Topological thermal Hall effect in spin liquids

GANG CHEN

Fudan University -> University of Hong Kong

Topological thermal Hall effect for topological excitations in spin liquid: Emergent Lorentz force on the spinons
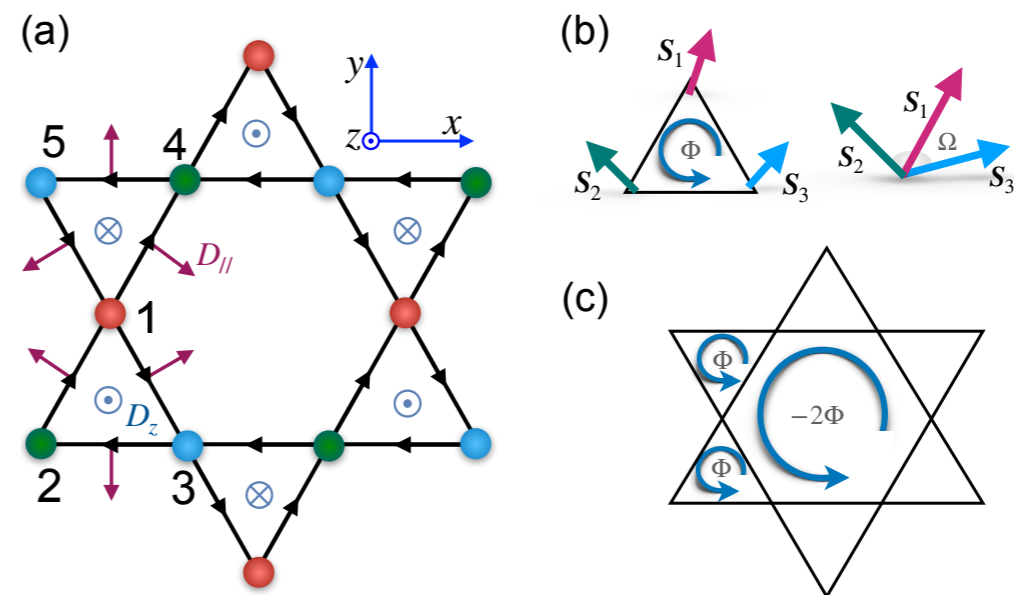
Yong Hao Gao[1] and Gang Chen[1,2*]

[1]*State Key Laboratory of Surface Physics and Department of Physics, Fudan University, Shanghai 200433, China and*
[2]*Department of Physics and Center of Theoretical and Computational Physics,*
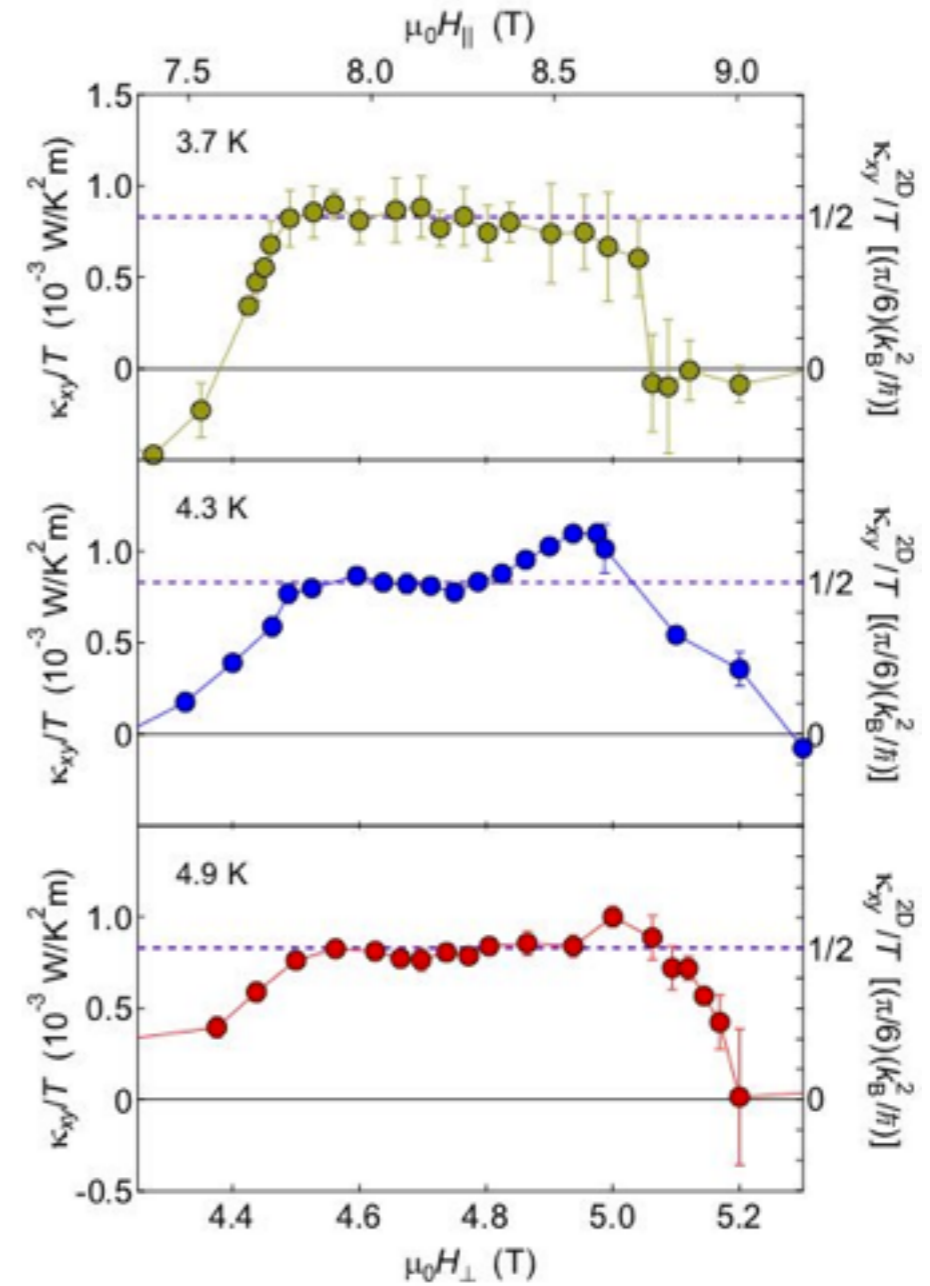*The University of Hong Kong, Pokfulam Road, Hong Kong, China*
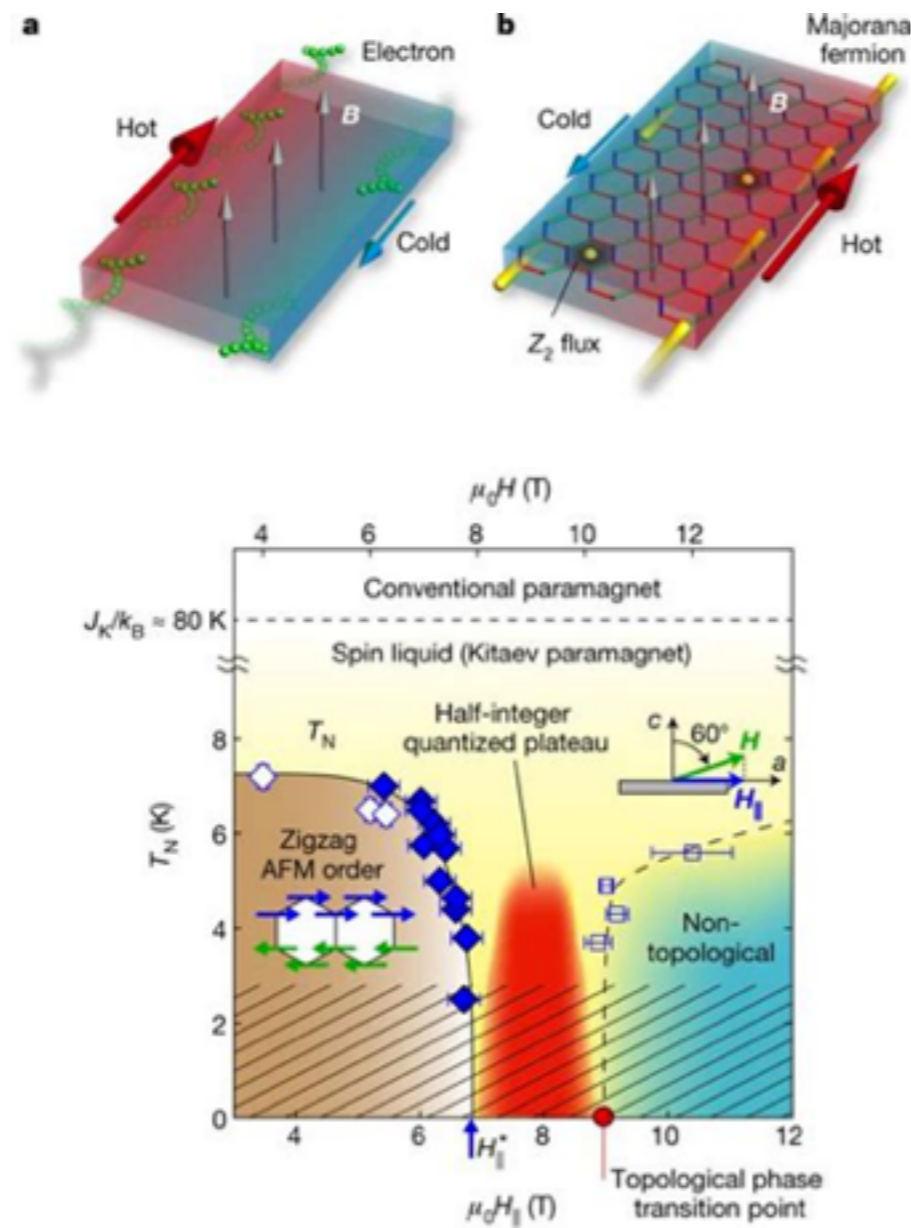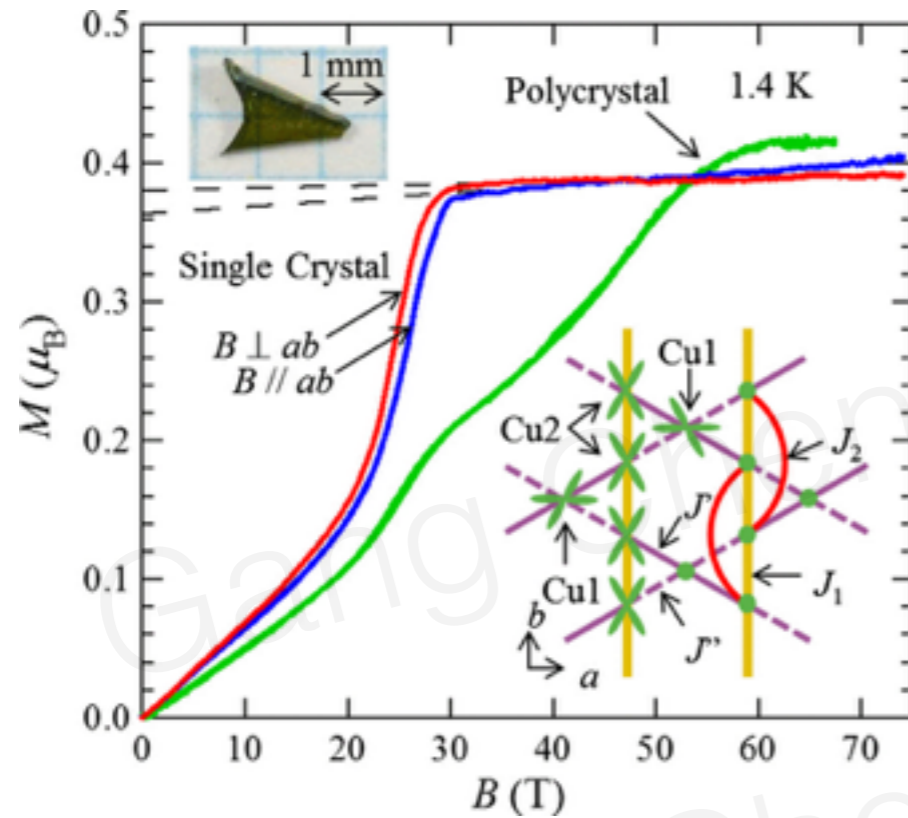
Yong Hao Gao (高永豪)
Fudan University

Yong Hao Gao, GC,
arXiv 1901.XXXXX

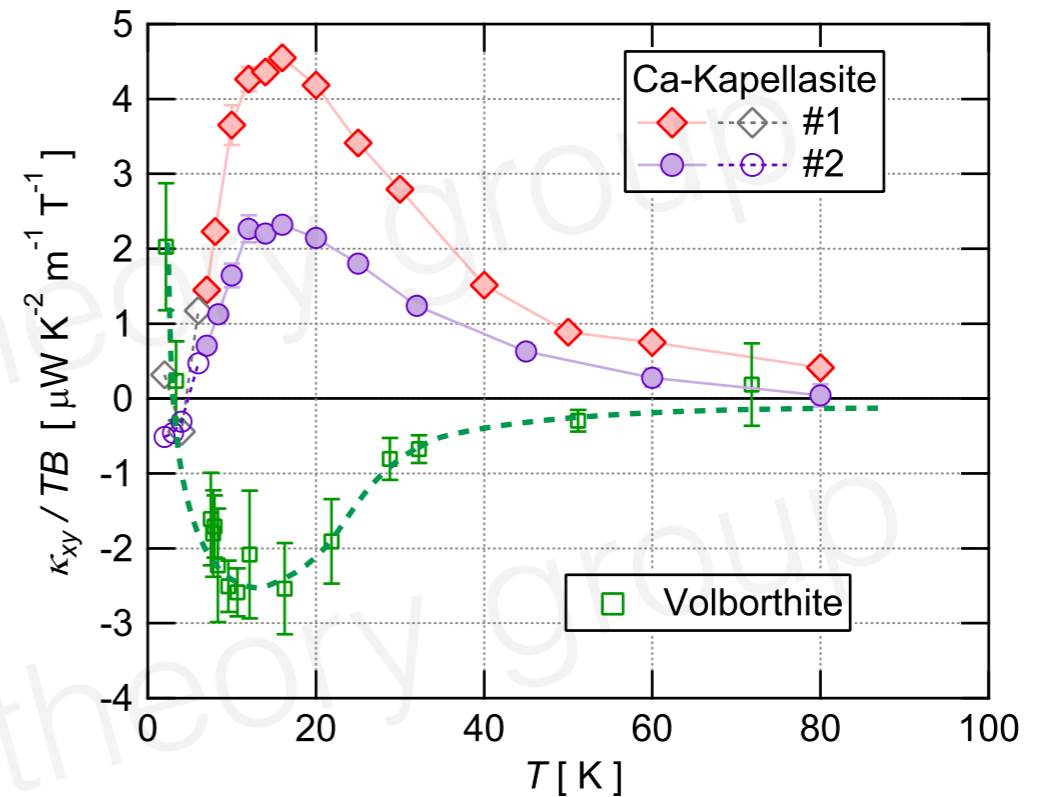# Quantized thermal Hall effect in RuCl₃ ?

Yuji Matsuda's group

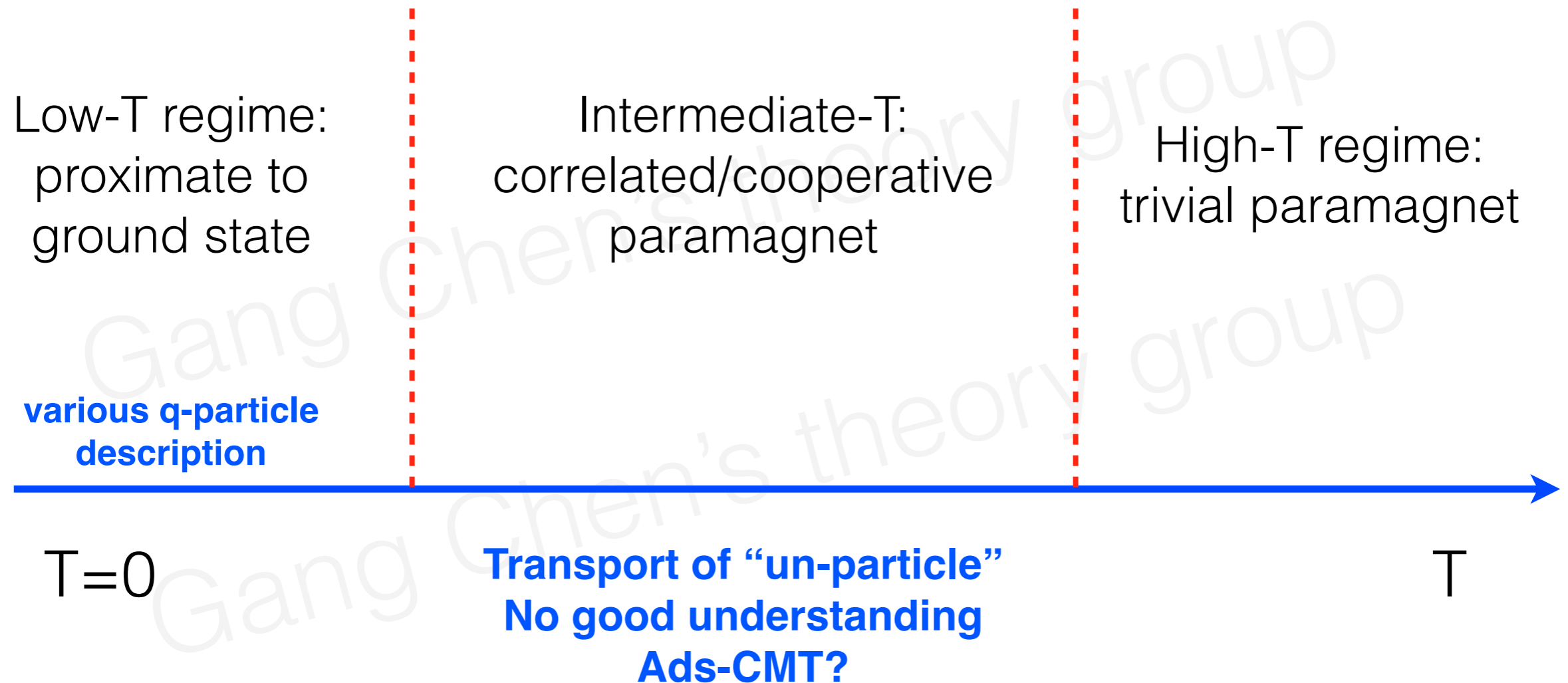# Large thermal Hall effect in spin-1/2 Kagome magnets

Z Hiroi's group

Minoru Yamashita's group
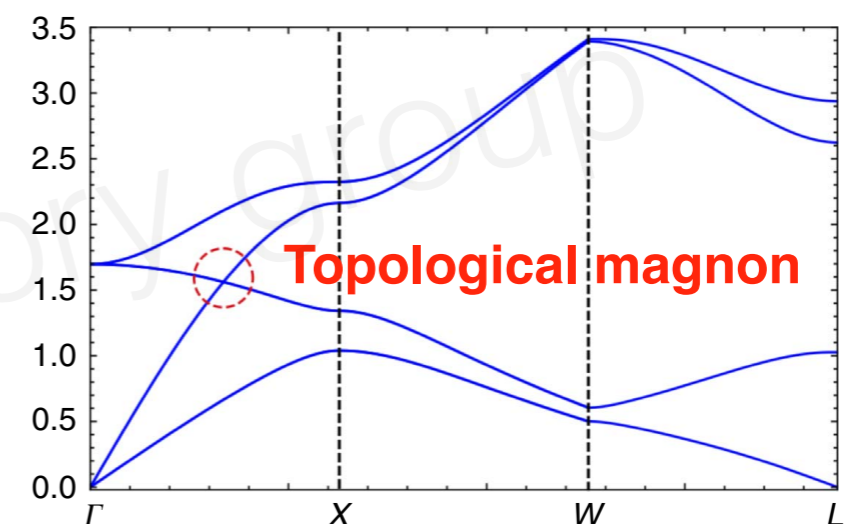
1. Why it is finite? All neutral excitations.
2. Non-monotonic.
3. Opposite signs in two materials.

Thermal transport in Mott insulator

Low-temperature regime 1: simple magnons

a Ground state

b Excited state
High energy

c Excited state
Low energy

Side view

Top view

Topological magnon
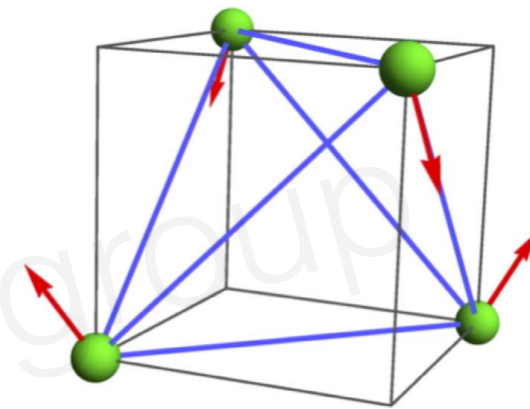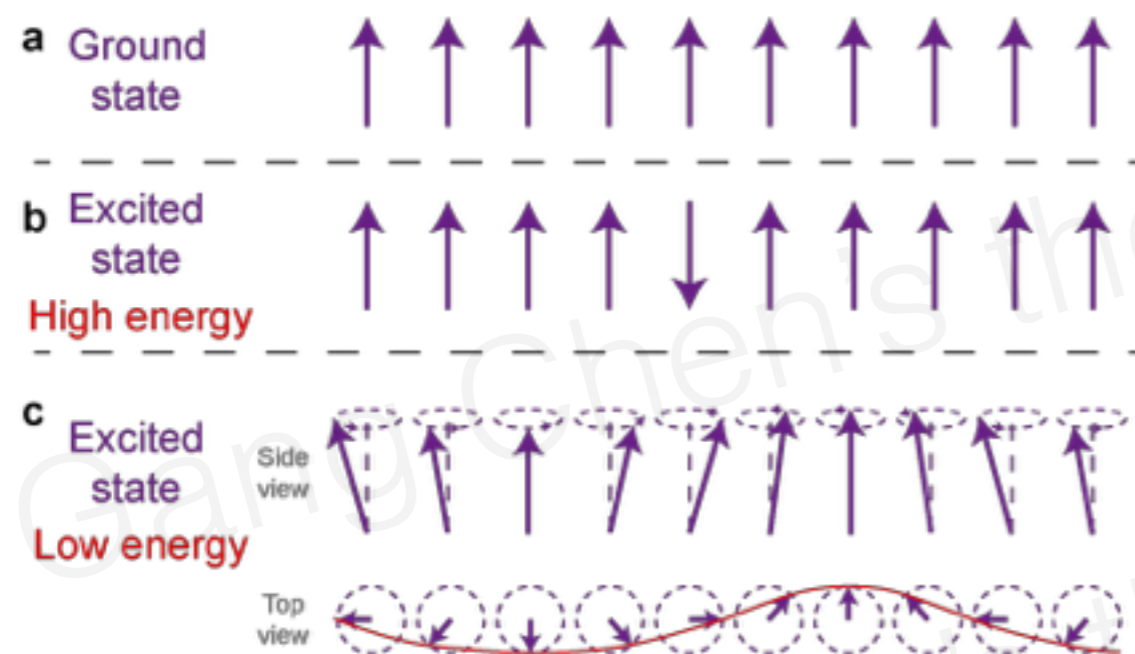
F-Y Li, YD Li, Kim, Balents, Yu, **GC**, Ncomms 2016

Magnetic orders and magnons

Magnon thermal Hall effect
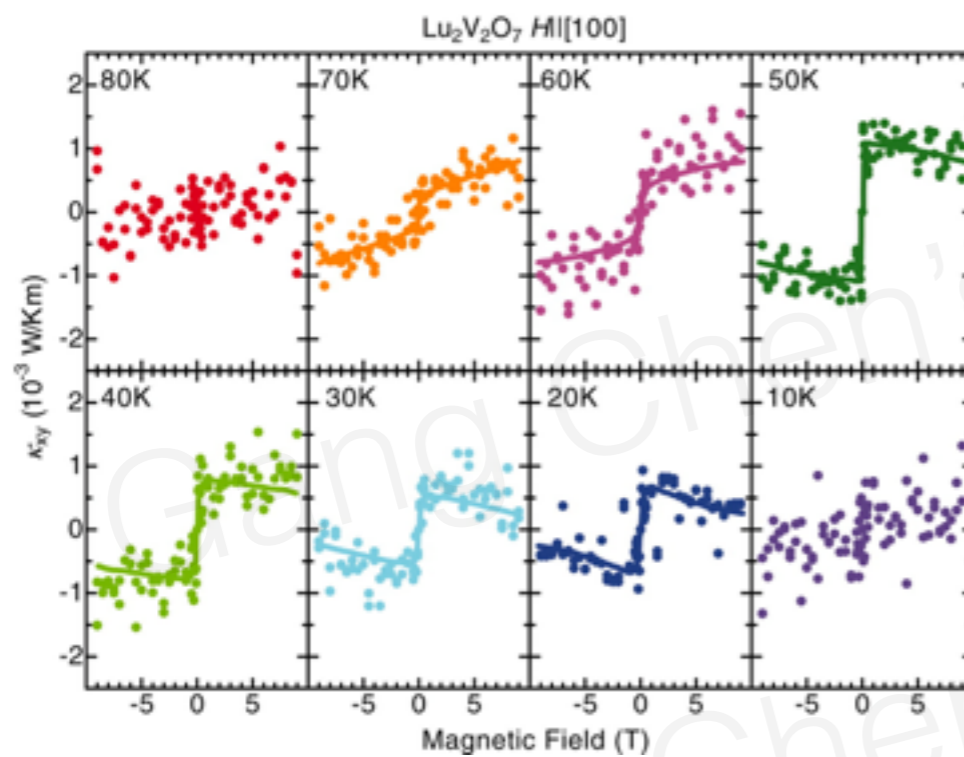
Lu2V2O7: pyrochlore ferromagnet

Nagaosa, Takura, et al
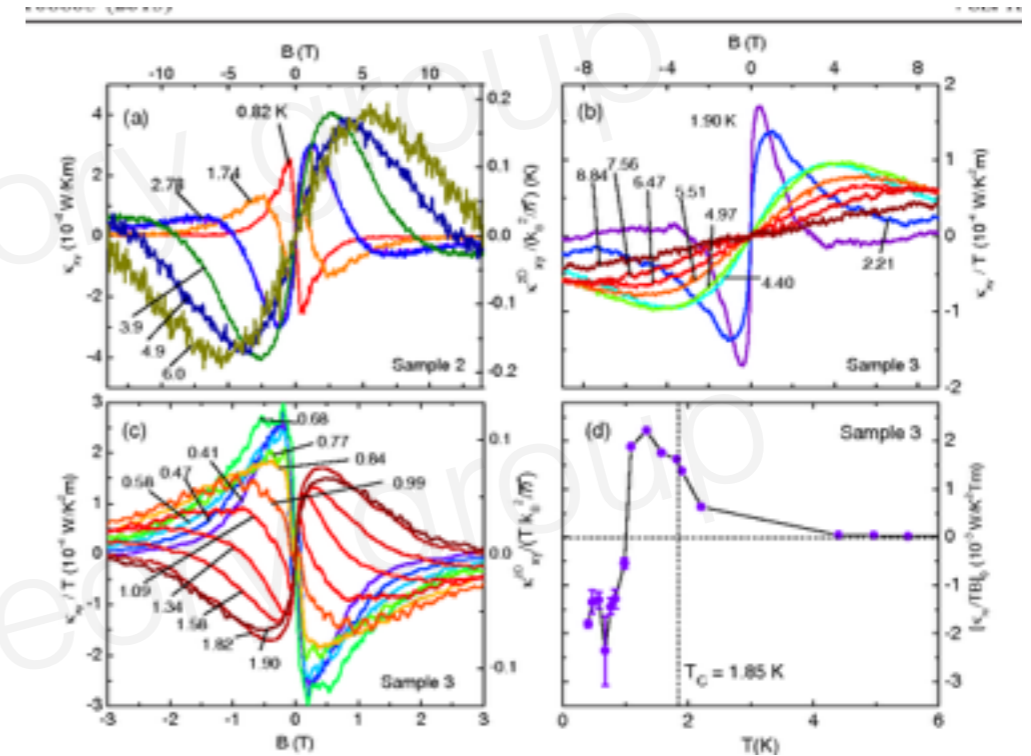
Cu(1-3, bdc): kagome ferromagnet

Young Lee, P Ong, et al

Low-temperature regime 2: quantum spin liquid

PW Anderson

RVB state for QSL

1970-1980s, lattice gauge theory was developing.
We now know that we need lattice gauge theory to **describe** QSLs.
Different branches of theoretical physics merge.

Gauge structure: deconfinement and fractionalization

Emergent gauge structure by fluctuating mean-field states

PW Anderson, Baskaran, Affleck, Xiao-Gang Wen, ...

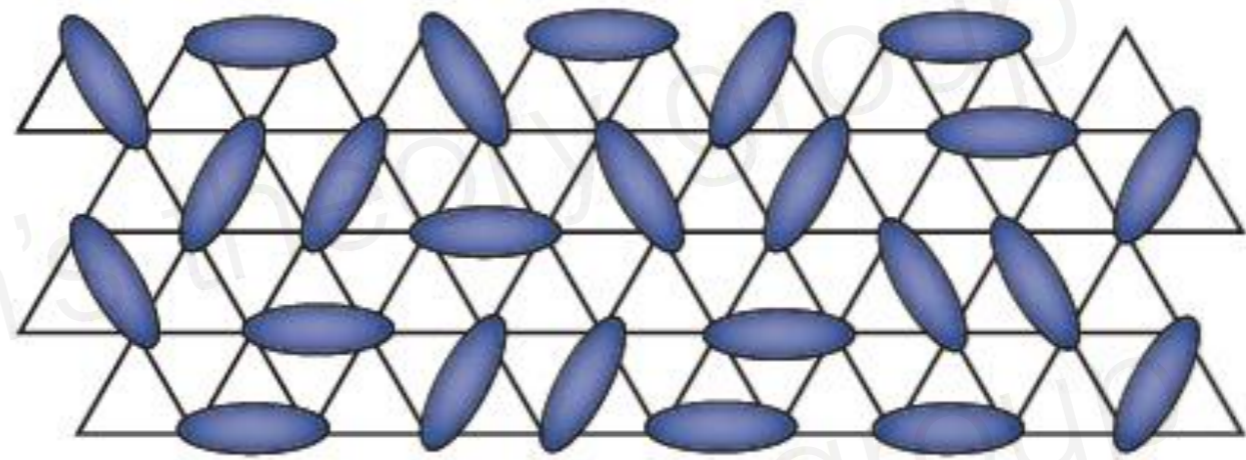"Cutting spin into halves, and glue them back by gauge fields." - Xiao-Gang Wen

We will concentrate on the spin liquid states of a pure
spin-1/2 model on a 2D square lattice

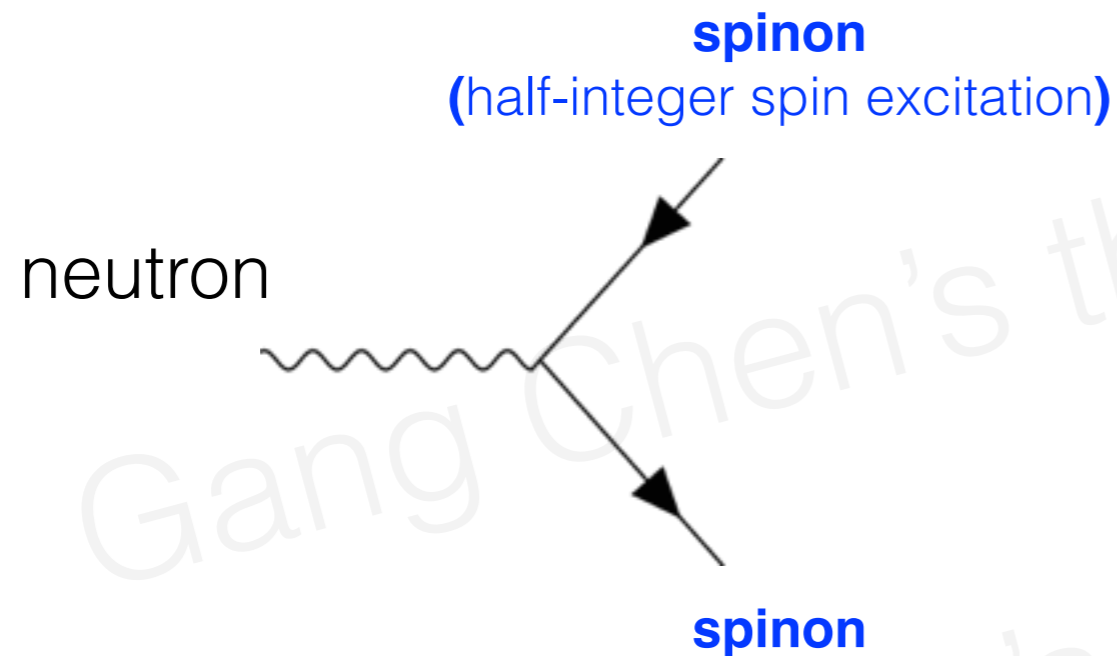$$H_{spin} = \sum_{<ij>} J_{ij} \boldsymbol{S_i} \cdot \boldsymbol{S_j} + ... \qquad (1)$$

$$\boldsymbol{S_i} = \frac{1}{2} f_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} f_{i\beta} \qquad (2)$$

In terms of the fermion operators the Hamiltonian Eq. (1)
can be rewritten as

$$H = \sum_{\langle ij \rangle} -\frac{1}{2} J_{ij} \left(f_{i\alpha}^\dagger f_{j\alpha} f_{j\beta}^\dagger f_{i\beta} + \frac{1}{2} f_{i\alpha}^\dagger f_{i\alpha} f_{j\beta}^\dagger f_{j\beta} \right) \qquad (3)$$

$$
\begin{aligned}
&H_{mean}\\
&= \sum_{\langle ij \rangle} -\frac{3}{8} J_{ij} \left[(\chi_{ji} f_{i\alpha}^\dagger f_{j\alpha} + \eta_{ij} f_{i\alpha}^\dagger f_{j\beta}^\dagger \ \epsilon_{\alpha\beta} + h.c) \right.\\
&\qquad\qquad \left. -|\chi_{ij}|^2 - |\eta_{ij}|^2 \right]\\
&+ \sum_{i} \left[a_0^3 (f_{i\alpha}^\dagger f_{i\alpha} - 1) + [(a_0^1 + ia_0^2) f_{i\alpha} f_{i\beta} \epsilon_{\alpha\beta} + h.c.] \right]
\end{aligned} \qquad (7)
$$

Consequence: spin fractionalization and continuum

spinon
(half-integer spin excitation)

neutron

spinon

$$P = q_1 + q_2$$

$$E = \omega(q_1) + \omega(q_2)$$

YbMgGaO$_4$
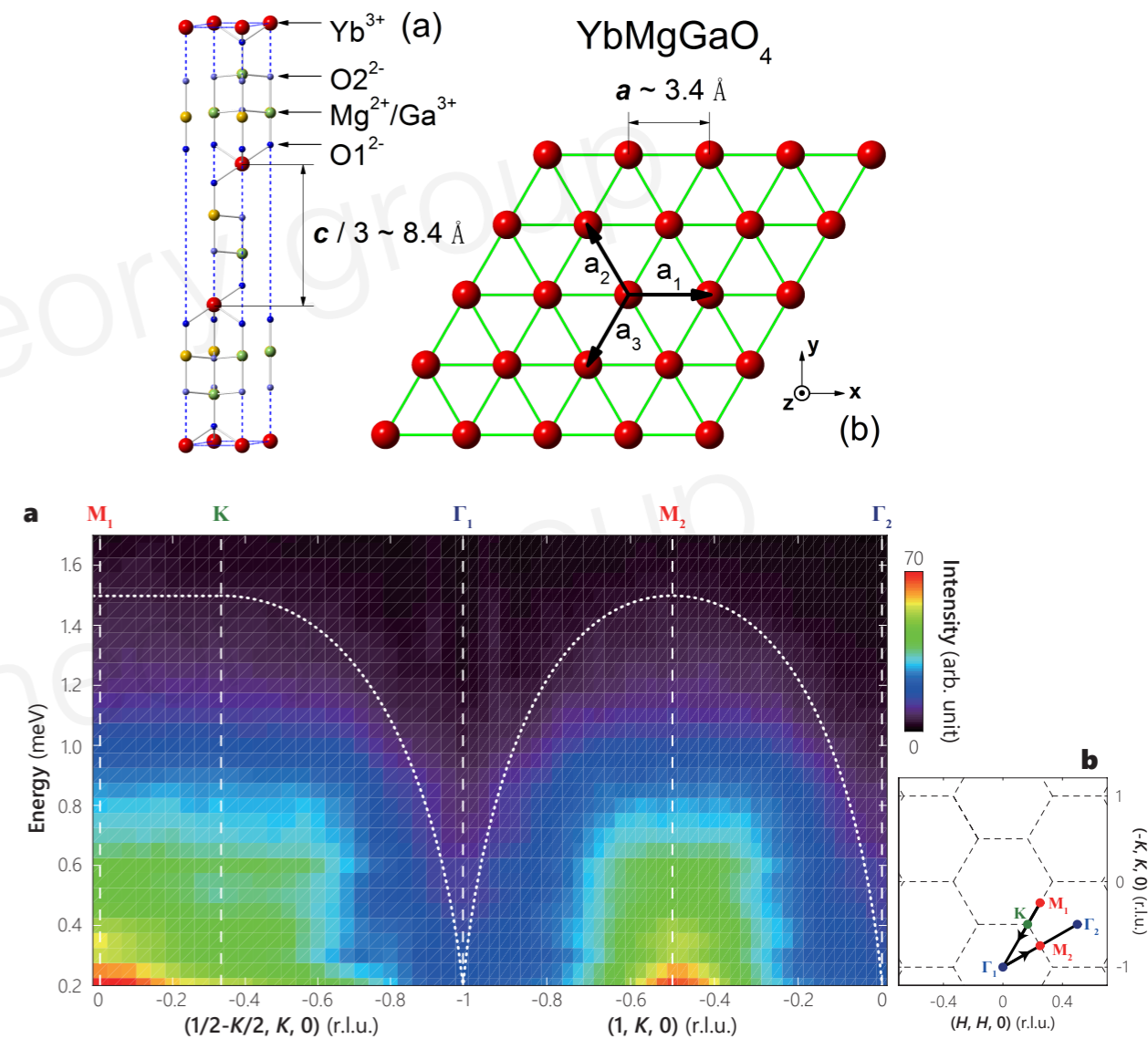
With Qingming Zhang, Jun Zhao
2015-2018

Consequence: symmetry fractionalization

XG Wen PRB, 2002

translation symmetry is realized projectively

$$T_\mu^m T_\nu^m (T_\mu^m)^{-1} (T_\nu^m)^{-1} = e^{i\pi} = -1.$$
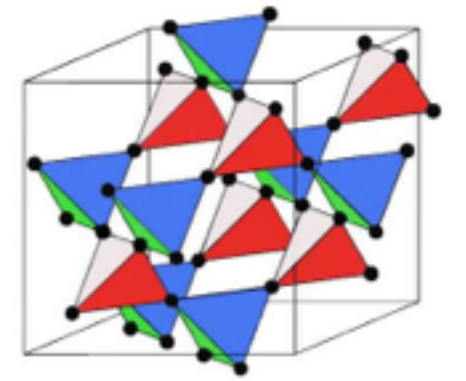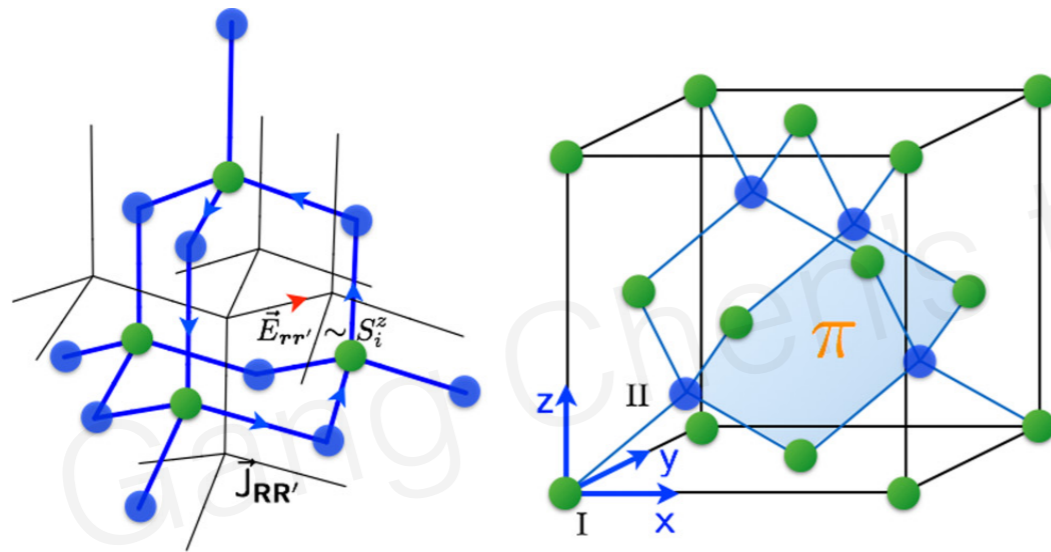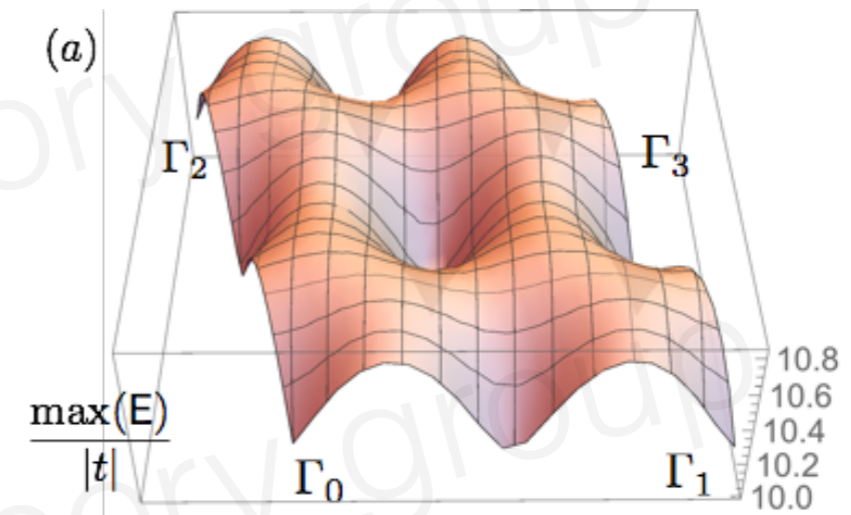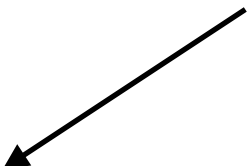

Enhanced spectral periodicity in monopole continuum for 3D U(1) QSL

GC, PRB, 96,195127 (2017)

Matter-gauge coupling?

Deconfinement and fractionalization are consequences of the matter-gauge coupling in the deconfined phase.

What is the direct evidence of the matter-gauge coupling?

$$H_{\text{dual}} = \boxed{-t \sum_{\langle \mathbf{RR'} \rangle} e^{-i2\pi\alpha_{\mathbf{RR'}}} \Phi_{\mathbf{R}}^{\dagger} \Phi_{\mathbf{R'}}} - \mu \sum_{\mathbf{R}} \Phi_{\mathbf{R}}^{\dagger} \Phi_{\mathbf{R}}$$

$$+ \frac{U}{2} \sum_{\lozenge^*} \left(curl\alpha - \frac{\eta_r}{2} \right)^2 - K \sum_{\langle \mathbf{RR'} \rangle} \cos B_{\mathbf{RR'}} + \cdots,$$

Electrons in fields: signature of matter-gauge coupling?

right-moving skipping orbit

cyclotron orbits

left-moving skipping orbit
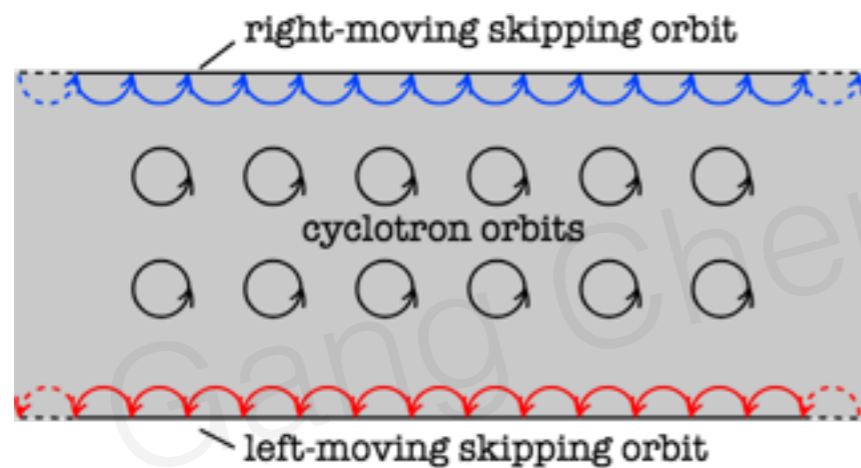
Cyclotron motion of electrons:
from Lorentz force

b

$\theta_1 = 6.5°$

S_{xy} (a.u.)

$\theta_1 = -6.5°$

B (T)
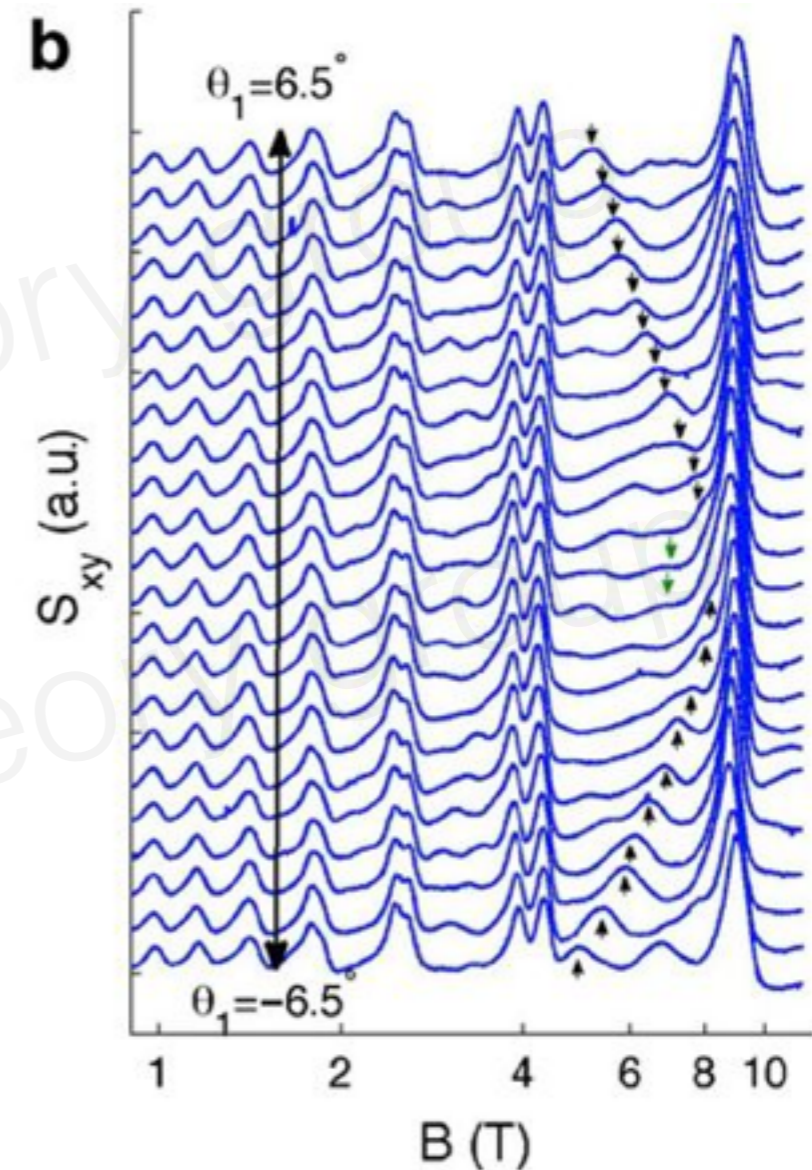
Quantum oscillation in bismuth

Consequence of matter-gauge coupling in QSL ?

AF
J_{ext}

spin liquid
spin gap; many singlets

spin liquid
no spin gap

J_4/J_2

~0.1

$\pi/2$−flux

no flux (spinon FS)

0

0.7

$g_4 J_4/(g_2 J_2)$

Orbital magnetic field effects in spin liquid with spinon Fermi sea:
Possible application to κ-(ET)$_2$Cu$_2$(CN)$_3$

spin liquid

AF

???

projected Fermi sea

J_4/J_2

Olexei I. Motrunich

Kavli Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106-4030

0.14

$$\hat{H}_{\mathrm{ring}} = J_2 \sum_{\bullet\!-\!\bullet} P_{12} + J_4 \sum \left(P_{1234} + P_{1234}^\dagger \right)$$

$|\chi_\phi|$

$\nu=n-1$

n

n+1

$1/\nu = 2\phi_\Delta^{int}/\pi$
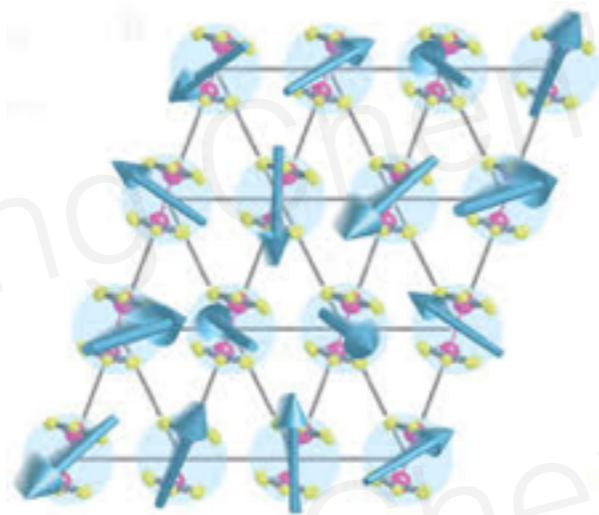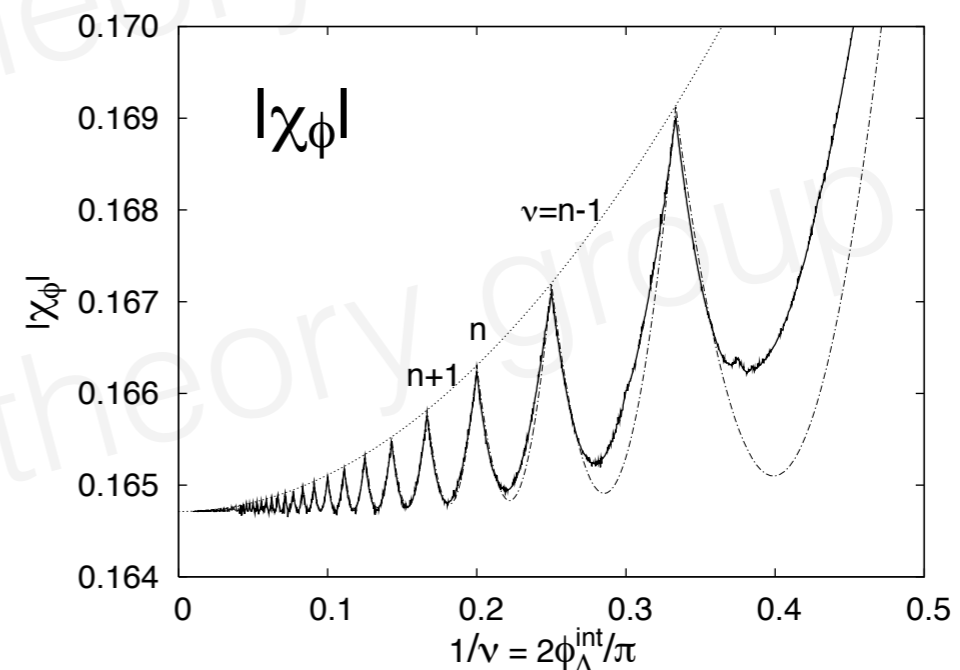
Quantum oscillation of spinon Fermi surface

Weak Mott insulators: spinons are not far from electrons.

Theory of the Thermal Hall Effect in Quantum Magnets

Hosho Katsura[1], Naoto Nagaosa[1,2], Patrick A Lee[3]

$$\mathcal{L} = \sum_{j,\sigma} f_{j\sigma}^\dagger (\partial_\tau - ia_j^0 - \mu) f_{j\sigma}$$
$$- \sum_{j,k} t_f e^{ia_{jk}} f_{j\sigma}^\dagger f_{k\sigma} + \mathcal{L}_g, \tag{7}$$

Following the previous works [14, 25], we take the spinon metal with a Fermi surface as a candidate for the 2D quantum spin liquid realized in κ-$(ET)_2Cu_2(CN)_3$[12]. In a magnetic field $F_{xy} = B_z$, the average of the gauge flux $\langle \mathcal{F}_{xy} \rangle = cF_{xy}$ is induced with c a constant of the order of unity because of the coupling between $\mathcal{F}_{xy}$ and F_{xy} in $\mathcal{L}_g$ [14, 24]. Therefore, the spinons are subject to the *effective magnetic field* $\langle \mathcal{F}_{xy} \rangle$ and to the Lorentz force.

カゴメ格子銅鉱物
ボルボサイト

1 mm

quantum spin on Cu²⁺ ion

Why there is thermal Hall effect in strong Mott insulator QSL?
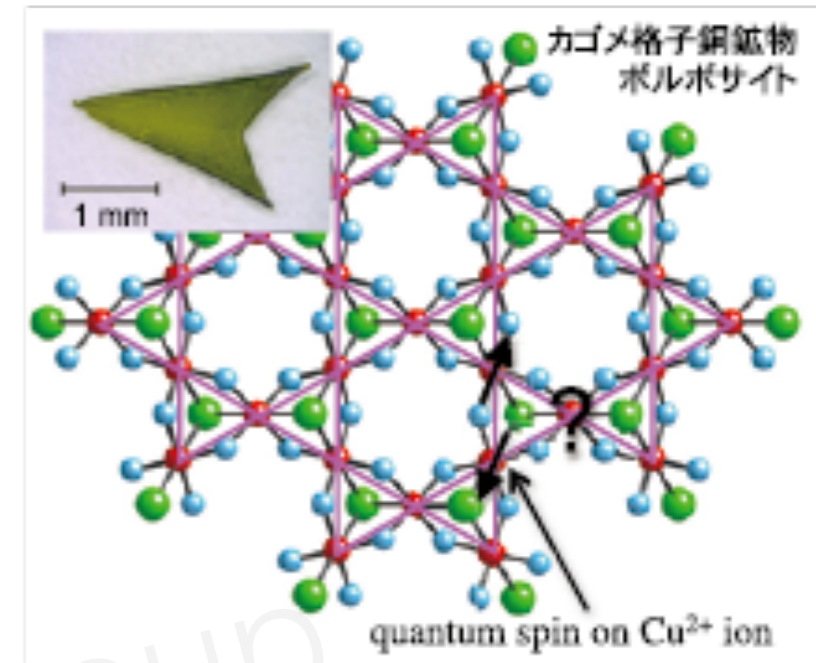
The D.O.F. are spins, not electrons.

The excitations are neutral spinons, do not carry external U(1) gauge charge.

There is only Zeeman coupling to the field.

How can magnetic field twist the spinon motion and create Hall effect?

Three cases of thermal Hall effects

1. Chiral spin liquids: quantized w/o field

2. Magnetic field changes the spinon band topology
   and creates chiral edge states: e.g. Kitaev spin liquid,
   (not much different from case 1), apply to QSL w/ gapped gauge.

3. External field comes to modify the internal continuous gauge field and
   thereby indirectly twists the motion of matter fields, and generate
   thermal Hall effects.

Plover Cove Reservoir
of Hong Kong

Built a fresh water reservoir in the sea (1960s)
using the existing landscapes
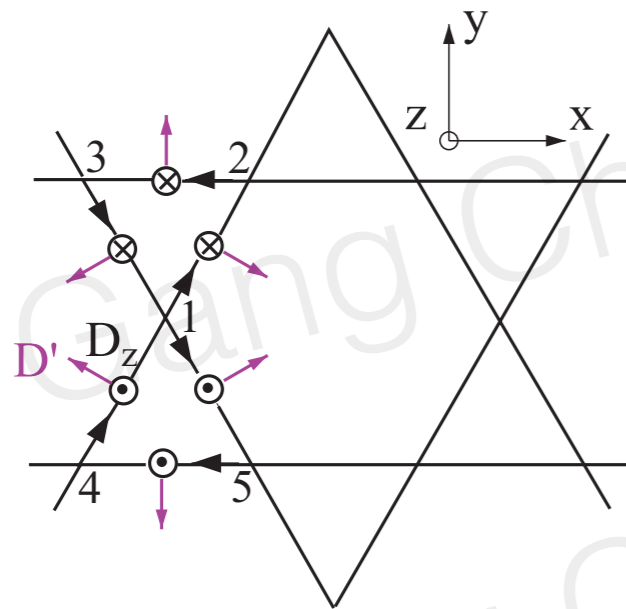
A distant observation by Patrick Lee and Nagaosa

Proposal to use neutron scattering to access scalar spin chirality fluctuations in kagome lattices

Patrick A. Lee[*]

Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA

Naoto Nagaosa

$$\langle f_\chi | S_z(\boldsymbol{r}_1)|i\rangle = -\sum_{jk} \frac{2D_{jk}}{\Delta_t}\langle \alpha_\chi | S_z(\boldsymbol{r}_1)\hat{z}\cdot \boldsymbol{S}(\boldsymbol{r}_j)\times \boldsymbol{S}(\boldsymbol{r}_k)|0\rangle.$$

$$\mathcal{H}_{\mathrm{DM}} = \sum_{\langle ij\rangle} \boldsymbol{D}_{ij}\cdot \boldsymbol{S}_i \times \boldsymbol{S}_j$$

Pairwise spin correlation contains a piece of gauge field correlation.

$$\sin\Phi = \tfrac{1}{2}\boldsymbol{S_1}\cdot(\boldsymbol{S_2}\times\boldsymbol{S_3})$$

XG Wen, F Wilczek, A Zee, PRB, 1989

Our observation: induced internal gauge flux and emergent Lorentz force

Yong Hao Gao
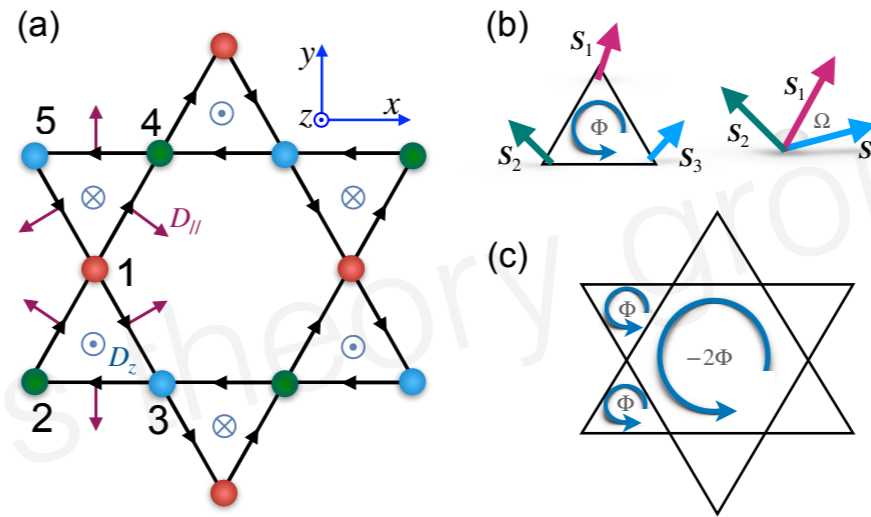
$$H = \sum_{i,j} J_{ij} \boldsymbol{S_i} \cdot \boldsymbol{S_j} + \sum_{i,j} \boldsymbol{D}_{ij} \cdot \boldsymbol{S_i} \times \boldsymbol{S_j} - \sum_i B S_i^z,$$

$$\langle \boldsymbol{S}_i \times \boldsymbol{S}_j \cdot \boldsymbol{S}_k \rangle \sim \langle \boldsymbol{S}_i \times \boldsymbol{S}_j \rangle \cdot \langle \boldsymbol{S}_k \rangle \neq 0$$
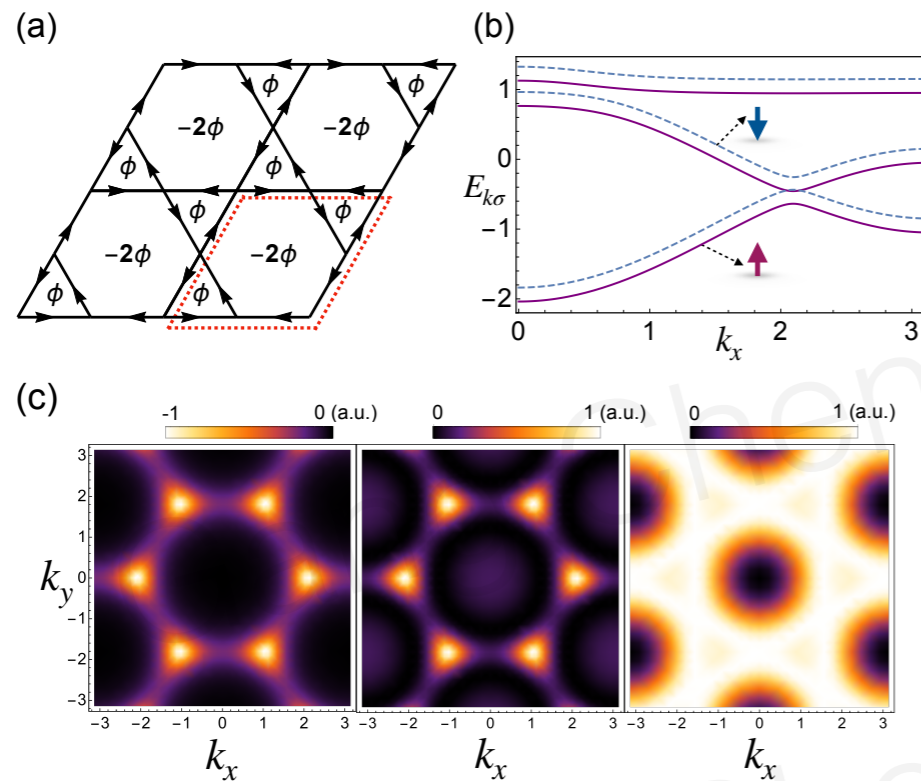
The combination of Zeeman coupling and DMI generates
an internal U(1) gauge flux distribution.

This provides a way to **control** emergent D.O.F. with external probes.

Topological thermal Hall effect

Yong Hao Gao

(a)

(b)

(c)

Spinon bands and
Berry curvatures

$$H_{\mathrm{MF}}[\phi] = -t \sum_{\langle ij \rangle} [e^{-i\phi/3} f_{i\sigma}^{\dagger} f_{j\sigma} + h.c.] - \mu \sum_{i} f_{i\sigma}^{\dagger} f_{i\sigma}$$

$$-B \sum_{i,\alpha\beta} f_{i\alpha}^{\dagger} \frac{\sigma_{\alpha\beta}^{z}}{2} f_{i\beta},$$
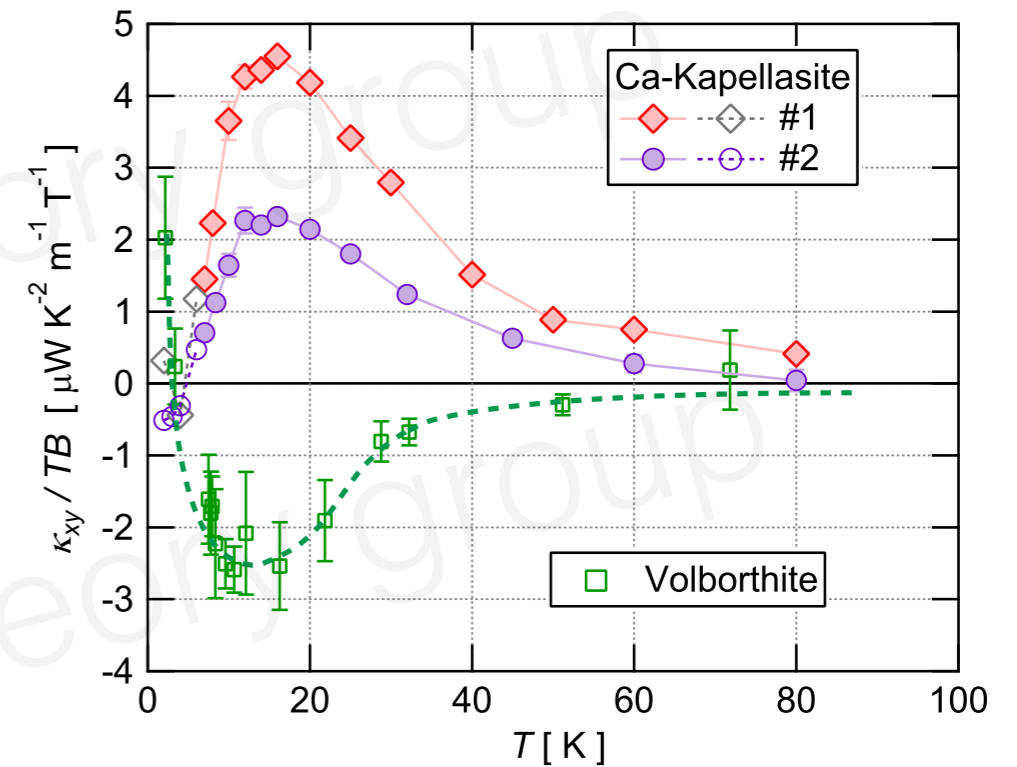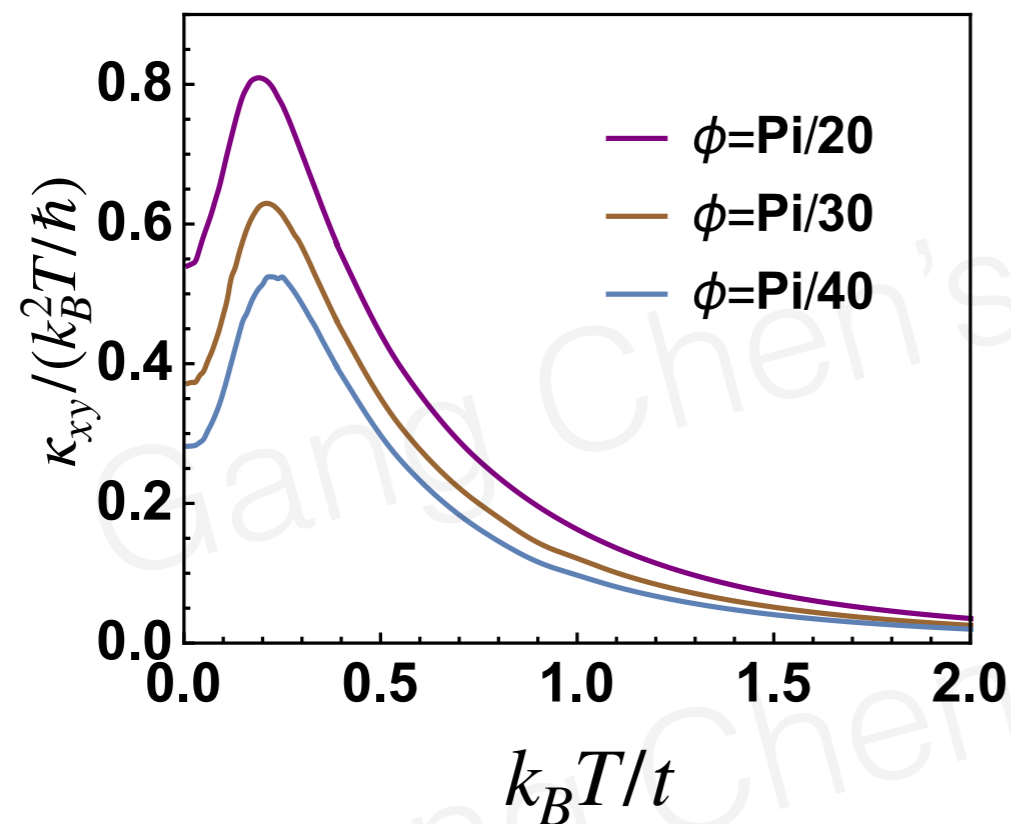
$$\kappa_{xy} = -\frac{1}{T} \int d\epsilon (\epsilon - \mu)^2 \frac{\partial f(\epsilon, \mu, T)}{\partial \epsilon} \sigma_{xy}(\epsilon).$$

$$\sigma_{xy}(\epsilon) = -\sum_{\boldsymbol{k},\sigma,\xi_{n,\boldsymbol{k}}<\epsilon} \Omega_{n,\boldsymbol{k},\sigma}$$
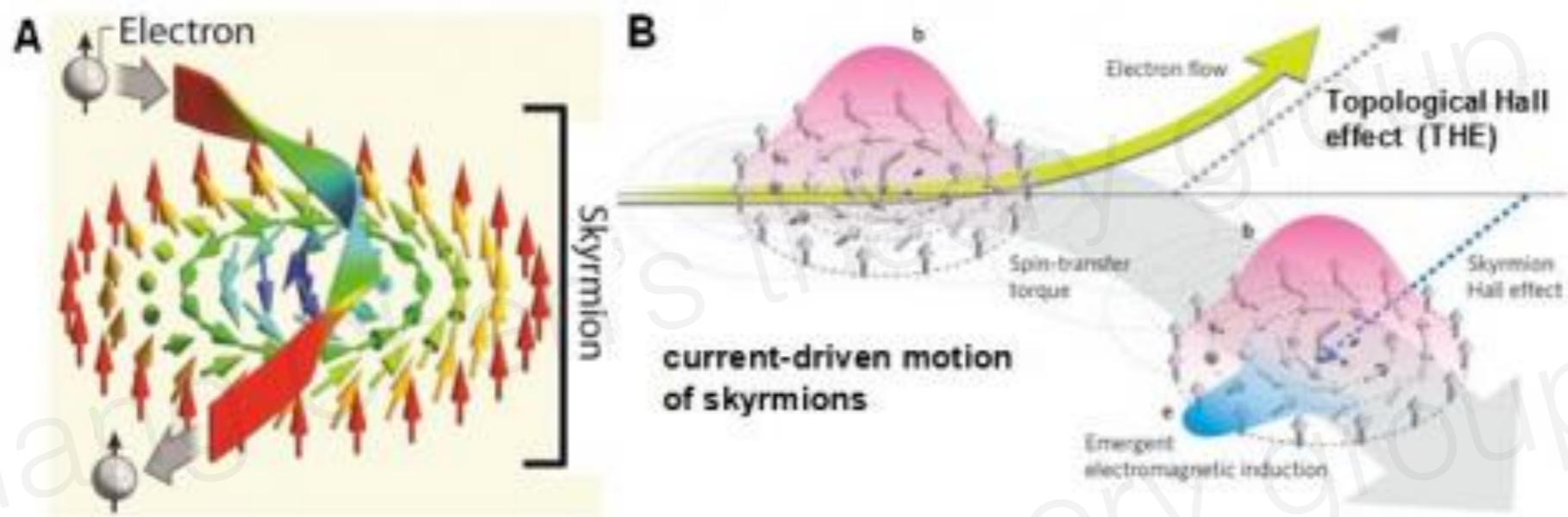
Topological thermal Hall effect

Yong Hao Gao

Minoru Yamashita's group

1. Why it is finite? All neutral excitations.
2. Non-monotonic.
3. Opposite signs in two materials.

Topological Hall effect in itinerant magnets

Nagaosa, Takura, etc 2013

also due to scalar spin chirality

Early contribution: Luttinger 1960s, Jinwu Ye, YB Kim, A Millis, PRL 1999

Other recent works

arXiv:1812.08792

Thermal Hall effect in square-lattice spin liquids: a Schwinger boson mean-field study

Rhine Samajdar,[1] Shubhayu Chatterjee,[1, 2] Subir Sachdev,[1, 3] and Mathias S. Scheurer[1]

Spin Thermal Hall Conductivity of a Kagome Antiferromagnet

Hayato Doki,[1] Masatoshi Akazawa,[1] Hyun-Yong Lee,[1] Jung Hoon Han,[2] Kaori Sugii,[1] Masaaki Shimozawa,[1]
Naoki Kawashima,[1] Migaku Oda,[3] Hiroyuki Yoshida,[3] and Minoru Yamashita[1,*]

[1]The Institute for Solid State Physics, The University of Tokyo, Kashiwa, 277-8581, Japan
[2]Department of Physics, Sungkyunkwan University, Suwon 16419, Korea
[3]Department of Physics, Faculty of Science, Hokkaido University, Sapporo 060-0810, Japan

quantum antiferromagnets [2,3,7,8,49–53]. In the SBMFT framework, spin is expressed by a pair of bosons $(b_{i\uparrow}, b_{i\downarrow})$ as $\mathbf{S}_i = \frac{1}{2} \sum_{\alpha,\beta=\uparrow,\downarrow} b_{i\alpha}^\dagger \boldsymbol{\sigma}_{\alpha\beta} b_{i\beta}$, where $\boldsymbol{\sigma}$ is the Pauli matrices. We decouple the Hamiltonian by taking a mean-field value of the bond operator $\chi_{ij} = \langle b_{i\sigma}^\dagger b_{j\sigma} \rangle$ and diagonalize it to

They use Schwinger bosons, then do mean-field decoupling, obtain mean-field spinon Hamiltonian, and calculate the spinon thermal Hall contribution.

Summary

1. We point out the physical origin of emergent Lorentz force on spinons and obtain the resulting topological thermal Hall effects.

2. We establish the connection between microscopic interactions and emergent D.O.F. and thus provide a scheme to control the emergent D.O.F.

3. Our results can be extended to other non-centrosymmetric QSLs with with Dzyaloshinskii-Moriya interaction.

4. **Thermal transports in Mott insulators are not well understood.**