“Magnetic” monopole condensation transition out of quantum spin ice: quantum spin ice in pyrochlore iridates

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Longer talk can be found at KITP website last Sep.
Job opportunities

• **Postdocs** are generously funded and will have tremendous freedom.

Shanghai, China
Pyrochlore Iridate and Pyrochlore Spin Ice

- Pyrochlore Iridates
- Pr$_2$Ir$_2$O$_7$
- Pyrochlore Spin Ice
Pyrochlore iridates: $\text{Pr}_2\text{Ir}_2\text{O}_7$

Many nice experimental works by S Nakatsuji, P Gegenwart, L Balicas, etc

Ref: D Pesin, L Balents, 2009, Xian-Gang Wan, etc 2010, Witczak-Krempa, Yong Baek Kim, SungBin Lee; Michael Hermele, Gang Chen, etc

$\text{R}_2\text{Ir}_2\text{O}_7$  

K Matsuhiira, M Wakeshima, Y Hinatsu, S. Takagi  
JPSJ, 2011
My proposal for Pr$_2$Ir$_2$O$_7$-delta

Pr local moments are close to a “magnetic” monopole condensation transition from quantum spin ice quantum spin liquid to an AFM long-range ordered state.

The Ir conduction electrons may drive the transition, but do not influence the nature of the phase transition.
Pr\textsubscript{2}Ir\textsubscript{2}O\textsubscript{7} a featureless disordered state near an ordered state

ARPES: quadratic band touching of Ir 5d electrons

Nakatsuji, etc
PRL 96, 087204 (2006)

metamagnetic transition

T Kondo, S Shin, etc 2014
B J Yang, Yong Baek Kim 2011
E G Moon, CK Xu, Y B Kim, L Balents, 2013

Expts are sample dependent.
Recently, some samples are found AFM ordered.
Quantum spin ice U(1) spin liquid

\[ H = J_{zz} \sum_{\langle ij \rangle} \tau_i^z \tau_j^z - J_{\pm} \sum_{\langle ij \rangle} (\tau_i^+ \tau_j^- + \tau_i^- \tau_j^+ ) + \cdots \]

Senthil Motrunich, 2002
Hermele, Fisher, Balents, 2003
Moessner, Huse, Isakov, YB Kim…

QSI (U(1) QSL) is an example of Xiao-Gang Wen’s string net condensed state. The physics of QSI is described by compact quantum electrodynamics.
Confinement transition out of U(1) quantum spin liquid

Spinons are deconfined.

Spinons are confined!

More generally, for non-Kramers’ doublet, the magnetic transition out of QSI MUST be a confinement transition, this may apply to Tb2Ti2O7.
Lattice gauge theory formalism: technical part

\[ E_{rr'} \sim \tau_i^z, e^{iA_{rr'}} \sim \tau_i^+ \]

Hermelo, Fisher, Balents, 2004

\[ H_{\text{ring}} = - \sum_{\mathcal{O}_p} \frac{K}{2} (\tau_1^+ \tau_2^- \tau_3^+ \tau_4^- \tau_5^+ \tau_6^- + h.c.) \]

\[ H_{\text{LGT}} = \sum_{\langle rr' \rangle} \frac{U}{2} (E_{rr'} - \frac{\epsilon_r}{2})^2 - \sum_{\mathcal{O}_d} K \cos(\text{curl } A) \]

\( H_{\text{LGT}} \) captures the **universal properties** of QSI.

- In an ordered state, \( <\tau_{z>}!=0, <\tau^+> \) is strongly fluctuating.
- In the gauge language, “E field” is static, “B magnetic field” is strongly fluctuating, the magnetic monopole (carrying magnetic charge) is condensed, which confines the electric charge carriers (spinons).
Electromagnetic duality

Monopole lives on dual diamond lattice, carry magnetic charge or dual U(1) gauge charge.

Insert monopole variables

\[ H_{\text{dual}} = \sum_{\mathcal{O}_d^*} \frac{U}{2} (\text{curl} \ a - \mathbf{E})^2 - \sum_{r,r'} K \cos B_{rr'} \]

- \[ \sum_{r,r'} t \cos(\theta_r - \theta_{r'} + 2\pi a_{rr'}). \]

Monopole loop current defines the magnetic order

\[ \tau^z_i \sim E_{rr'} \sim \sum_{rr' \in \mathcal{O}_d^*} J_{rr'}, \]

Proximate magnetic order generically breaks translation symmetry.

Motrunich, Senthil 2005, Bergman, Fiete, Balents 2006

Monopole hopping on dual lattice

Gang Chen’s theory group
Implication for Pr$_2$Ir$_2$O$_7$-delta

FIG. 2: (color online) Temperature dependence of elastic neutron scattering intensity of Pr$_{2+x}$Ir$_{2-x}$O$_{7-δ}$ at the position of the $q_m = (100)$ reflection. The intensity measured at $T = 2$ K was subtracted as a background. Curve: Ising mean-field theory fit to the data, which yields a transition temperature of $T_M = 0.93(1)$ K. Inset: sketch of the 2-in/2-out magnetic structure.

Ising order is discovered in some samples. (MacLaughlin, etc, 2015)
Subsidiary order and weak divergence

\[ L = \sum_a \left[ \left| (\partial^\mu - i\tilde{a}^\mu)\phi_a \right|^2 + m^2|\phi_a|^2 \right] + \frac{F_{\mu\nu}}{2} + u_0 \left( \sum_a |\phi_a|^2 \right)^2 + u_1 \sum_{a\neq b} |\phi_a|^2|\phi_b|^2 + \cdots, \]

The critical theory is described by gapless monopoles coupled with a fluctuating U(1) gauge field in 3+1D.

A unusual weak divergence \( \chi(Q) \sim -\ln T \) “subsidiary order” (Kivelson)!
More experimental prediction for Pr$_2$Ir$_2$O$_7$-delta

Particle-hole excitations are centered at **Gamma** point

Emergent gauge photons are near the **suppressed pinch points**

The energy scales are different, maybe inelastic neutron scattering can work.
Sign problem free model for quantum Monte Carlo

$$H_1 = \sum_{\langle ij \rangle} J_z \tau_i^z \tau_j^z - J_\perp (\tau_i^+ \tau_j^- + h.c.)$$

$$+ \sum_{\langle\langle ij \rangle\rangle} J_{3z} \tau_i^z \tau_j^z,$$

where $J_{3z}$ is the third neighbor Ising exchange.

The schematic phase diagram shows the transition between different spin orders as a function of $J_\perp/J_z$. The diagram includes:
- AFM Ising
- FM Ising
- Transverse spin order
- U(1) QSL
- U(1) QSL

The diagram illustrates the continuous line degeneracy for the Tisza method.
Summary

• I have studied the phase diagram near quantum spin ice quantum spin liquid.

• Using field theoretic technique, I have obtained the structure of the magnetic states and the nature of the magnetic transition.

• I use the theoretical results to explain the puzzling experiments in Pr$_2$Ir$_2$O$_7$ and Yb$_2$Ti$_2$O$_7$. It implies the disordered phase is a quantum spin ice U(1) quantum spin liquid.

Ref: Gang Chen, arXiv:1602.02230, longer talk can be found at KITP website last Sep.

Work in progress: sign problem free model that demonstrates both proximate and unproximate magnetic transition out of QSI QSL.
Thank you!

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Implication for $\text{Yb}_2\text{Ti}_2\text{O}_7$

\begin{itemize}
  \item $Q = (000)$ state
  \item but with some spin reorientation
\end{itemize}

Savary, Balents, PRL, PRB 2012
Coldea, etc 2015, Gingras et al 2015

YTO: First order transition to $Q=0$ FM state.
Theoretical framework: compact QED and electromagnetic duality

Ref: Gang Chen, arXiv:1602.02230, longer talk can be found at KITP website last Sep.
Lattice gauge theory formalism: technical part

\[ E_{rr'} \sim \tau^z_i, \ e^{iA_{rr'}} \sim \tau^+_i \]

Hermene, Fisher, Balents, 2004

\[ H_{\text{ring}} = -\sum_{\Omega_p} \frac{K}{2} (\tau^+_1 \tau^-_2 \tau^+_3 \tau^-_4 \tau^+_5 \tau^-_6 + h.c.), \]

\[ H_{\text{LGT}} = \sum_{\langle rr' \rangle} \frac{U}{2} (E_{rr'} - \frac{\epsilon_r}{2})^2 - \sum_{\Omega_d} K \cos(\text{curl} A), \]

\[ H_{\text{LGT}} \text{ captures the universal properties of QSI.} \]

- In an ordered state, \( <\tau_z> \neq 0, <\tau^+> \) is strongly fluctuating.

- In the gauge language, \( \mathbf{E} \) field is static, \( \mathbf{B} \) magnetic field is strongly fluctuating, the magnetic monopole (carrying magnetic charge) is condensed, which confines the electric charge carriers (spinons).
Electromagnetic duality

Monopole lives on dual diamond lattice, carry magnetic charge or dual U(1) gauge charge.

To study monopole physics, we need to use a technique called “duality” to make it explicit.

![Diagram of dual lattice with arrows indicating fields and monopole variables]

Insert monopole variables

\[
H_{\text{dual}} = \sum_{\partial_a} \frac{U}{2} (\text{curl } a - \vec{E})^2 - \sum_{r,r'} K \cos B_{rr'}
\]

- B magnetic field is strongly fluctuating, the fluctuation of dual U(1) gauge field is weak.

Motrunich, Senthil 2005, Bergman, Fiete, Balents 2006
Analogy with Boson-vortex duality

Balents, et al. ensemble with fixed boson number (average filling $f$). We will typically do the latter, except in Secs. 2.2, 2.3, and the first part of Sec. 3, where we work at fixed chemical potential.

2.2. Mott states at integral filling

"0" Mott

$\bar{n}/U = 0$

"1" Mott

$\bar{n}/U = 1$

"2" Mott

$\bar{n}/U = 2$

**Fig. 1. Schematic phase diagram of boson rotor model, with on-site interactions only.** The shaded regions indicate where there is a large near-degeneracy of states with different boson densities, and the system is highly susceptible to off-site interactions.

Neglecting terms in $H'$, the zero temperature phase diagram of $H$ is well-known. It takes the schematic form in Fig. 1. For $t/U \ll 1$, the system is in a Mott insulating ground state, with $\langle \hat{n}_i \rangle = N$, i.e., one over site. This phase persists inside the "lobes" drawn in the figure. There is a gap to the lowest-lying excited states, which may be thought of as single extra/missing bosons (which delocalize into plane-waves). For large $t/U$, the ground state is a superfluid (SF in the figure), with $\langle e^{i\hat{\phi}_i} \rangle = \Psi_{sf} \neq 0$, and the density $f = \langle \hat{n}_i \rangle$ varies smoothly with parameters in an unquantized fashion. There is no excitation gap, and the lowest-lying excitations are acoustic "phonons" or "phason"s, the Goldstone modes of the broken U(1) symmetry of the superfluid.

2.3. Mott states at non-integral filling

We now return to the shaded regions of the phase diagram in Fig. 1, where states with different boson density are nearly degenerate. Indeed, in the simple model with $H' = 0$, for $n = N + 1/2$, states with any average density between $N$ and $N + 1$ are degenerate. For $t/U = 0$, the eigenvalue of $\hat{n}_i = N$ or $\hat{n}_i = N + 1$ can be independent chosen on each site. The omitted terms in $H'$ will then clearly determine the nature of the ground states appearing in the shaded region. Generally, Mott insulating states appear at rational fractional fillings, $f = p/q$, with $p, q$ relatively prime. For $q > 1$, these are boson "crystals" or charge density waves. Mott states with increasing $q$ are expected to require longer-range interactions in $H'$ for their stabilization.

For example, in the vicinity of $n = N + 1/2$, we can adopt a pseudo-spin description, with $S_z^i = \hat{n}_i - N - 1/2 = \pm 1/2$. In the limit of $U \to \infty$ (or $t/U \ll 1$), one can then replace $H \to -t \sum_{\langle ij \rangle} (S_i^+ S_j^- + S_i^- S_j^+)$, with $\langle ij \rangle$ indicating the sum is taken over nearest neighbors.

Fisher et al, PRB (1989)

Balents, et al, 2005
Physical observables are gauge invariant

- Monopole loop current defines the magnetic order

\[ \tau_i^z \sim E_{rr'} \sim \sum_{rr' \in \Omega^*_d} J_{rr'} \]

Proximate magnetic order generically breaks translation symmetry.

Q = 2\Pi(001)
Critical theory for proximate ordering transition

\[
L = \sum_a \left[ |(\partial_\mu - i \tilde{a}_\mu) \phi_a|^2 + m^2 |\phi_a|^2 \right] + \frac{F_{\mu\nu}^2}{2} + u_0 \left( \sum_a |\phi_a|^2 \right)^2 + \cdots,
\]

where

- \( g \) is the mass of the monopole.
- The critical theory is described by multicomponent bosons coupled with a fluctuating U(1) gauge field in 3+1D.
- A unusual weak divergence

\[ \chi(Q) \sim - \ln T \]

“subsidiary order”!
Ir conduction electrons

Ir conduction electron Fermi surface does not modify the critical property.

Ordering wavevector $|Q| >> K_F$, Yukawa coupling and Landau damping is suppressed.

Lohneysen, A Rosch, Vojta, Wolfle, RMP 2007

But deep in the ordered regime, magnetic order influences the conduction electron bands.

Yao-Dong Li, **GC**, in preparation, 2016