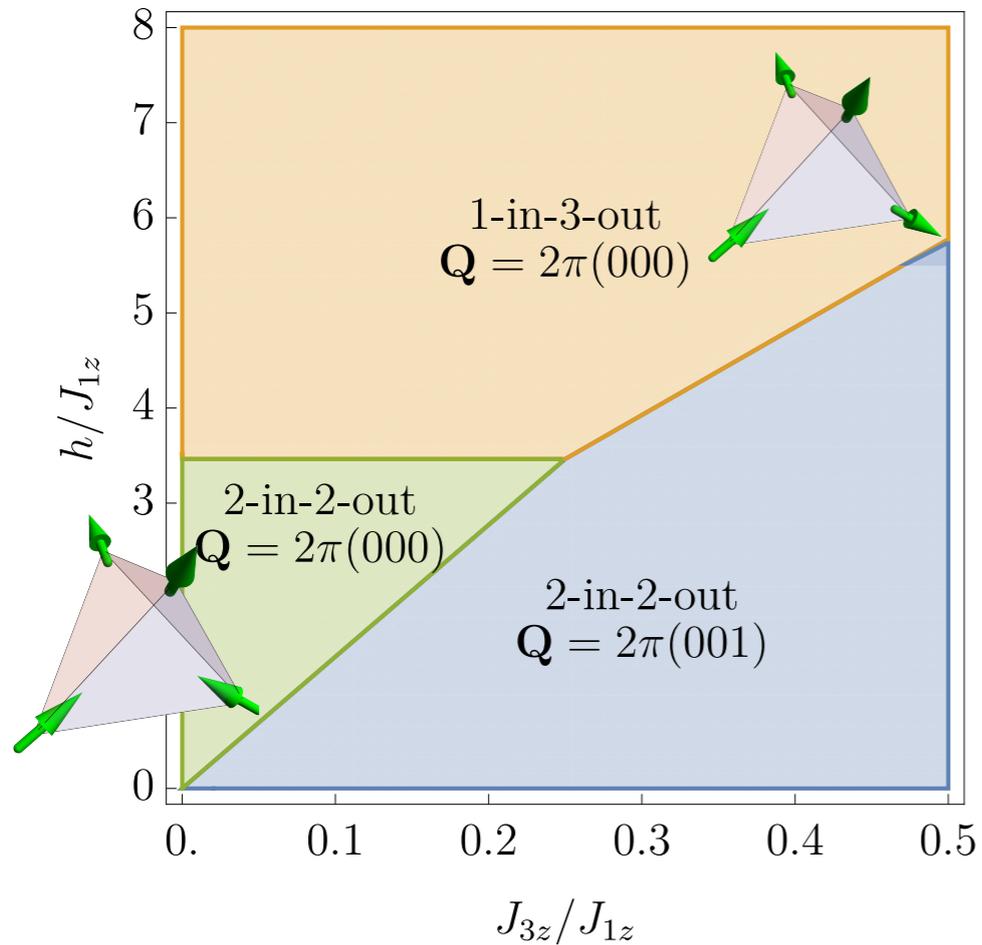
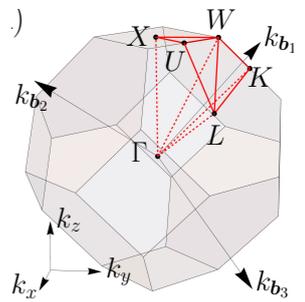
$$\tilde{\mathcal{T}}^2 = -1 \text{ at } \tilde{\Gamma} \text{ point}$$



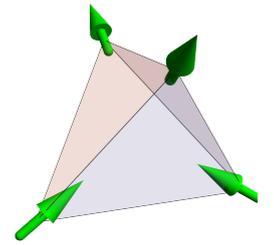
in addition, there are Weyl nodes whose existence does not require symmetry

Pr magnetic order transfers its time reversal symmetry breaking to Ir Luttinger semimetal.

# Band engineering by external magnetic field

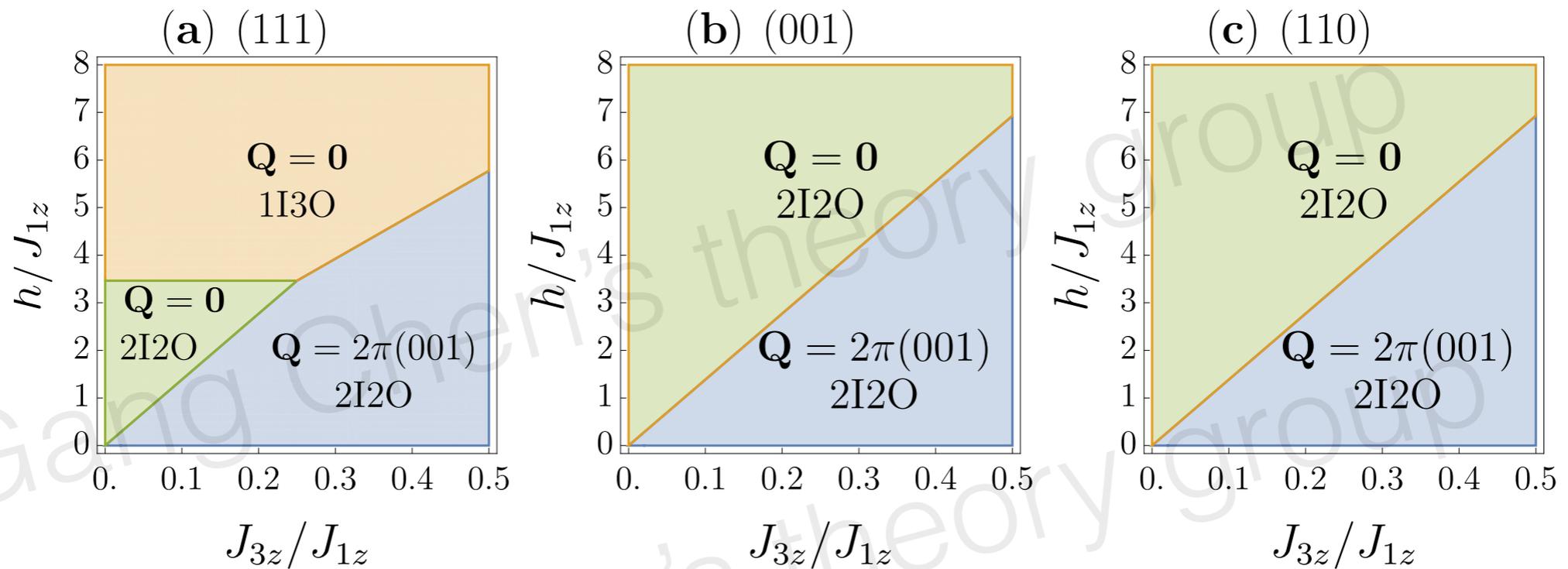


example: double Weyl nodes



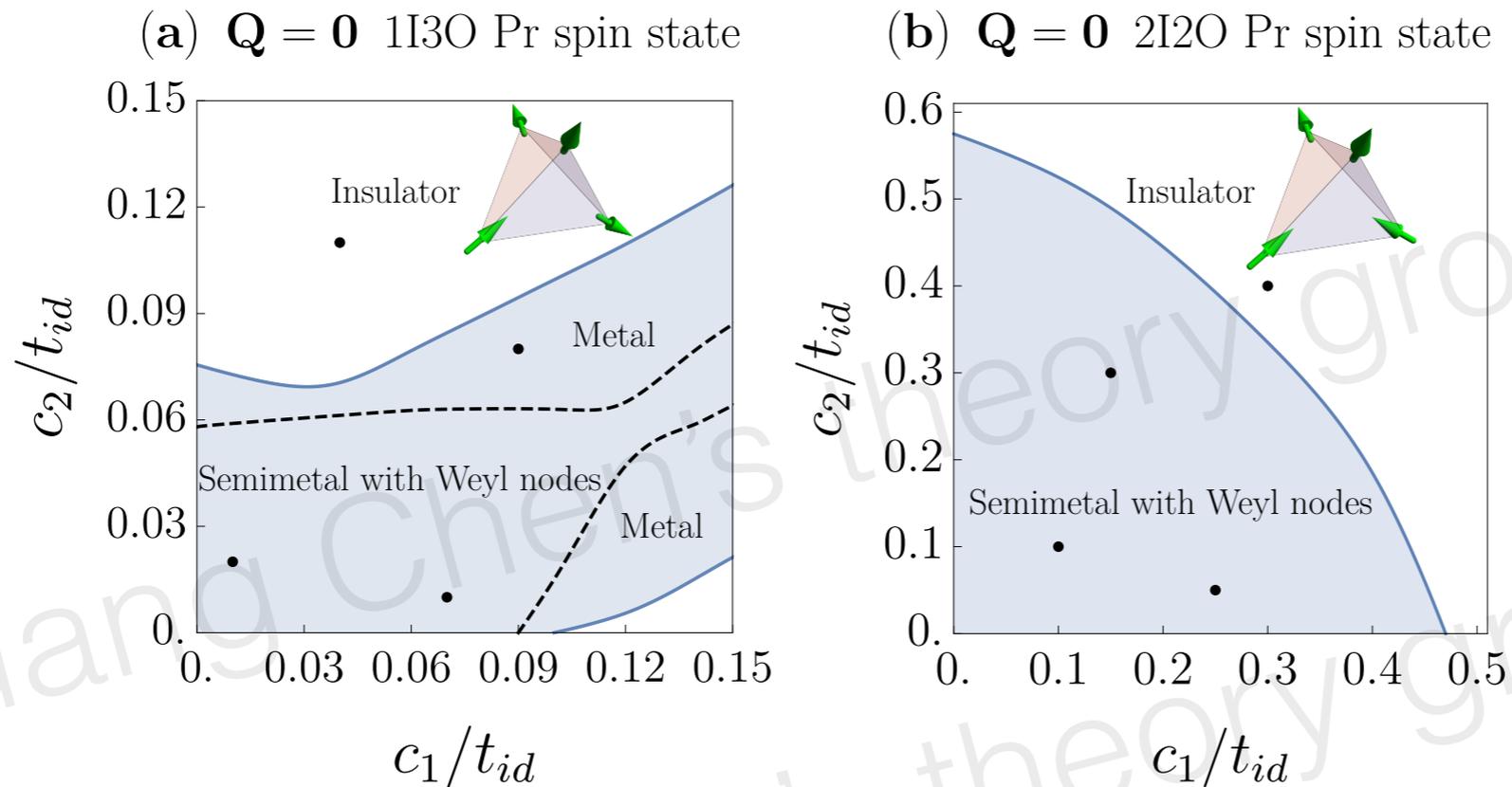
1. Magnetic field primarily couples to Pr moments, modifies Pr spin state, thereby indirectly influence the Ir band structure,
2. Field immediately removes the Dirac band touching,
3. Field induces Weyl nodes on the Ir band structure as well, anomalous Hall effect

# Quantum control under magnetic field



The Pr magnetic state under different direction magnetic field

# 111 magnetic field, Ir band structure



Magnetic field modifies the Pr magnetic structure, thereby modifies the Ir band structure.

We predict that external magnetic field destroy the symmetry protected Dirac band touching, and Weyl nodes still persist and give to anomalous Hall effect.

Xu-Ping Yao, Gang Chen, arXiv 1712.06534

3. Proximate phase transition out of U(1) quantum spin liquid

# Some $\text{Pr}_2\text{Ir}_2\text{O}_7$ sample does order magnetically

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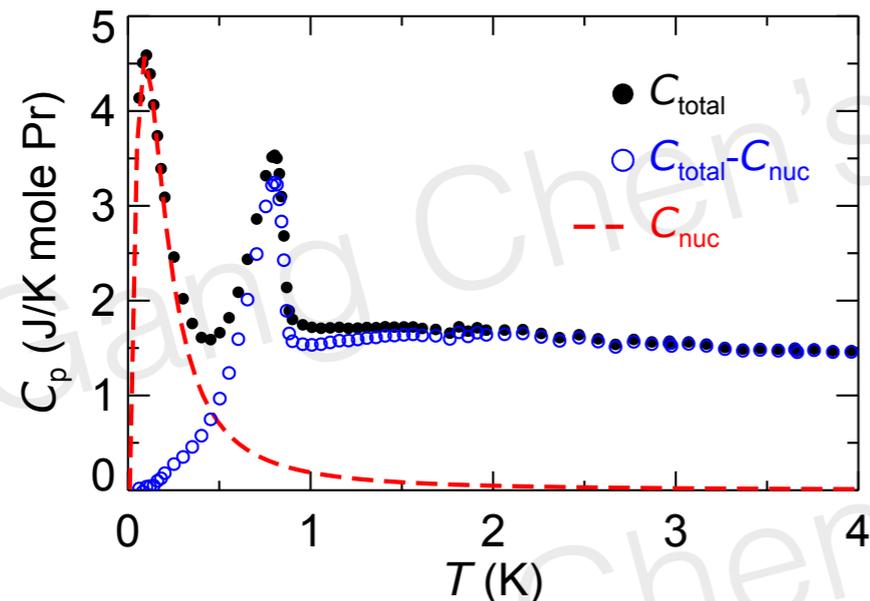


FIG. 1. (color online) Temperature dependence of the specific heat of  $\text{Pr}_{2+x}\text{Ir}_{2-x}\text{O}_{7-\delta}$  in zero field. Filled circles: experimental total specific heat. Dashed curve: calculated specific

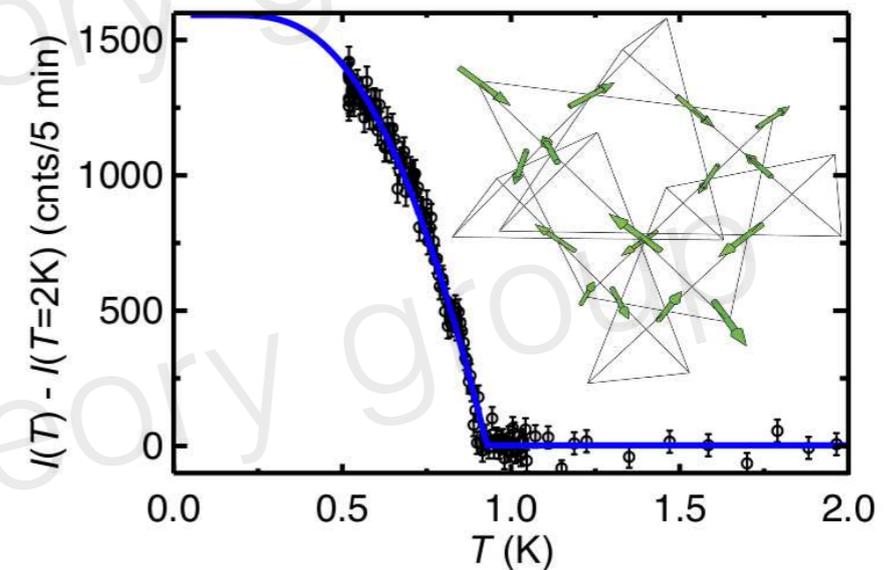
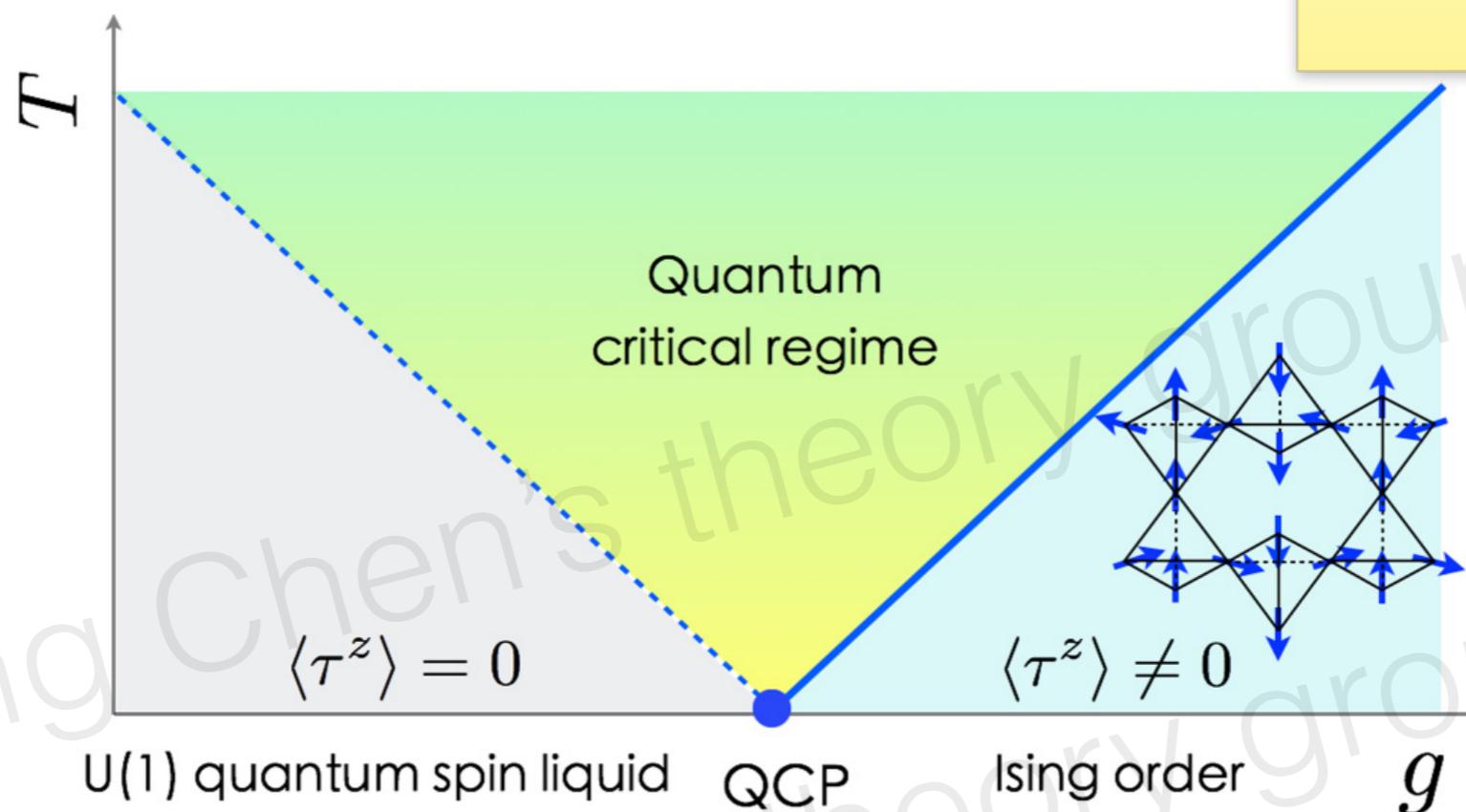


FIG. 2. (color online) Temperature dependence of elastic neutron scattering intensity of  $\text{Pr}_{2+x}\text{Ir}_{2-x}\text{O}_{7-\delta}$  at the position of the  $\mathbf{q}_m = (100)$  reflection. The intensity measured at  $T = 2$  K

actually “Melko-Hertog-Gingras” spin state  
(obtained numerically for a **different and classical** system)

# Our suggestion

there may be some jump in logic

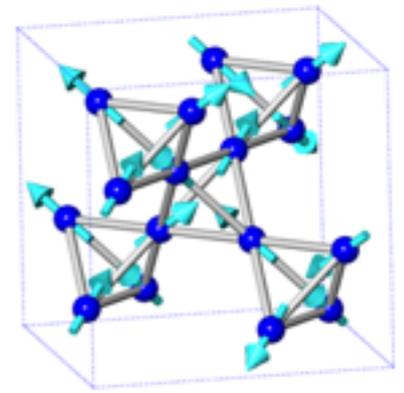


The Pr subsystem is proximate to a quantum phase transition from pyrochlore ice U(1) QSL to Ising magnetic order.

Microscopics: different samples have different Fermi energy, induces different RKKY interaction between Pr local moments.

**GC, PRB 94, 205107 (2016)**

# Pyrochlore ice U(1) spin liquid



spin ice

Hermele, Fisher, Balents, 2004

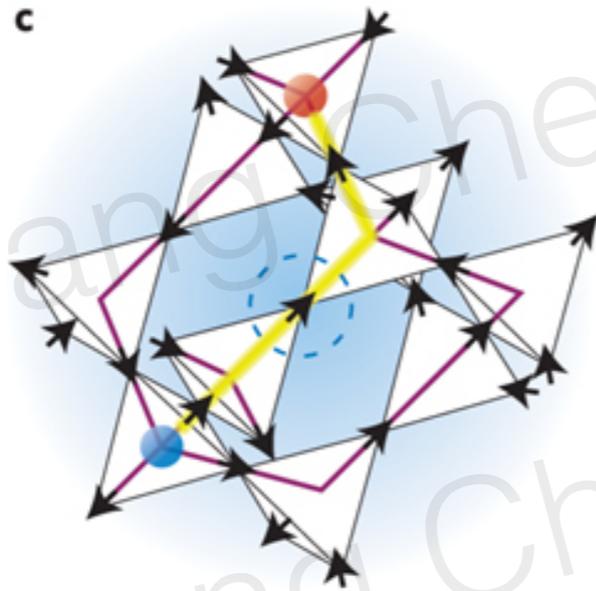
$$H = \sum_{\langle ij \rangle} J_{zz} \tau_i^z \tau_j^z - J_{\pm} (\tau_i^+ \tau_j^- + h.c.)$$

$$\tau^z \sim E$$

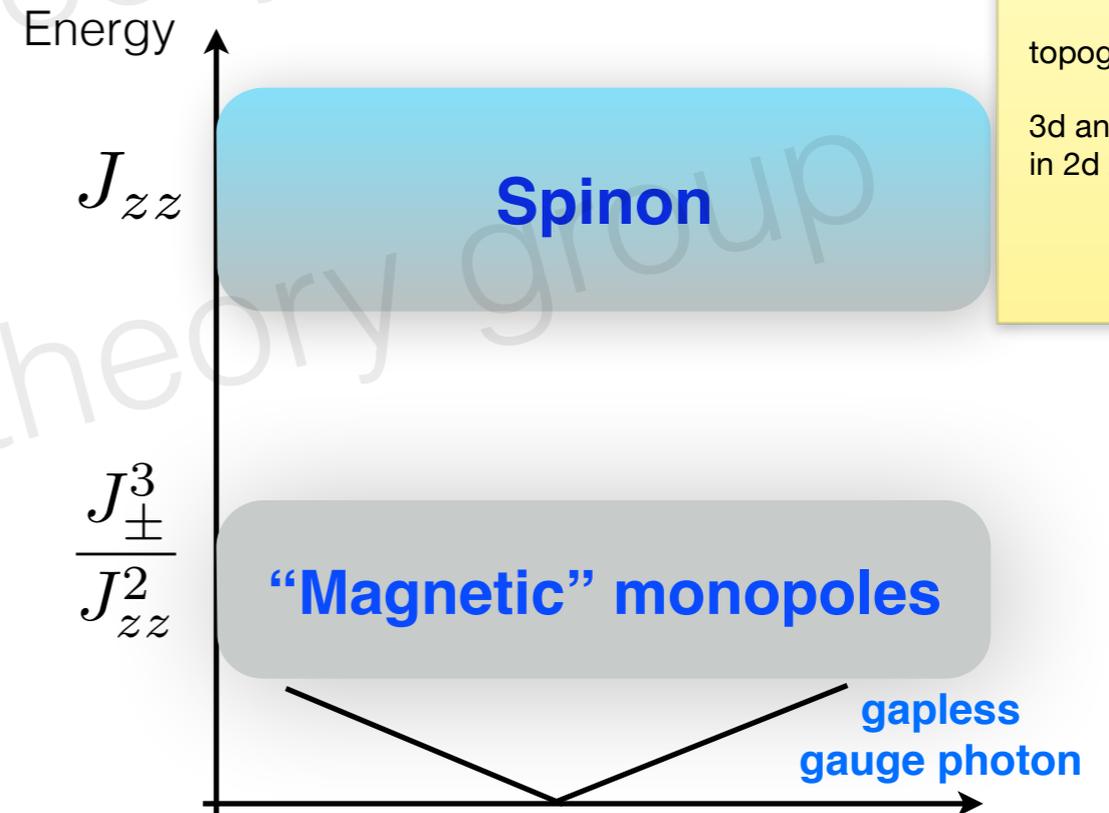
emergent electric field

$$\tau^{\pm} \sim e^{\pm iA}$$

emergent vector gauge potential



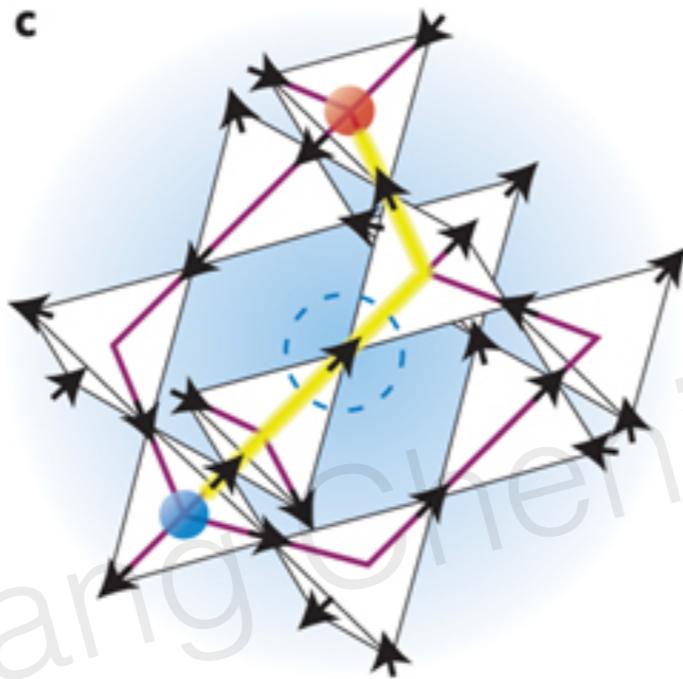
Balents, Savary, Gingras, SB Lee, Moessner, Onoda, Shannon.....



it is like topological  
topological defect of v  
3d analogue of vison  
in 2d z2 spin liquid.

It is an example of XG Wen's string-net condensed phase.  
Fluctuating closed string  $\rightarrow$  lights/photon, ends of open strings  $\rightarrow$  particles (spinon,..)  
It is like topological order.

# Electromagnetic duality and monopole condensation



$$\mathcal{T} : \tau^z \rightarrow -\tau^z$$

$$\mathcal{T} : \tau^{x,y} \rightarrow +\tau^{x,y}$$

Magnetic transition of non-Kramers doublet implies confinement transition.

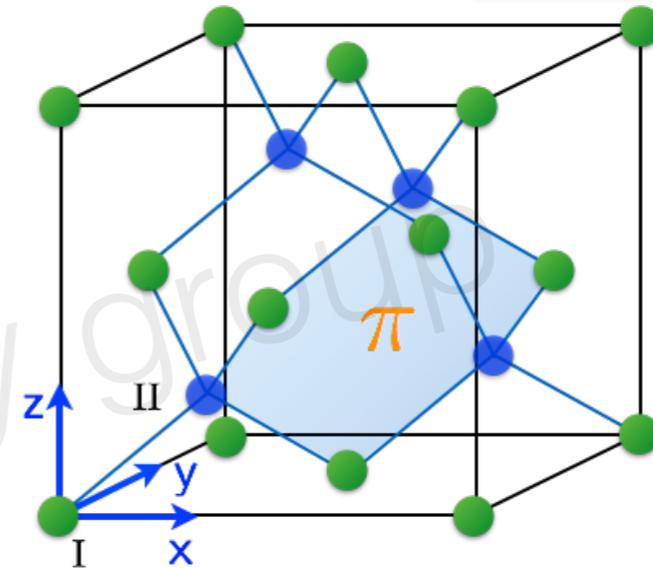
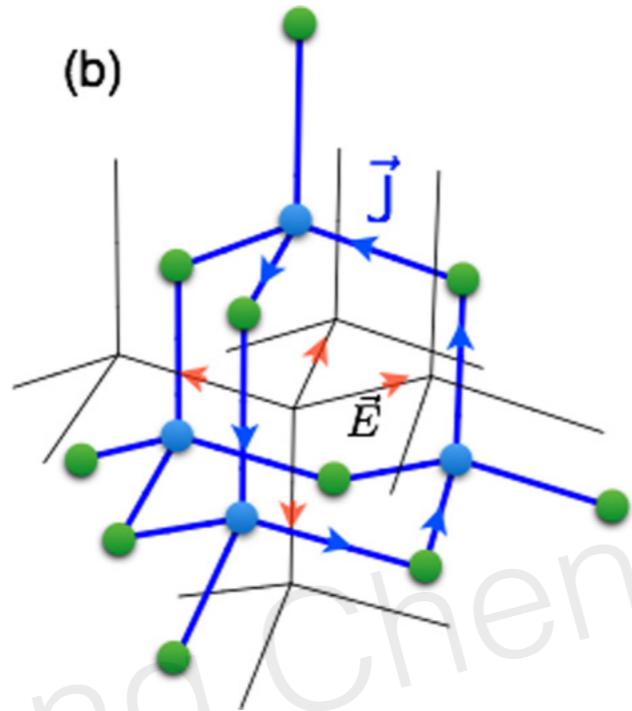
Confinement transition is obtained by magnetic monopole condensation.

“Fictitious” string now has a finite tension.  
Electric field gets a static value.  
Spinons are confined.

Motrunich Senthil 2005,  
DL Bergman, G Fiete, Balents 2005  
GC PRB 94, 205107 (2016)

# Monopole condensation and magnetic order

like Ginzburg Landau,  
except the gauge is internal and emergent



$$H_{\text{dual}} = \sum_{\text{Hex}_d^*} \frac{U}{2} (\text{curl } a - \bar{E})^2 - \sum_{\mathbf{r}, \mathbf{r}'} K \cos B_{\mathbf{r}\mathbf{r}'} - \sum_{\langle \mathbf{r}, \mathbf{r}' \rangle} t e^{-i2\pi \bar{a}_{\mathbf{r}\mathbf{r}'}} \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r}'} - \mu \sum_{\mathbf{r}} \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r}},$$

Electromagnetic duality  
and right-hand rule

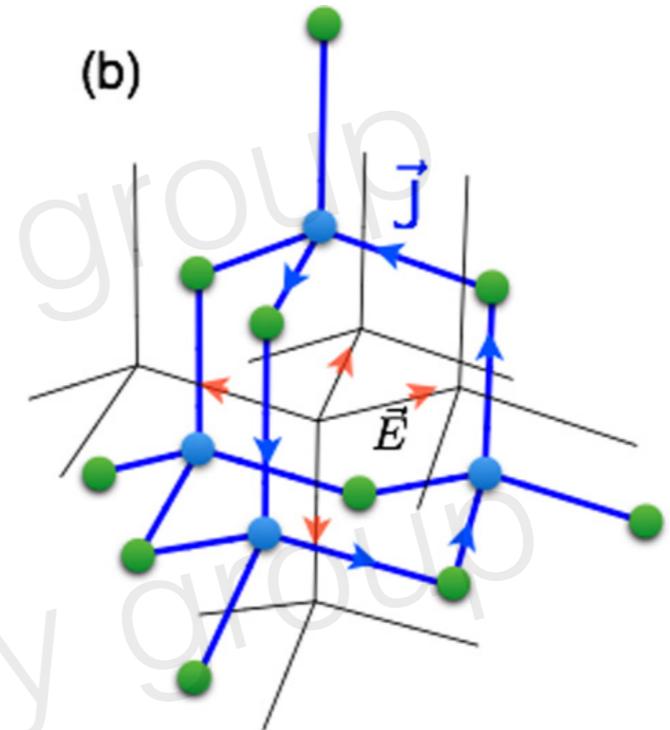
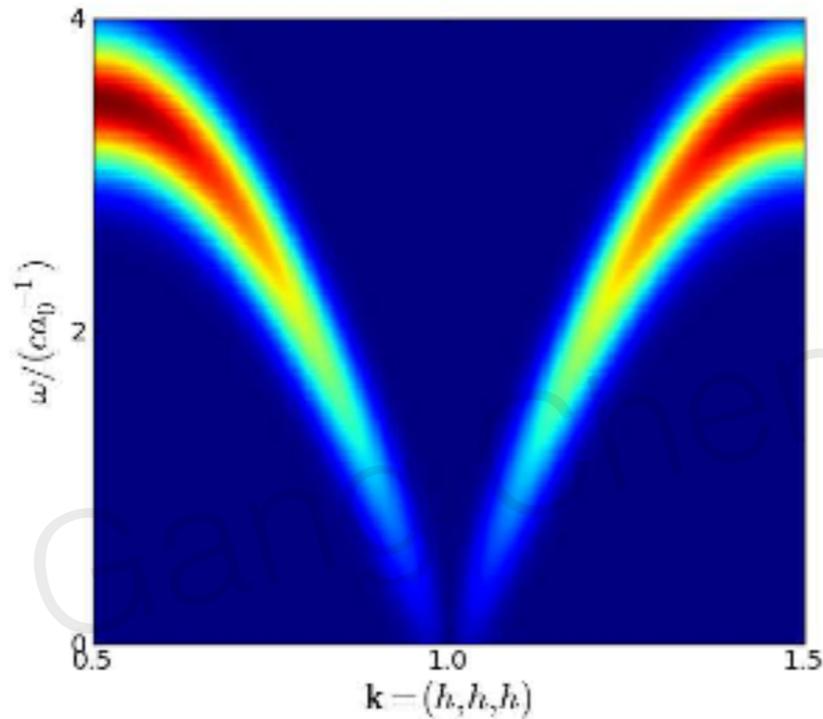
$$\tau_i^z \sim E_{\mathbf{r}\mathbf{r}'} \sim \sum_{\mathbf{r}\mathbf{r}' \in \text{Hex}_d^*} \mathbf{J}_{\mathbf{r}\mathbf{r}'},$$

This theory gives the same magnetic order in Pr<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub>,  
the magnetic order has to break translation symmetry.

GC, PRB 94, 205107 (2016)

# Where to see Dirac's magnetic monopoles?

Let's forget Pr<sub>2</sub>Ir<sub>2</sub>O<sub>7</sub>. Now given a U(1) QSL, what should we measure?



Electromagnetic duality

$\tau^z \sim E$  (emergent electric field)

$$\text{Im}[E_{-\mathbf{k}, -\omega}^\alpha E_{\mathbf{k}, \omega}^\beta] \propto \left[ \delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{\mathbf{k}^2} \right] \omega \delta(\omega - v|\mathbf{k}|),$$

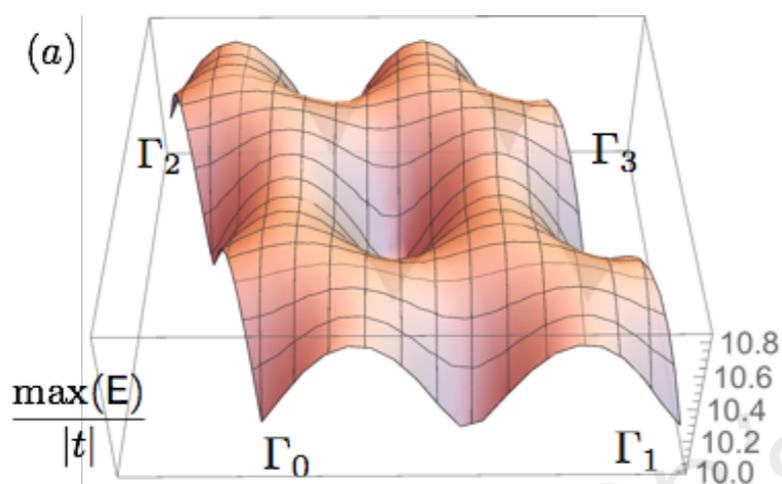
Low energy theory (Savary, Balents)

Electric loop current  $\rightarrow$  Magnetic field  
Magnetic loop current  $\rightarrow$  Electric field

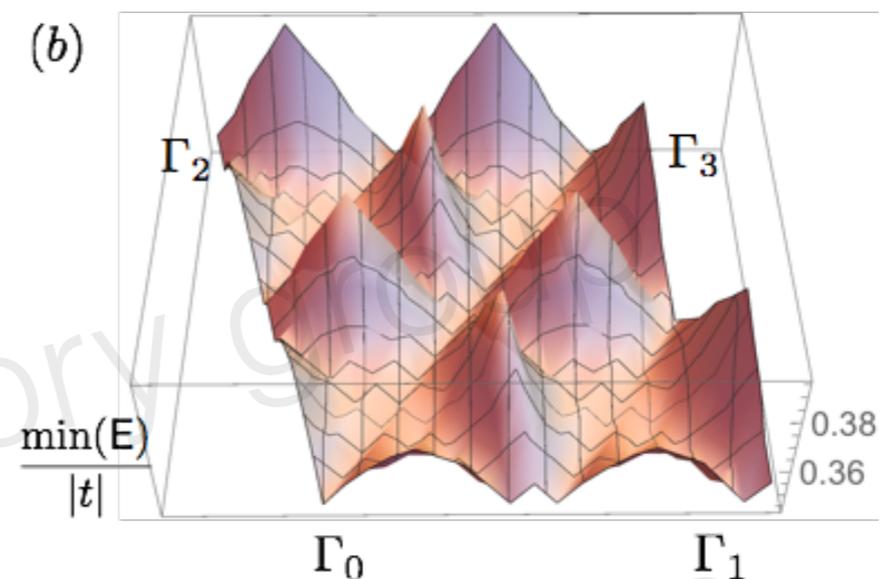
at higher energy, detect monopole continuum

GC, PRB 96, 195127 (2017)

# Spectral periodicity of monopole continuum



Upper excitation edge of monopole continuum



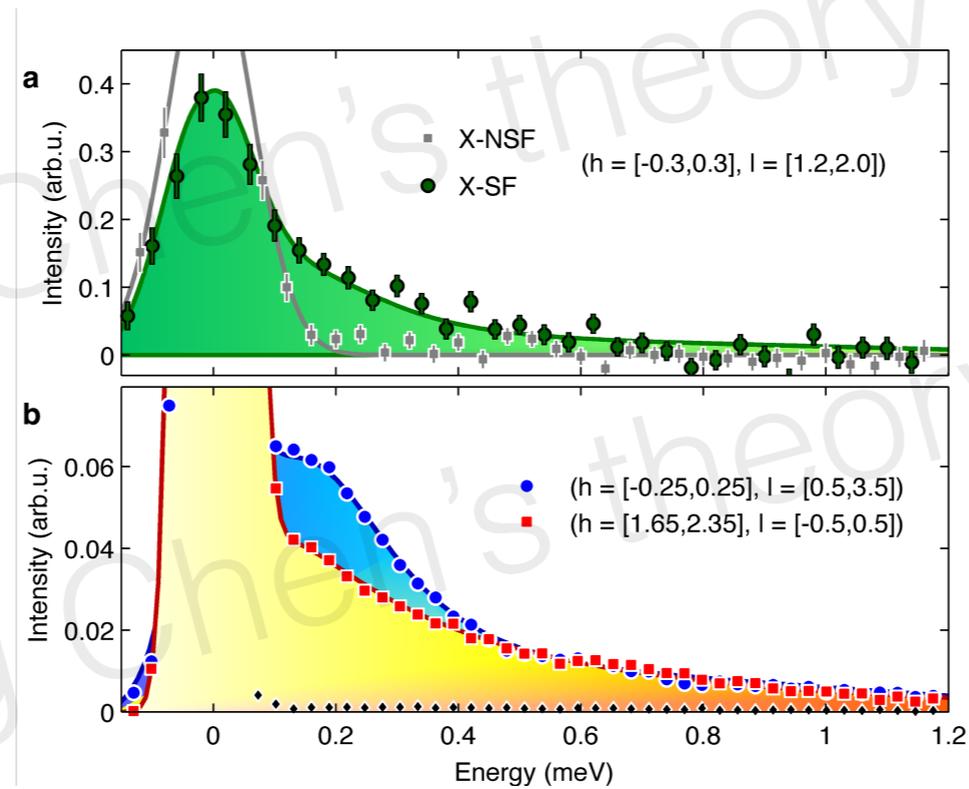
Lower excitation edge of monopole continuum

Projective realization of translation symmetry for monopoles leads to enhanced spectral periodicity of monopole continuum.

General connection was first understood by Xiao-Gang Wen 2001,2002.  
Essin, Hermele, 2015, GC PRB 96, 195127 (2017)

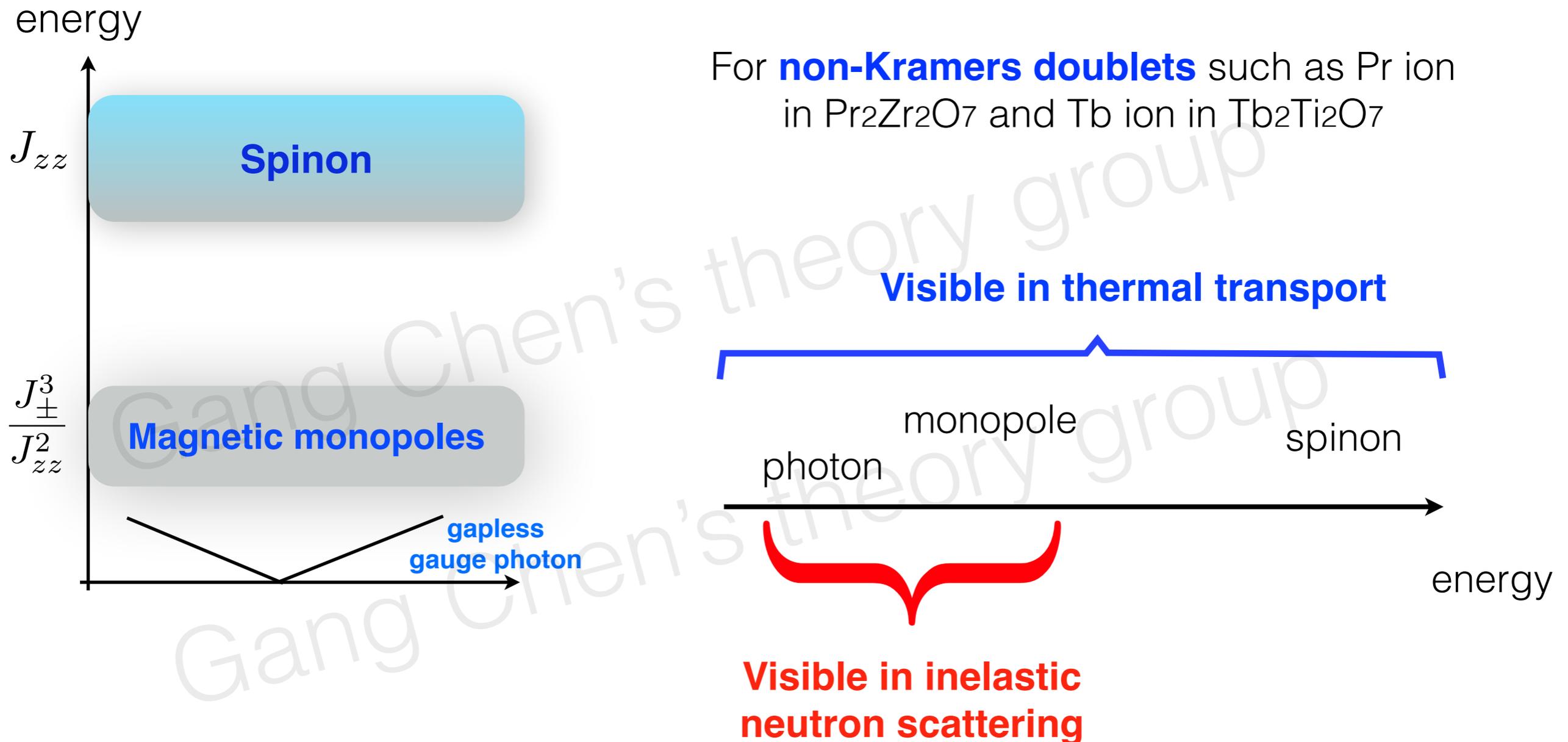
# Monopole continuum in $\text{Pr}_2\text{Hf}_2\text{O}_7$ ?

In fact, continuum has been observed in  $\text{Pr}_2\text{Hf}_2\text{O}_7$



( R. Sibille, et al, arXiv 1706.03604).

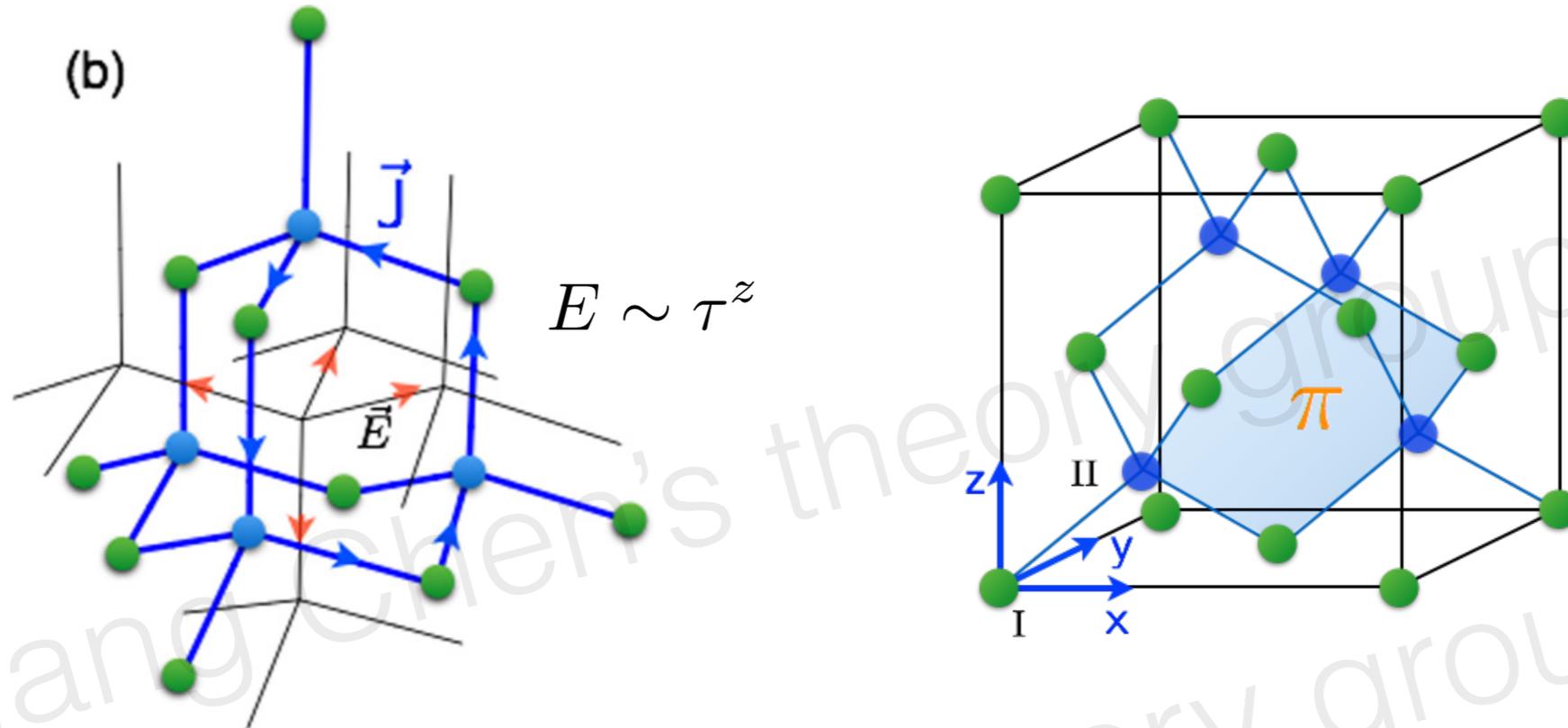
# Further suggestions for experiments 1



GC, PRB 96, 195127 (2017)

Thermal transport in spin ice, Yuji Matsuda, 2016

# Further suggestions for experiments 2



$$H_{\text{Zeeman}} = \vec{B} \cdot \sum_i \tau_i^z \hat{z}_i$$

The weak magnetic field polarizes  $\tau^z$  slightly, and thus modifies the background electric field distribution. This further modulates monopole band structure, creating “**Hofstadter**” monopole band, which may be detectable in inelastic neutron.

GC, PRB 96, 195127 (2017)

# Summary

1. We point out the Pr local moment is proximate to a quantum phase transition from U(1) QSL to the Ising magnetic order in  $\text{Pr}_2\text{Ir}_2\text{O}_7$ .
2. We point out the presence of monopole continuum in inelastic neutron scattering, and predict the enhanced spectral periodicity.
3. We predict the band structure reconstruction of the Ir conduction electrons by the Pr magnetic order. We predict symmetry protected Dirac band touching and topologically protected Weyl nodes.
4. This work points out the interesting interplay of conduction electron and local moments in **hybrid quantum materials**.