

“Magnetic” monopole condensation transition out of quantum spin ice:
quantum spin ice in **pyrochlore iridates**

Gang Chen

Fudan University, Shanghai

Gang Chen, arXiv:1602.02230.

Longer talk can be found at KITP website last Sep.



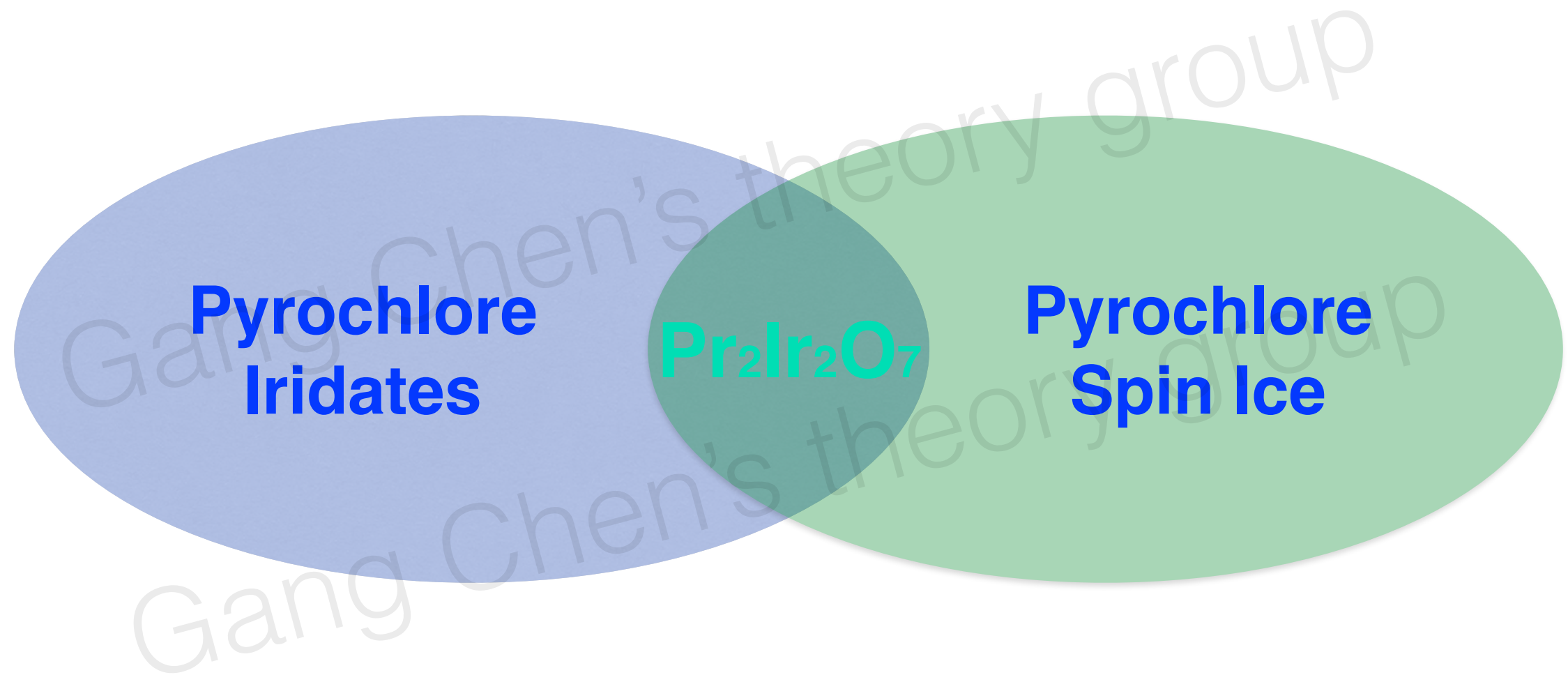
Job opportunities

- **Postdocs** are generously funded and will have tremendous freedom.

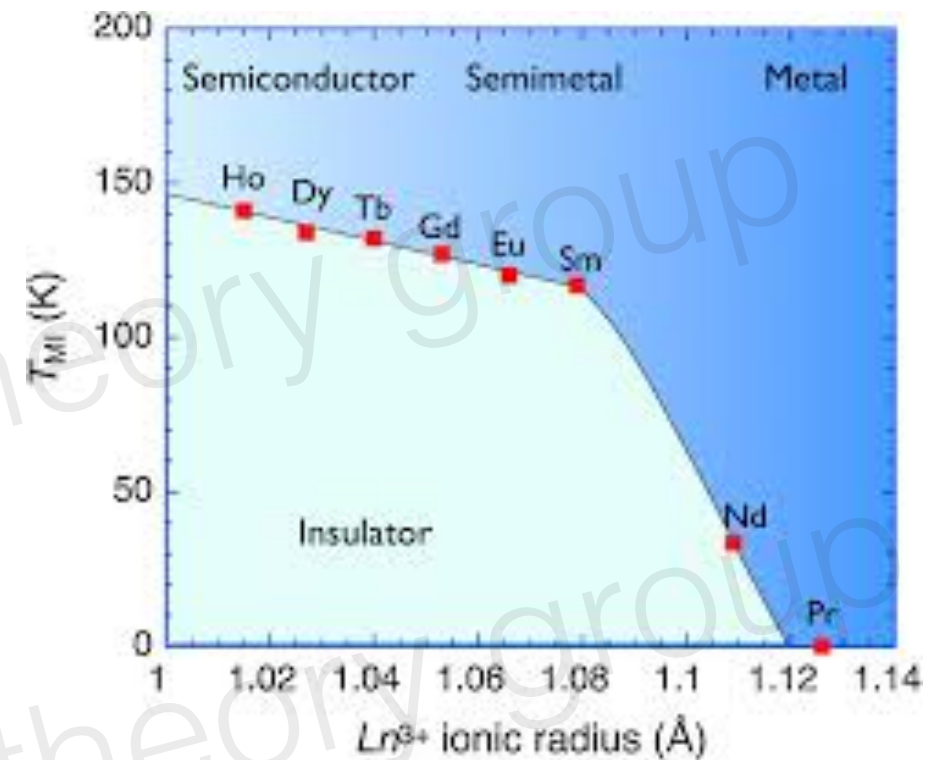
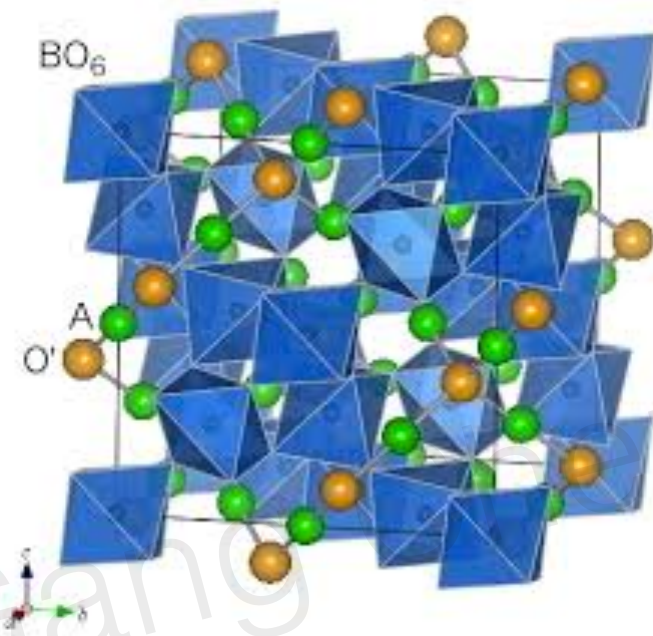


Shanghai, China

Pyrochlore Iridate and Pyrochlore Spin Ice



Pyrochlore iridates: $\text{Pr}_2\text{Ir}_2\text{O}_7$

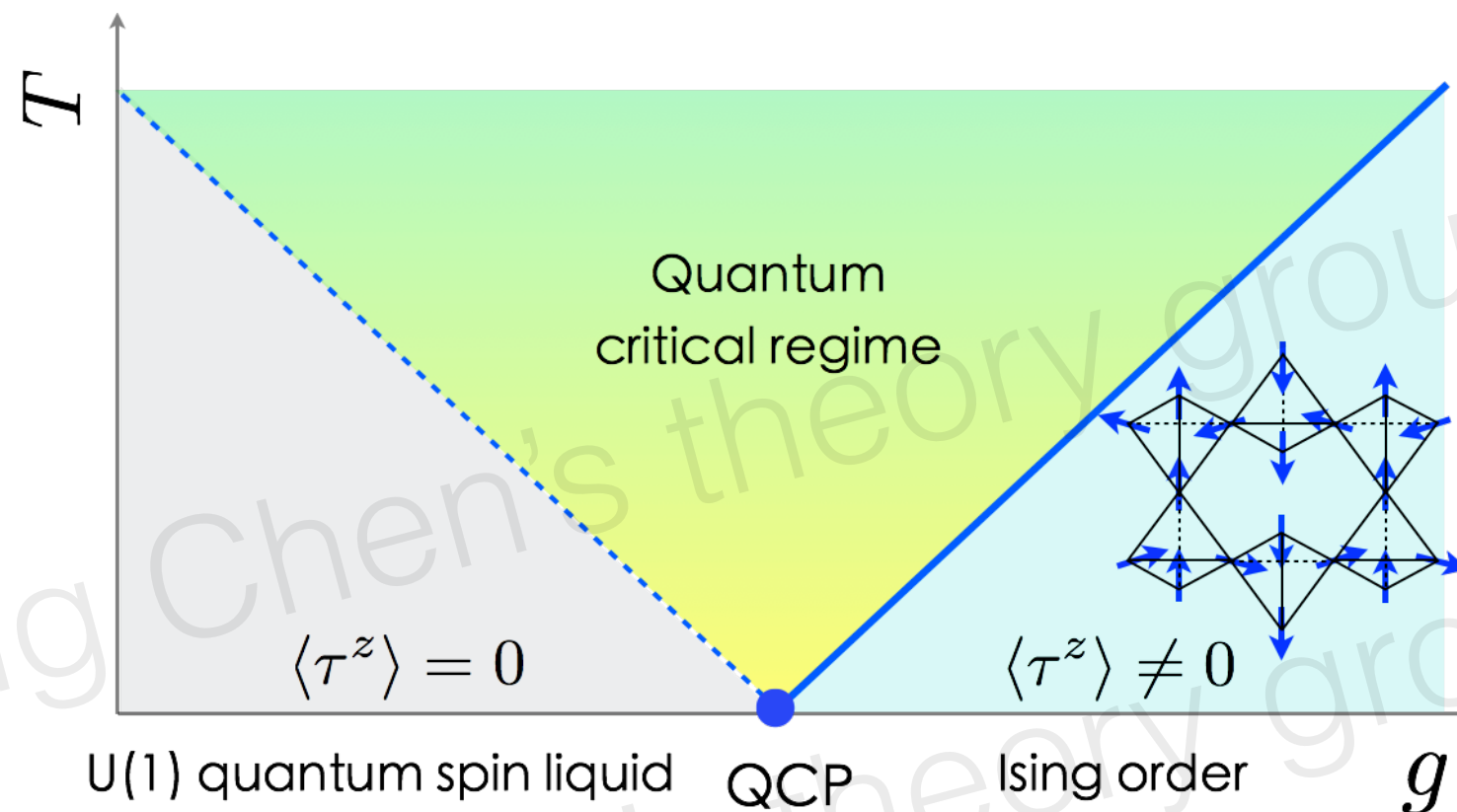


K Matsuhira, M Wakeshima, Y Hinatsu, S. Takagi
JPSJ, 2011

Many nice experimental works by S Nakatsuji, P Gegenwart, L Balicas, etc

Ref: D Pesin, L Balents, 2009, Xian-Gang Wan, etc 2010, Witczak-Krempa, Yong Baek Kim, SungBin Lee;
Michael Hermele, Gang Chen, etc

My proposal for $\text{Pr}_2\text{Ir}_2\text{O}_{7-\delta}$

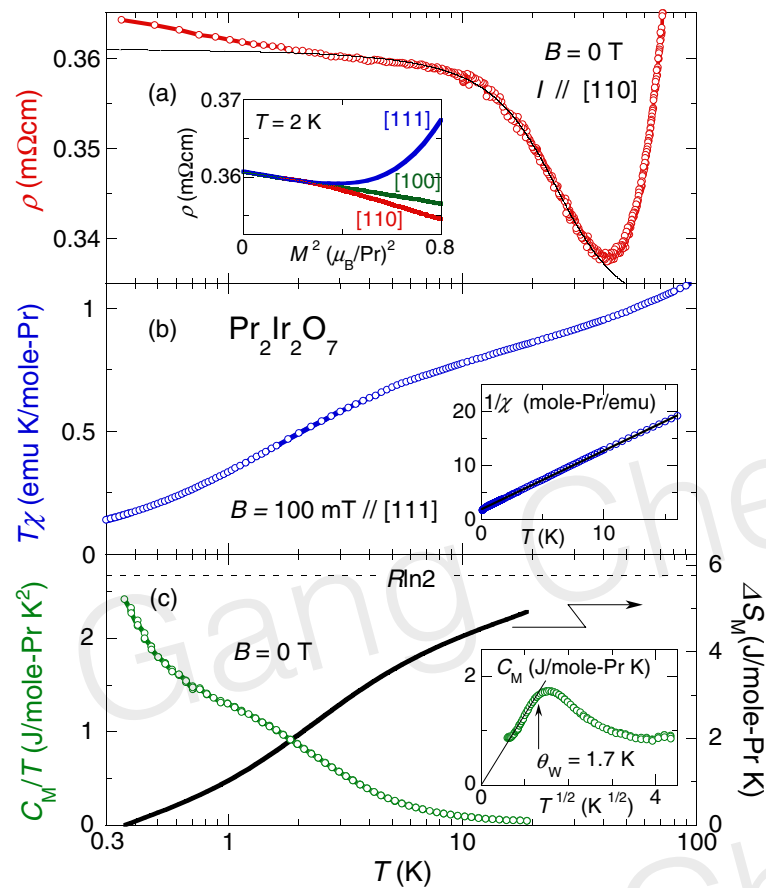


Pr local moments are close to a “magnetic” monopole condensation transition from quantum spin ice quantum spin liquid to an AFM long-range ordered state.

The Ir conduction electrons may drive the transition, but do not influence the nature of the phase transition.

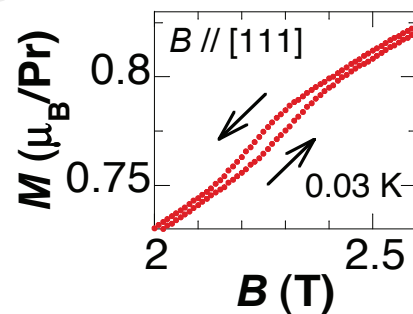
Pr₂Ir₂O₇ a featureless disordered state near an ordered state

ARPES: quadratic band touching of Ir 5d electrons

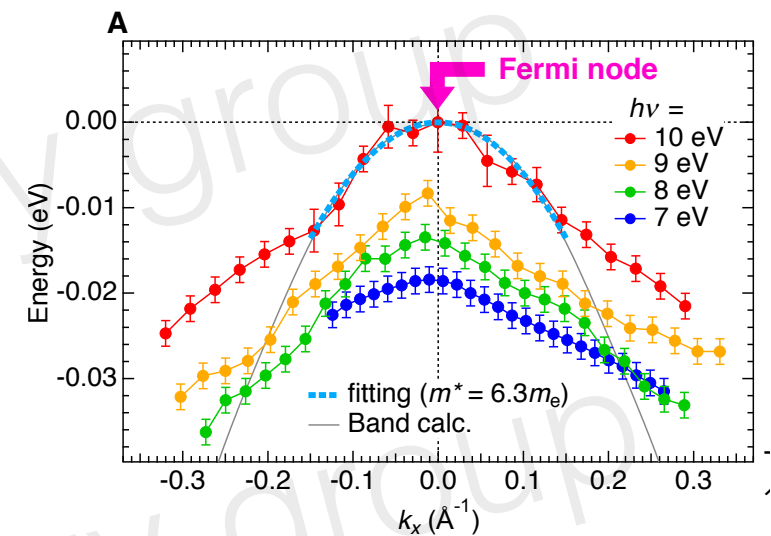


Nakatsuji, etc

PRL **96**, 087204 (2006)



metamagnetic transition



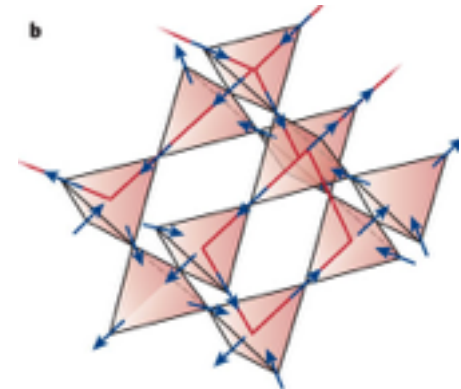
Valence band approaches the Fermi energy at few meV resolution

T Kondo, S Shin, etc 2014

B J Yang, Yong Baek Kim 2011

E G Moon, CK Xu, Y B Kim, L Balents, 2013

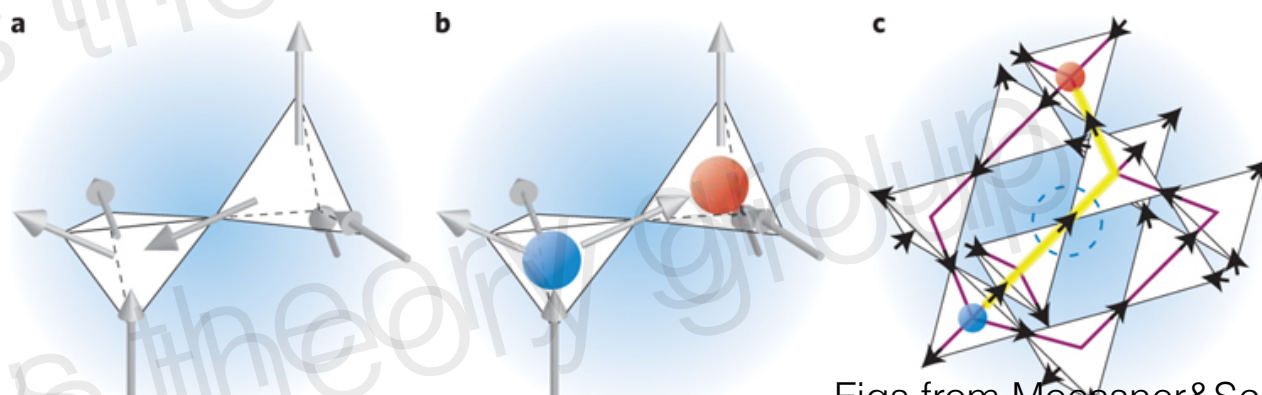
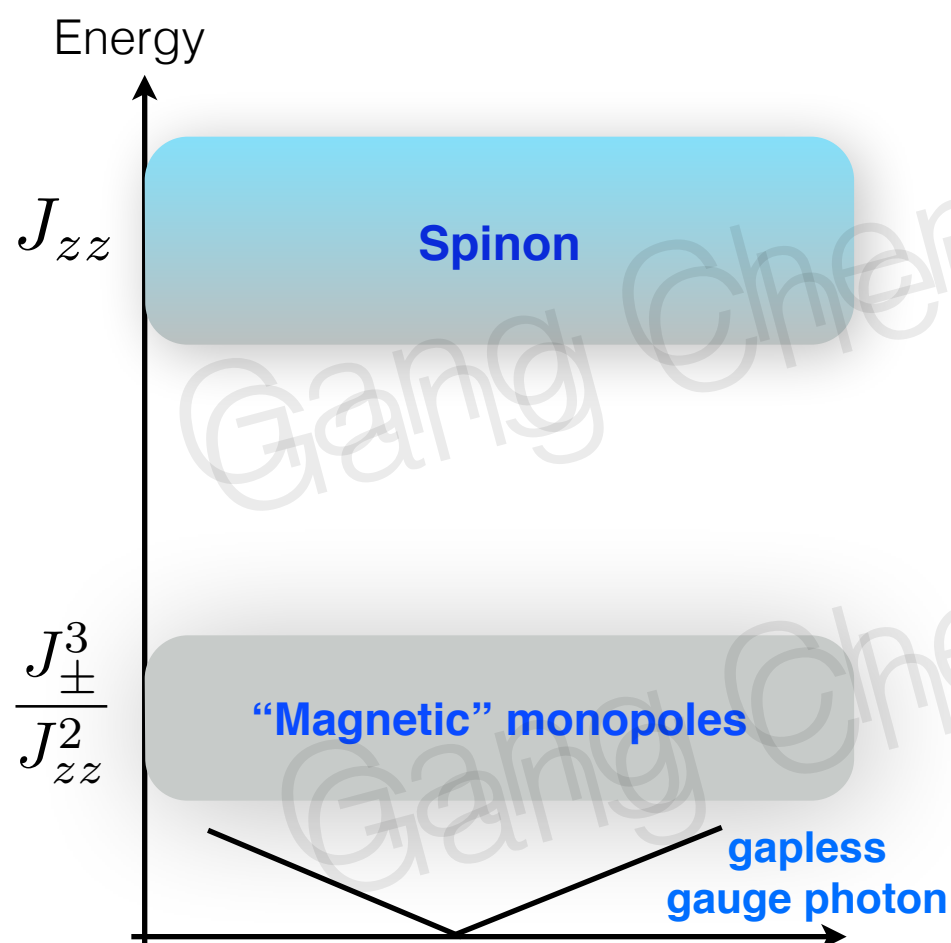
Expts are **sample dependent**,
Recently, some samples are found AFM ordered.



Quantum spin ice U(1) spin liquid

$$H = J_{zz} \sum_{\langle ij \rangle} \tau_i^z \tau_j^z - J_{\pm} \sum_{\langle ij \rangle} (\tau_i^+ \tau_j^- + \tau_i^- \tau_j^+) + \dots$$

Senthil Motrunich, 2002
 Hermele, Fisher, Balents, 2003
 Moessner, Huse, Isakov, YB Kim...

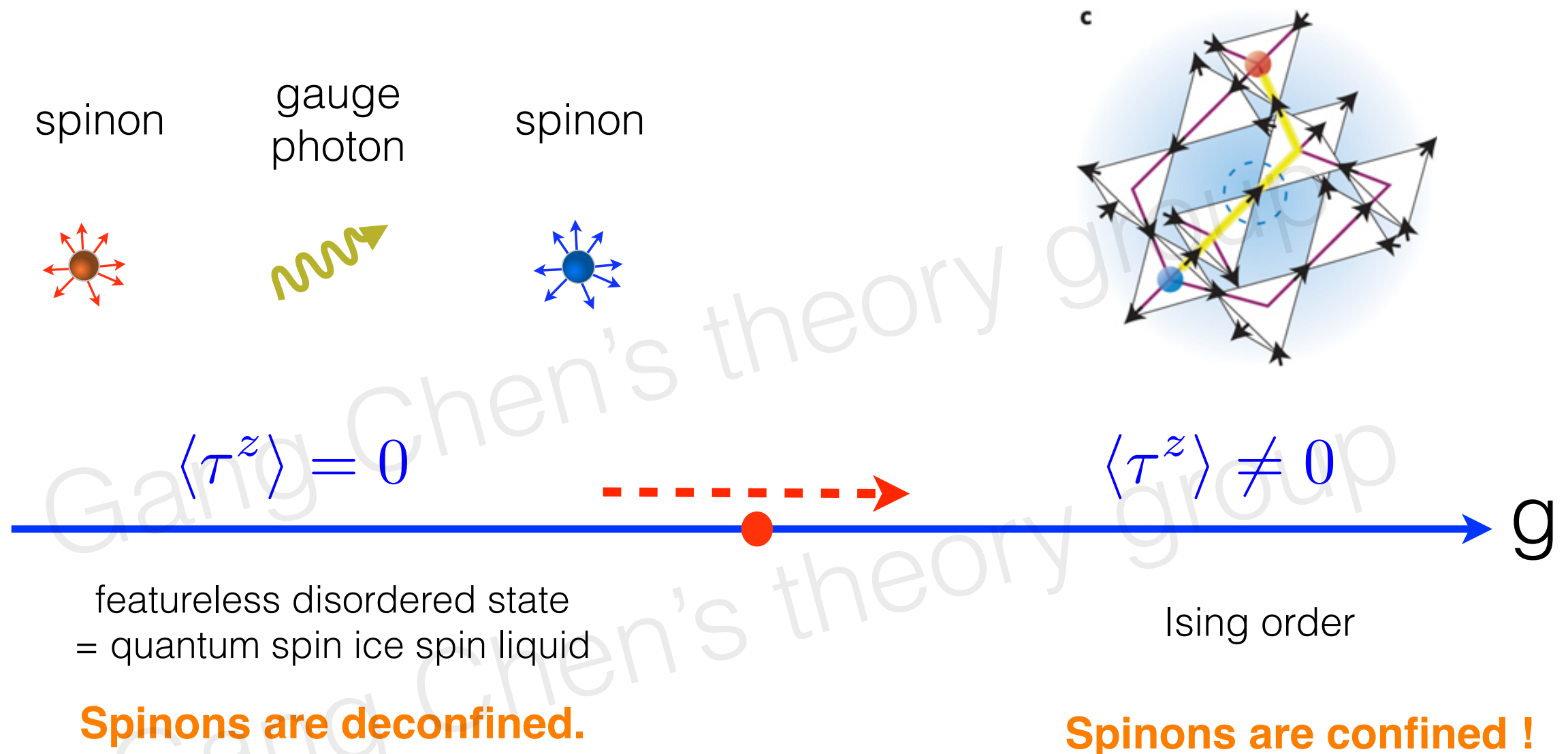


Figs from Moessner&Schiffer,2009

Spinon deconfinement

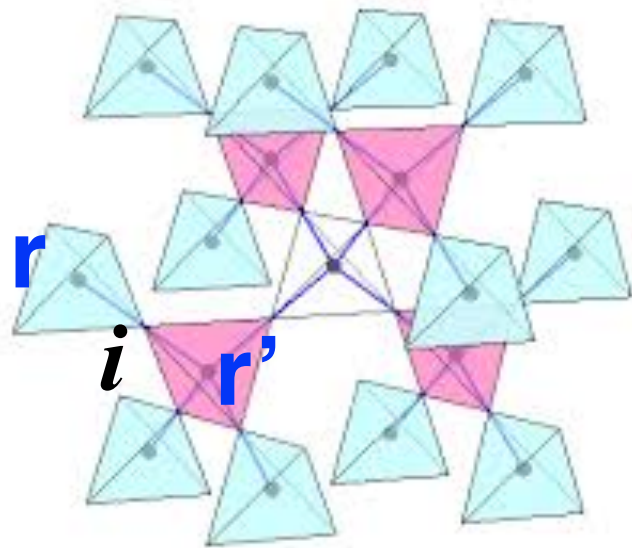
QSI (U(1) QSL) is an example of Xiao-Gang Wen's string net condensed state.
 The physics of QSI is described by **compact quantum electrodynamics**.

Confinement transition out of U(1) quantum spin liquid



More generally, for **non-Kramers' doublet**, the magnetic transition out of QSI **MUST** be a confinement transition, this may apply to Tb₂Ti₂O₇.

Lattice gauge theory formalism: technical part

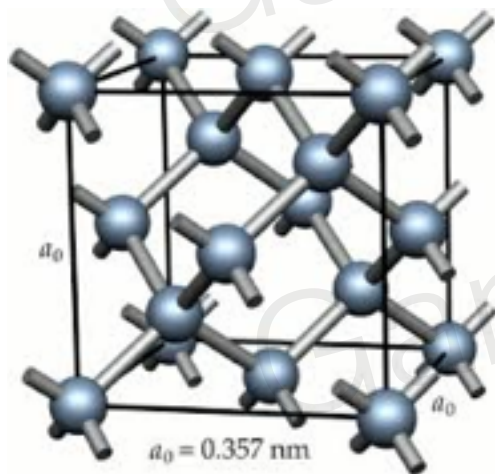


$$E_{\mathbf{r}\mathbf{r}'} \sim \tau_i^z, e^{iA_{\mathbf{r}\mathbf{r}'}} \sim \tau_i^+ \quad \text{Hermele, Fisher, Balents, 2004}$$

$$H_{\text{ring}} = - \sum_{\hexagon_p} \frac{K}{2} (\tau_1^+ \tau_2^- \tau_3^+ \tau_4^- \tau_5^+ \tau_6^- + h.c.),$$

$$H_{\text{LGT}} = \sum_{\langle \mathbf{r}\mathbf{r}' \rangle} \frac{U}{2} (E_{\mathbf{r}\mathbf{r}'} - \frac{\epsilon_{\mathbf{r}}}{2})^2 - \sum_{\hexagon_d} K \cos(\text{curl } A),$$

H_{LGT} captures the **universal properties** of QSI.



diamond lattice

- In an ordered state, $\langle \tau_z \rangle \neq 0$, $\langle \tau^+ \rangle$ is strongly fluctuating.
- In the gauge language, “**E field**” is static, “**B magnetic field**” is strongly fluctuating, the magnetic monopole (carrying magnetic charge) is condensed, which confines the electric charge carriers (spinons).

Electromagnetic duality

Monopole lives on dual diamond lattice, carry magnetic charge or dual U(1) gauge charge.

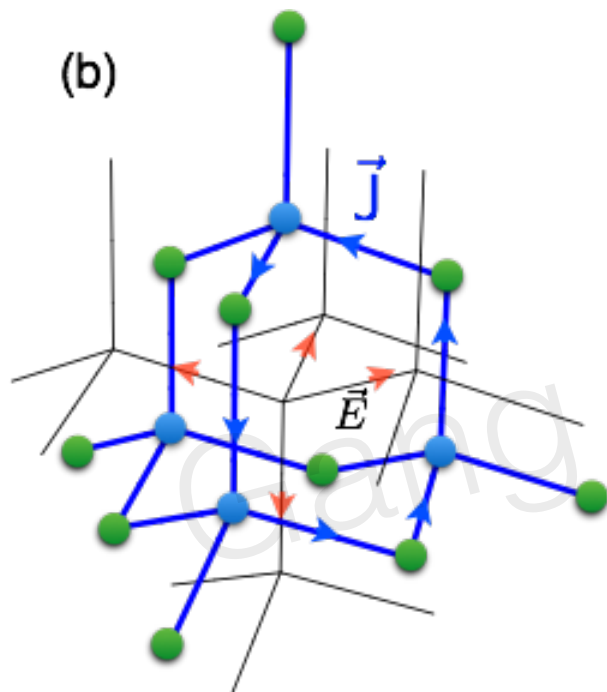
Insert monopole variables

$$H_{\text{dual}} = \sum_{\hexagon_d^*} \frac{U}{2} (\text{curl } a - \vec{E})^2 - \sum_{\mathbf{r}, \mathbf{r}'} K \cos B_{\mathbf{r}\mathbf{r}'} - \sum_{\mathbf{r}, \mathbf{r}'} t \cos(\theta_{\mathbf{r}} - \theta_{\mathbf{r}'} + 2\pi a_{\mathbf{r}\mathbf{r}'}).$$

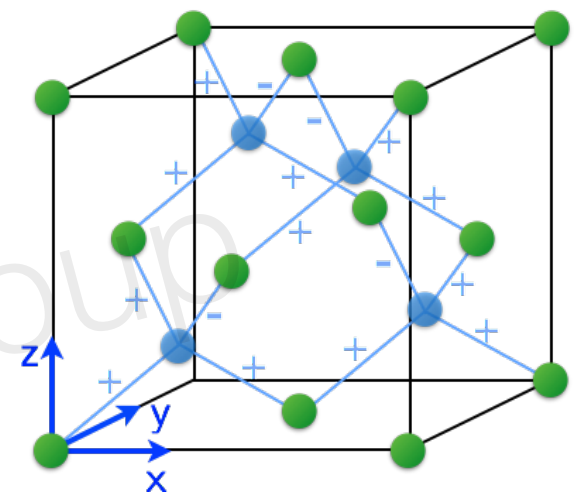
Monopole loop current defines the magnetic order

$$\tau_i^z \sim E_{\mathbf{r}\mathbf{r}'} \sim \sum_{\mathbf{r}\mathbf{r}' \in \hexagon_d^*} \mathbf{J}_{\mathbf{r}\mathbf{r}'},$$

Proximate magnetic order generically breaks translation symmetry.



Motrunich, Senthil 2005,
Bergman, Fiete, Balents 2006



monopole hopping on
dual lattice

Implication for $\text{Pr}_2\text{Ir}_2\text{O}_{7-\delta}$

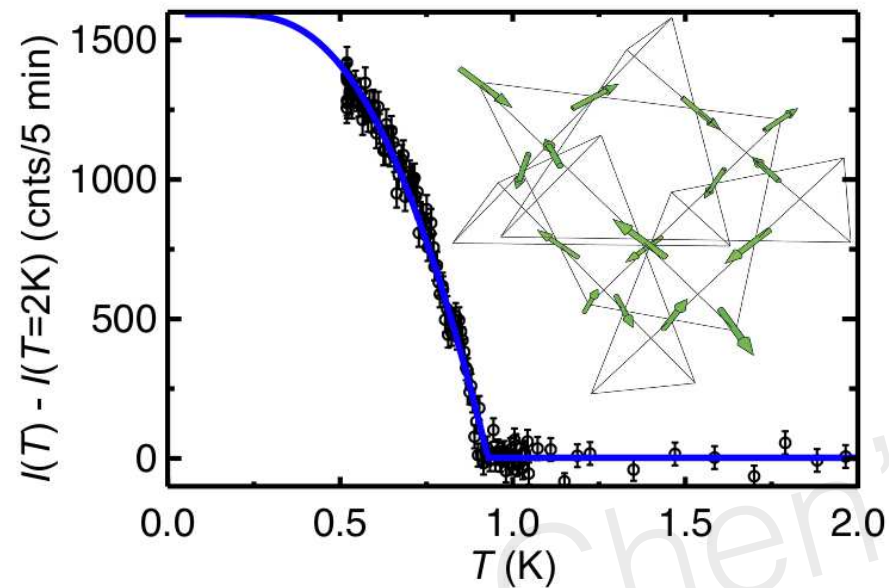
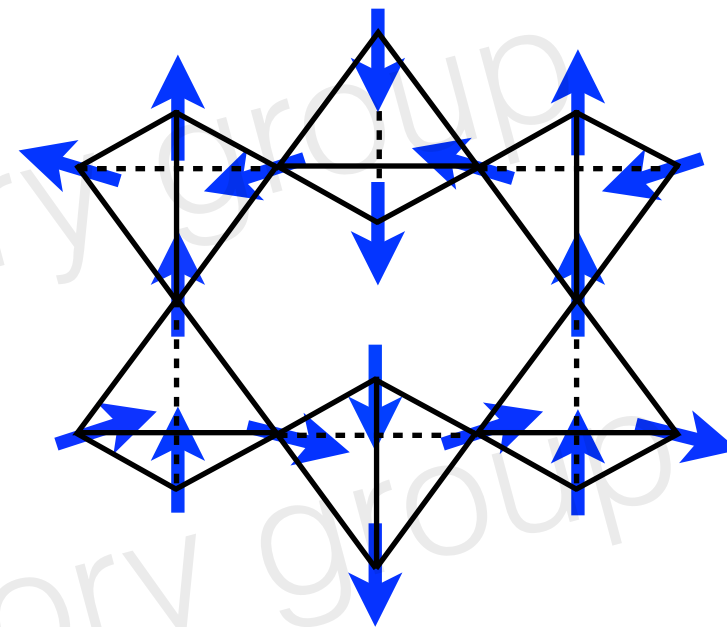


FIG. 2. (color online) Temperature dependence of elastic neutron scattering intensity of $\text{Pr}_{2+x}\text{Ir}_{2-x}\text{O}_{7-\delta}$ at the position of the $\mathbf{q}_m = (100)$ reflection. The intensity measured at $T = 2$ K was subtracted as a background. Curve: Ising mean-field theory fit to the data, which yields a transition temperature of $T_M = 0.93(1)$ K. Inset: sketch of the 2-in/2-out magnetic structure.

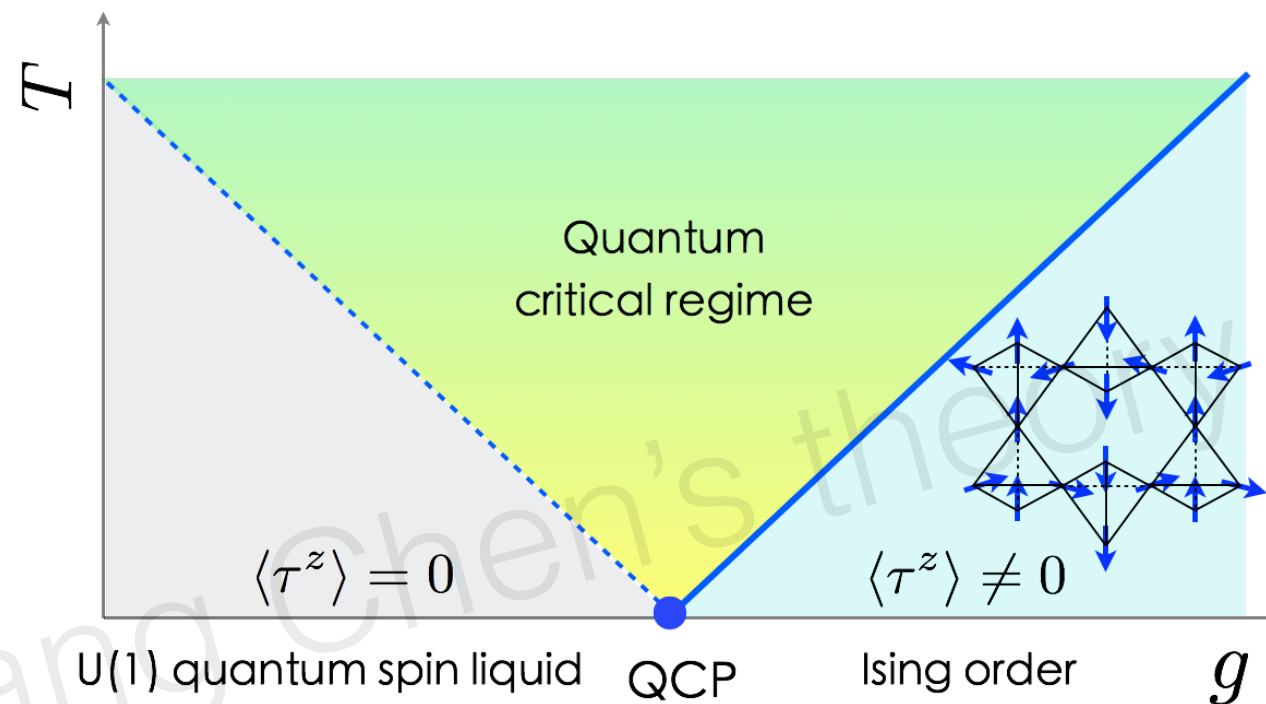
Ising order is discovered in some samples. (MacLaughlin, etc, 2015)



**Ising order,
 $\mathbf{Q} = 2\pi(001)$**

break translation symmetry

Subsidiary order and weak divergence



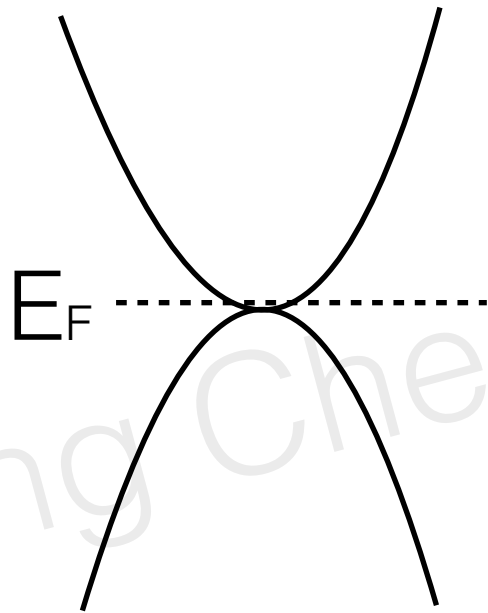
g is the mass of the monopole

$$L = \sum_a \left[|(\partial_\mu - i\tilde{a}_\mu)\phi_a|^2 + m^2|\phi_a|^2 \right] + \frac{F_{\mu\nu}^2}{2} + u_0 \left(\sum_a |\phi_a|^2 \right)^2 + u_1 \sum_{a \neq b} |\phi_a|^2 |\phi_b|^2 + \dots,$$

The critical theory is described by gapless monopoles coupled with a fluctuating U(1) gauge field in 3+1D.

a unusual weak divergence $\chi(Q) \sim -\ln T$ “subsidiary order” (Kivelson) !

More experimental prediction for $\text{Pr}_2\text{Ir}_2\text{O}_{7-\delta}$



Particle-hole excitations are centered at **Gamma** point

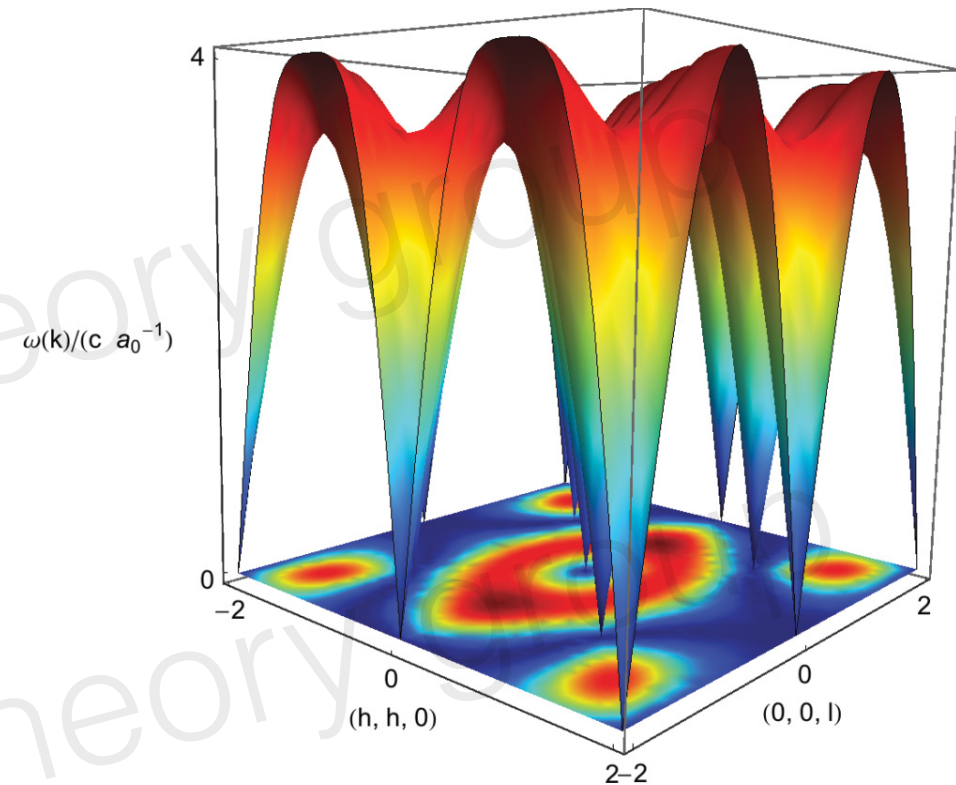
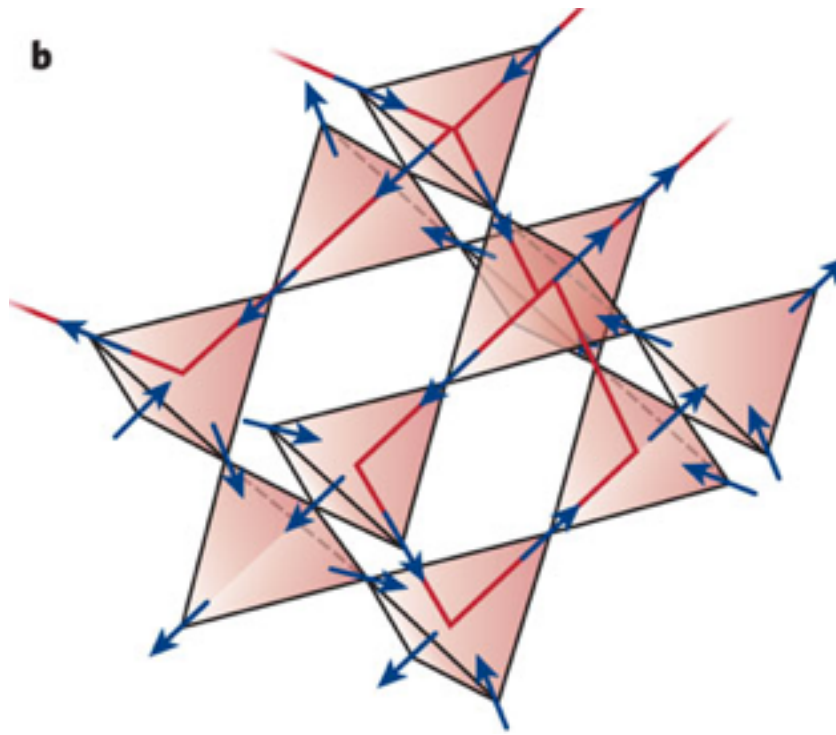


Fig from Benton, etc, PRB 2012

Emergent gauge photons are near the **suppressed pinch points**

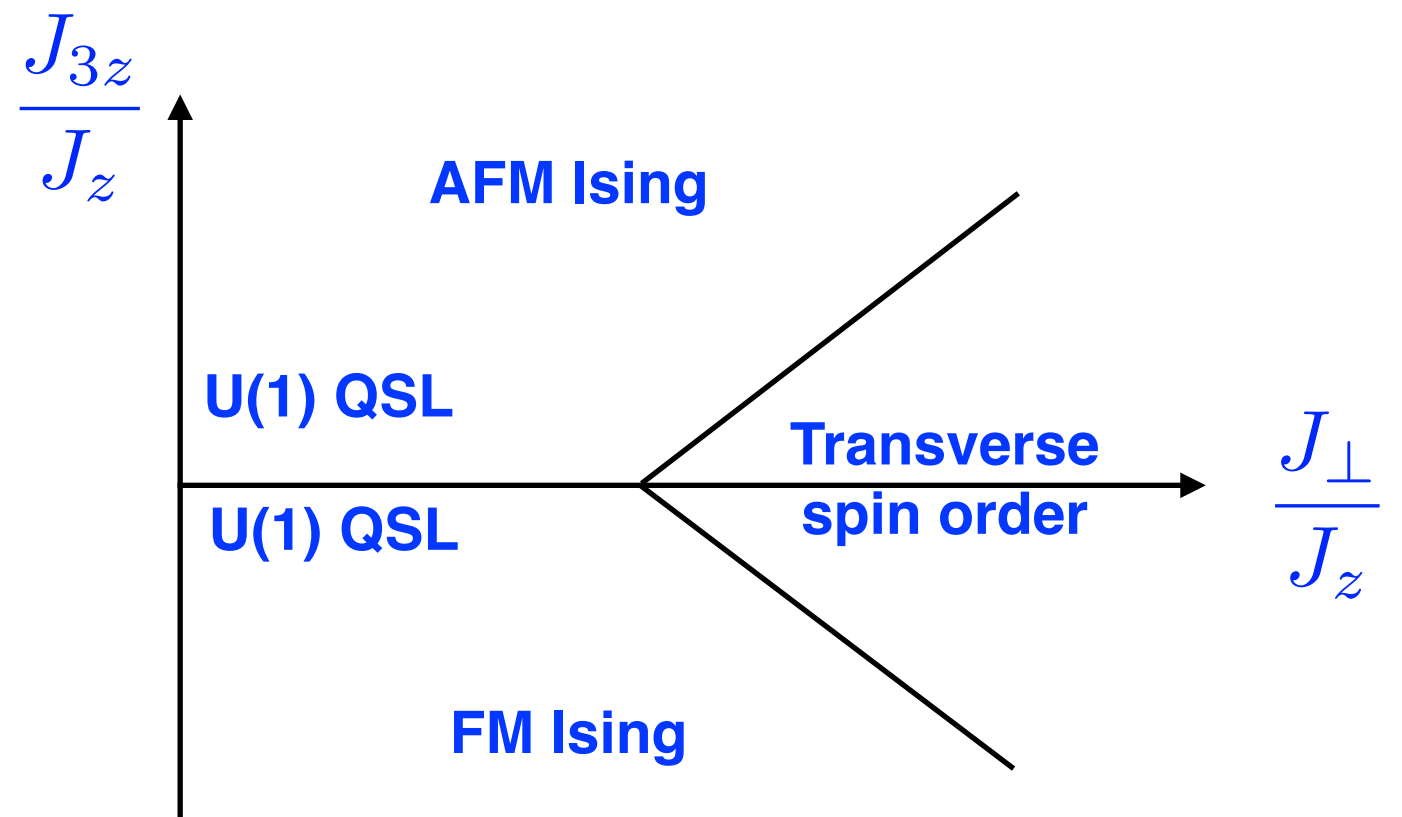
The energy scales are different, maybe inelastic neutron scattering can work.

Sign problem free model for quantum Monte Carlo



$$H_1 = \sum_{\langle ij \rangle} J_z \tau_i^z \tau_j^z - J_{\perp} (\tau_i^+ \tau_j^- + h.c.)$$

$$+ \sum_{\langle\langle\langle ij \rangle\rangle\rangle} J_{3z} \tau_i^z \tau_j^z,$$



schematic phase diagram

Summary

- I have studied the phase diagram near quantum spin ice quantum spin liquid.
- Using field theoretic technique, I have obtained the structure of the magnetic states and the nature of the magnetic transition.
- I use the theoretical results to explain the puzzling experiments in $\text{Pr}_2\text{Ir}_2\text{O}_7$ and $\text{Yb}_2\text{Ti}_2\text{O}_7$. It implies the disordered phase is a quantum spin ice U(1) quantum spin liquid.

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Work in progress: sign problem free model that demonstrates both proximate and unproximate magnetic transition out of QSI QSL.

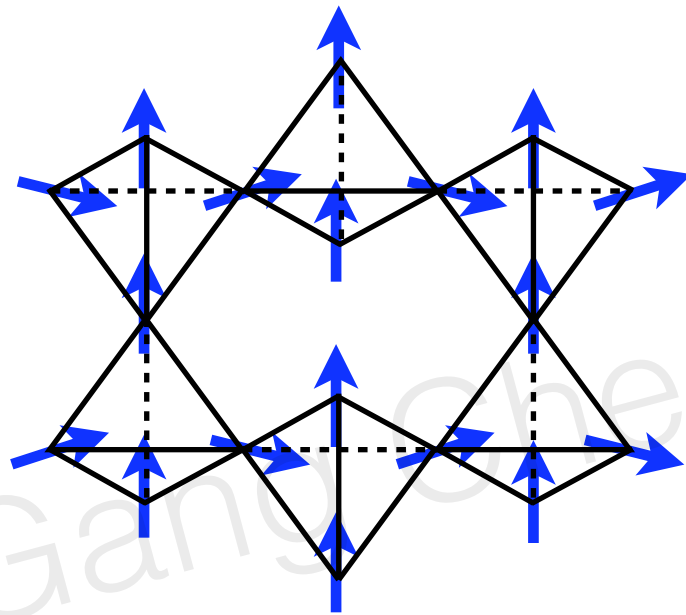
Thank you !

Email: gchen_physics@fudan.edu.cn

Implication for $\text{Yb}_2\text{Ti}_2\text{O}_7$

First-order magnetic transition in $\text{Yb}_2\text{Ti}_2\text{O}_7$

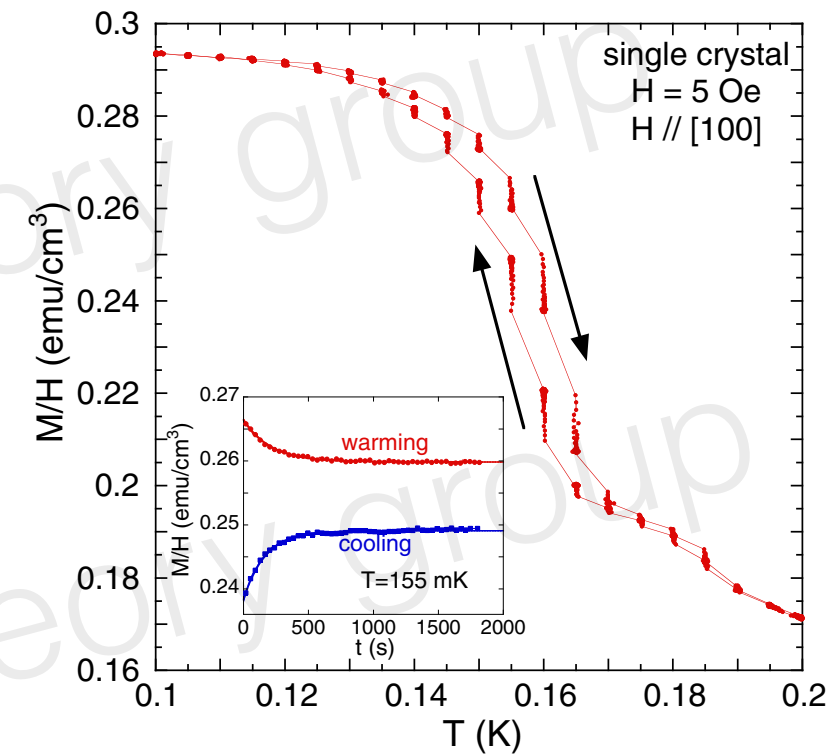
E. Lhotel,^{1,*} S. R. Giblin,² M. R. Lees,³ G. Balakrishnan,³ L. J. Chang,⁴ and Y. Yasui⁵



$Q = (000)$ state

but with some spin reorientation

Savary, Balents, PRL, PRB 2012
Coldea, etc 2015, Gingras et al 2015



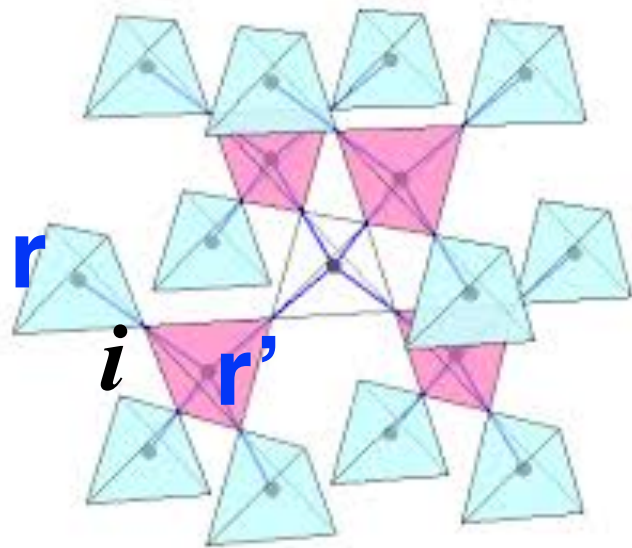
$\text{Yb}_2\text{Ti}_2\text{O}_7$

YTO: First order transition to **$Q=0$ FM state.**

Theoretical framework: compact QED and electromagnetic duality

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Lattice gauge theory formalism: technical part

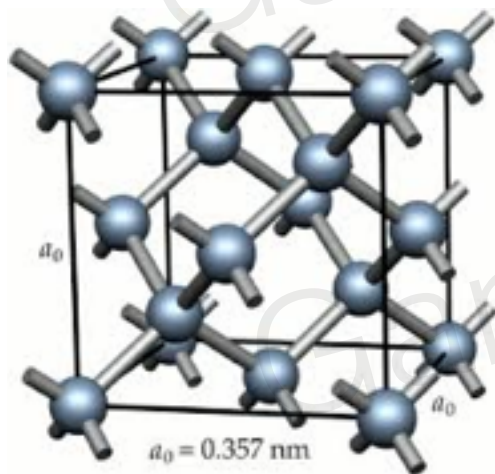


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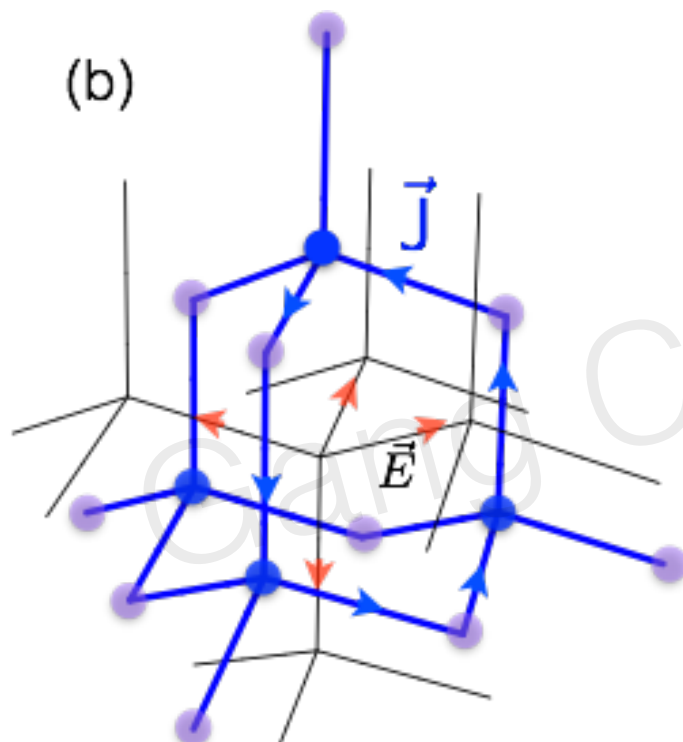
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Electromagnetic duality

Monopole lives on dual diamond lattice, carry magnetic charge or dual U(1) gauge charge.

To study monopole physics, we need to use a technique called “duality” to make it explicit.



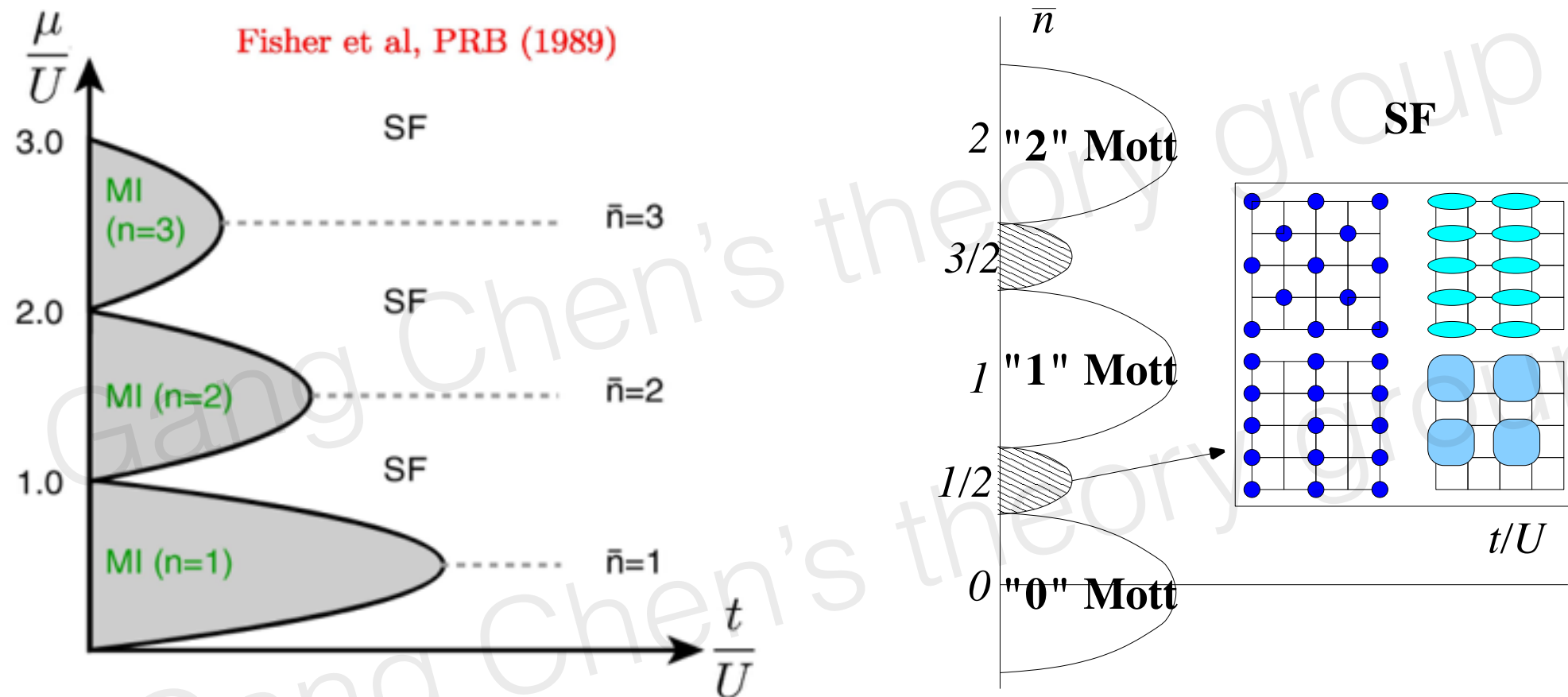
insert monopole variables

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- **B magnetic field** is strongly fluctuating, the fluctuation of dual U(1) gauge field is weak.

Motrunich, Senthil 2005,
Bergman, Fiete, Balents 2006

Analogy with Boson-vortex duality



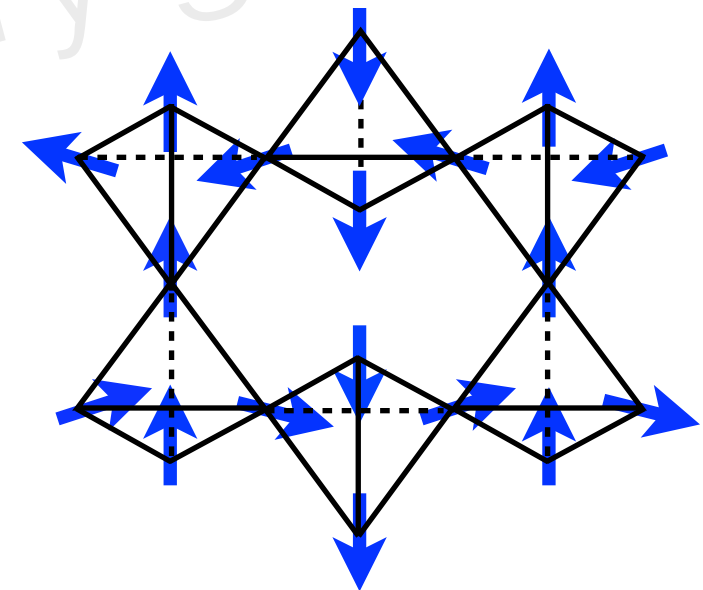
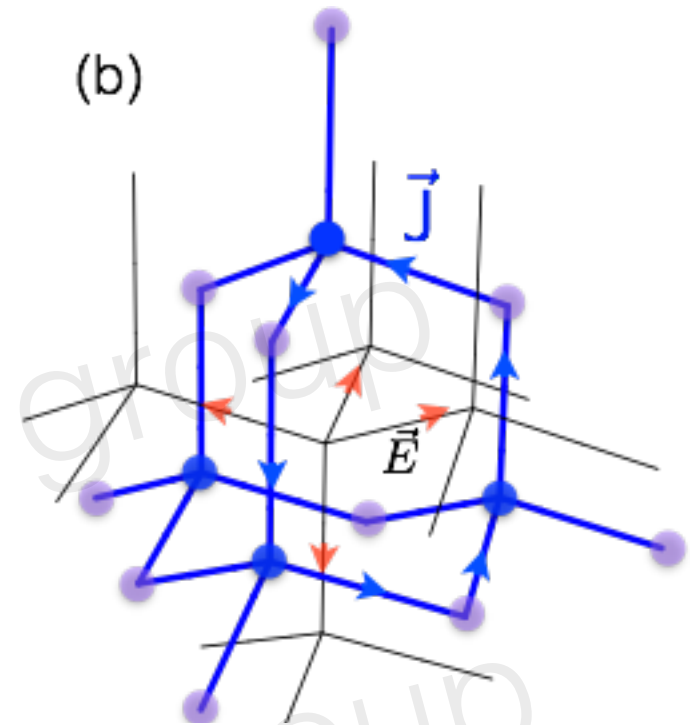
Balents, et al, 2005

Physical observables are gauge invariant

- Monopole loop current defines the magnetic order

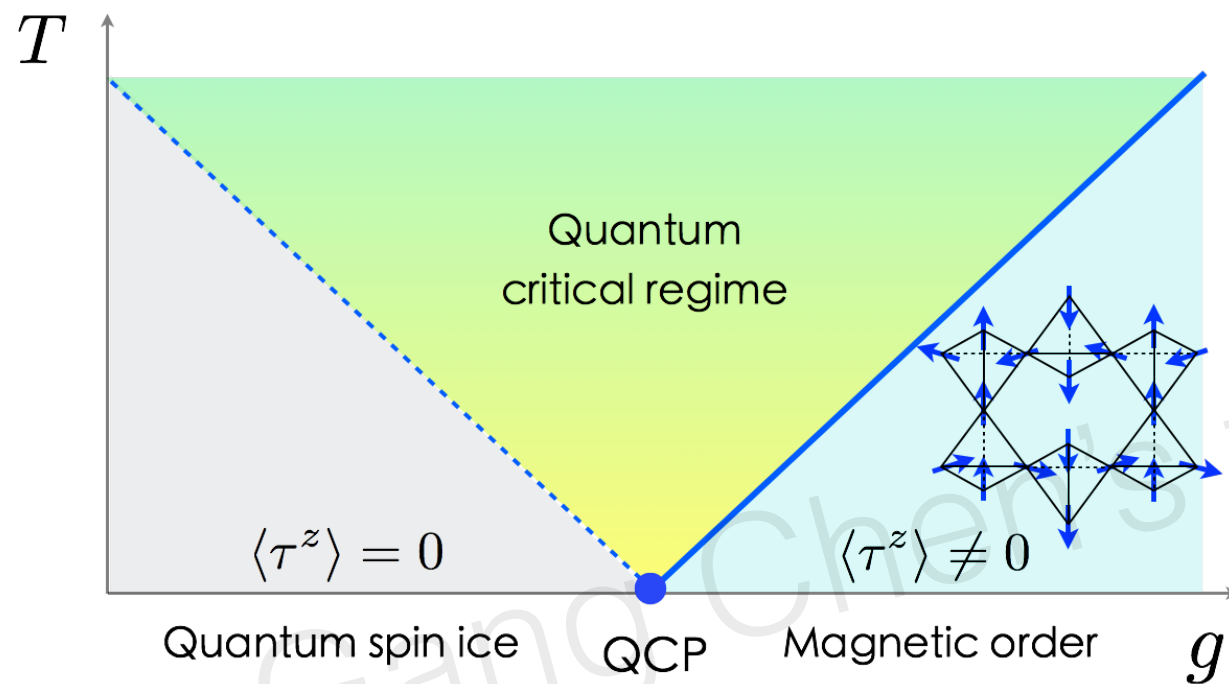
$$\tau_i^z \sim E_{\mathbf{r}\mathbf{r}'} \sim \sum_{\mathbf{r}' \in \hexagon_d^*} \mathbf{J}_{\mathbf{r}\mathbf{r}'},$$

Proximate magnetic order generically breaks translation symmetry.



$$Q = 2\pi(001)$$

Critical theory for proximate ordering transition



g is the mass of the monopole

Determined by **projective symmetry group**

$$L = \sum_a \left[|(\partial_\mu - i\tilde{a}_\mu)\phi_a|^2 + m^2|\phi_a|^2 \right] + \frac{F_{\mu\nu}^2}{2} + u_0 \left(\sum_a |\phi_a|^2 \right)^2 + \dots,$$

The critical theory is described by multicomponent bosons coupled with a fluctuating U(1) gauge field in 3+1D.

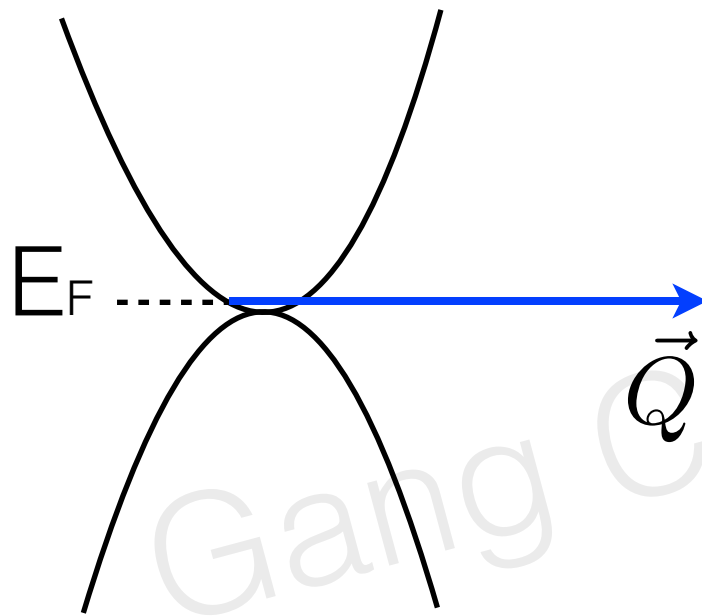
a unusual weak divergence

$$\chi(Q) \sim -\ln T$$

“subsidiary order” !

Ir conduction electrons

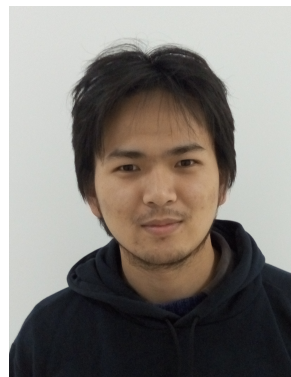
Ir conduction electron Fermi surface does not modify the critical property.



Ordering wavevector $|Q| \gg K_F$, Yukawa coupling and Landau damping is suppressed.

Lohneysen, A Rosch, Vojta, Wolfle, **RMP** 2007

But deep in the ordered regime, magnetic order influences the conduction electron bands.



Yao-Dong Li, **GC**, in preparation, 2016