Monopole condensation transition out of quantum spin ice

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Many thanks to

Prof Claudio Castelnovo
Prof David Logan
Prof Roderich Moessner

My purpose of this talk

1. Explain the puzzling experiments in quantum spin ice candidate materials.

2. Use proximate ordered phase and proximate phase transition to indirectly identify quantum spin ice.

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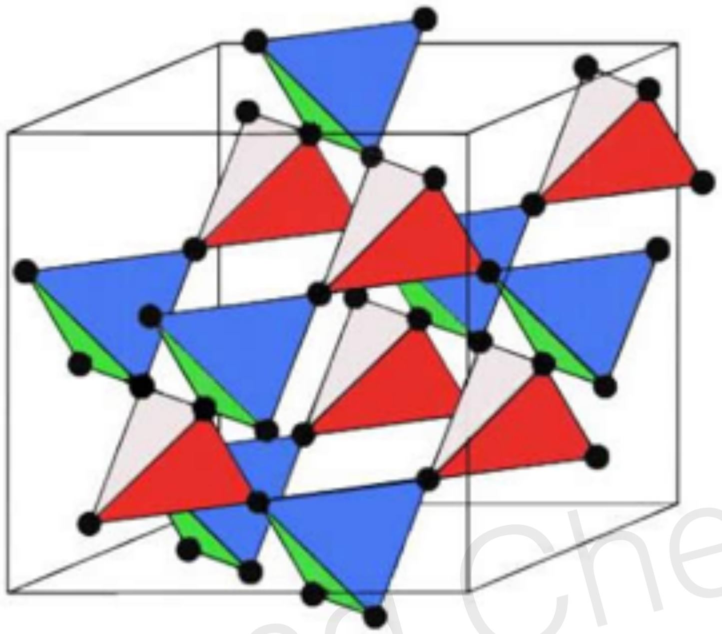
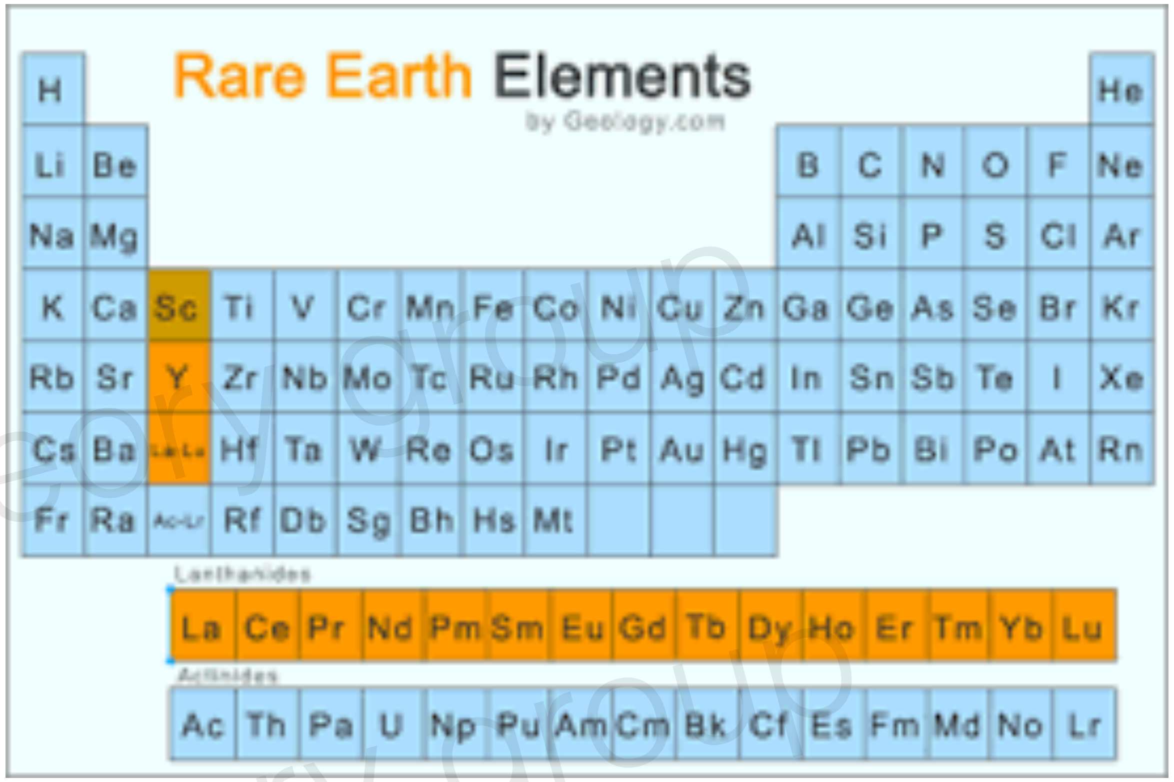
Outline

- Introduction: does quantum spin ice exist in nature?
- Magnetic transition of quantum spin ice is confinement transition of compact QED
- Monopole condensation and proximate phases
- Summary

Spin ice in rare-earth pyrochlores

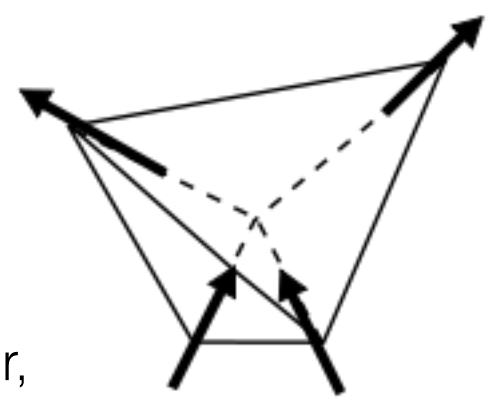


$RE_2M_2O_7$

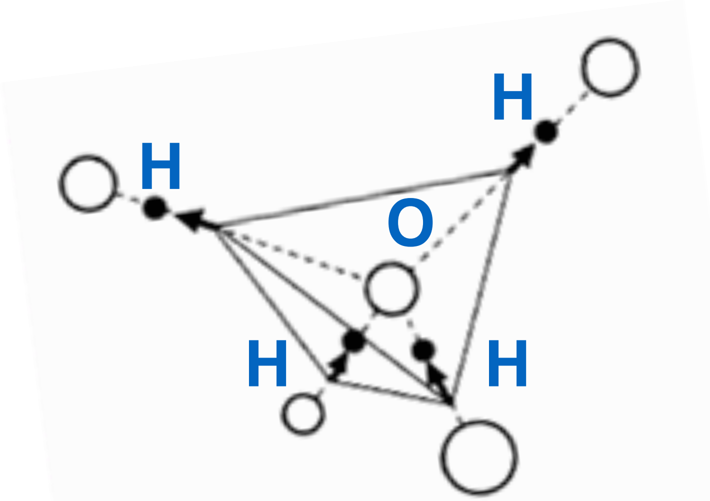


$$H = J_{zz}\sum_{\langle i,j\rangle} S_i^z S_j^z + \text{dipolar}$$

Castelnovo, Chalker,Gingras, Moessner, Sondhi, Schiffer, …….



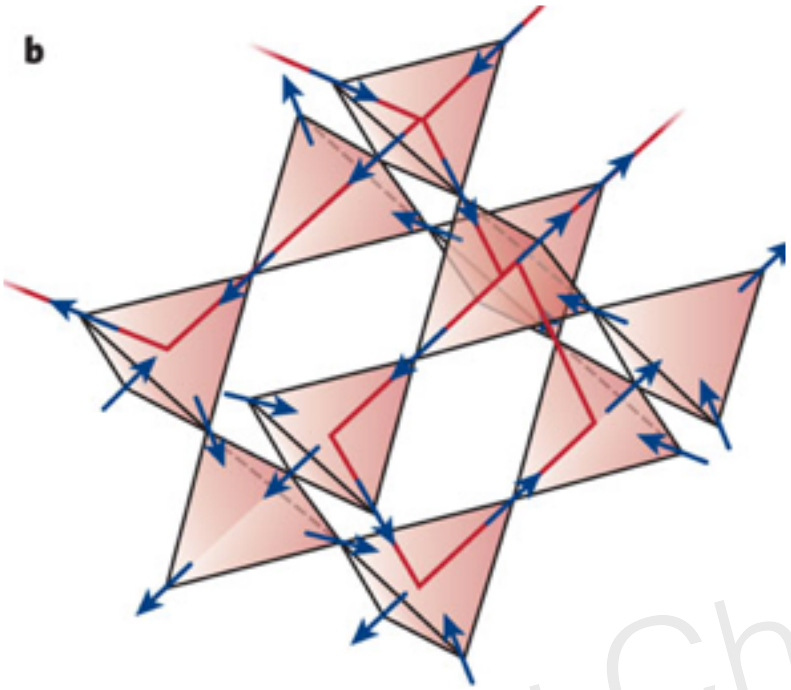
2-in 2-out
spin ice rule



2-in 2-out
water ice rule

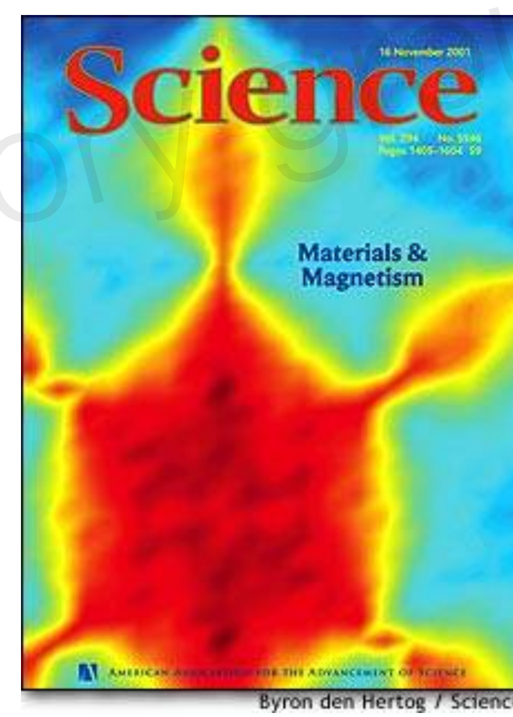
from wiki

Classical spin ice



- The "2-in 2-out" states are extensively degenerate.
- At T < Jzz, the system **thermally** fluctuates within the ice manifold, leading to classical spin ice and interesting experimental discoveries.

Gang Chen's theory group



Pinch points in spin correlation

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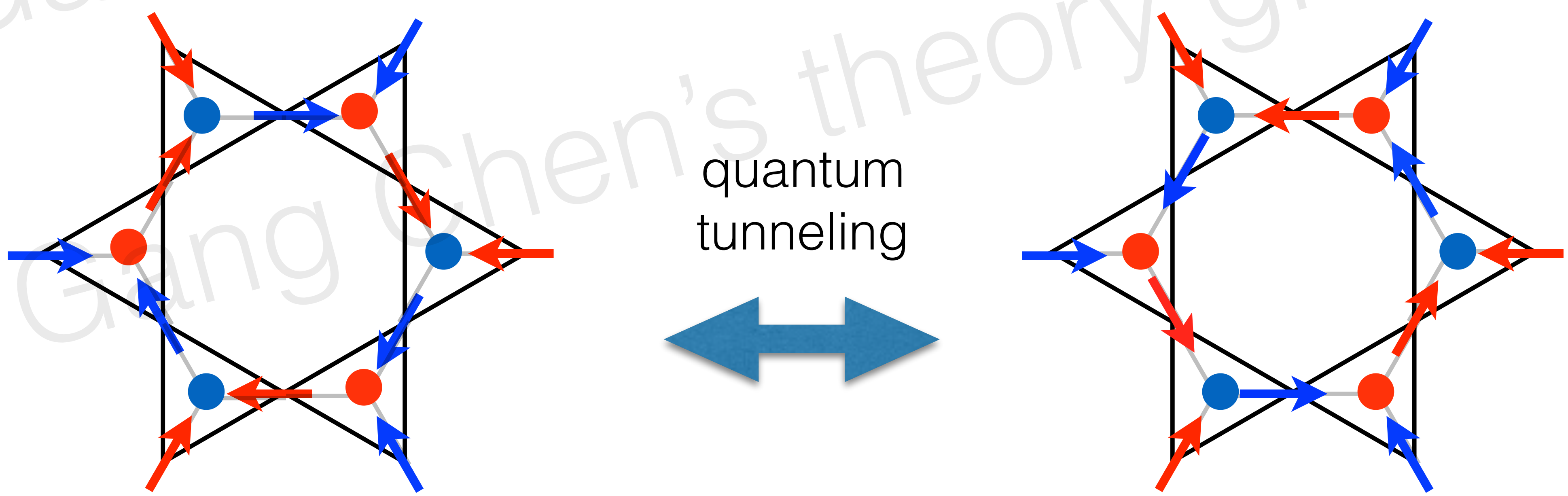
Quantum fluctuation leads to (disordered) quantum spin ice

$$H = J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z$$
$$- J_{\pm} \sum_{\langle i,j \rangle} (S_i^+ S_j^- + S_i^- S_j^+) \;\; + \cdots\cdots$$

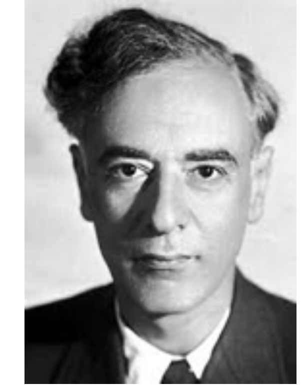
Hermele, Fisher, Balents,
Moessner, Isakov….

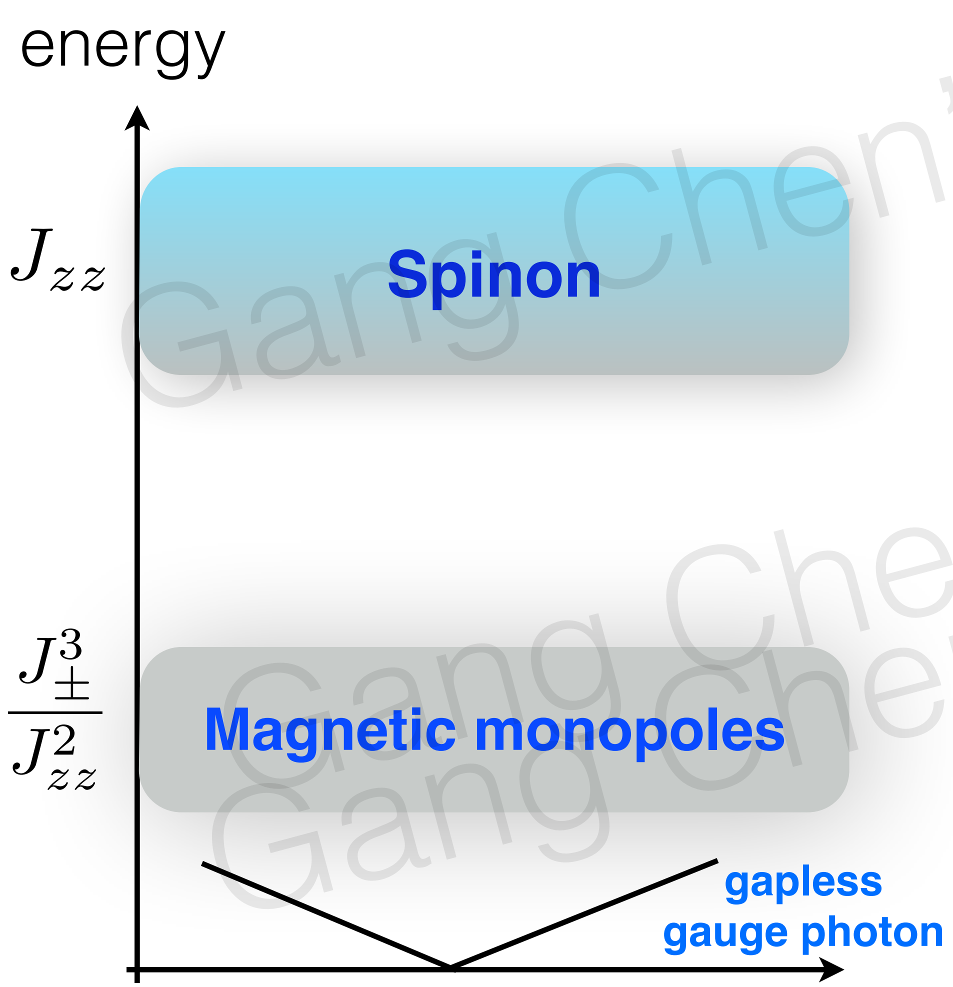
- Pretty much one can add any term to create **quantum** tunneling, as long as it is not too large to induce magnetic order, the **ground state** is a quantum spin ice !

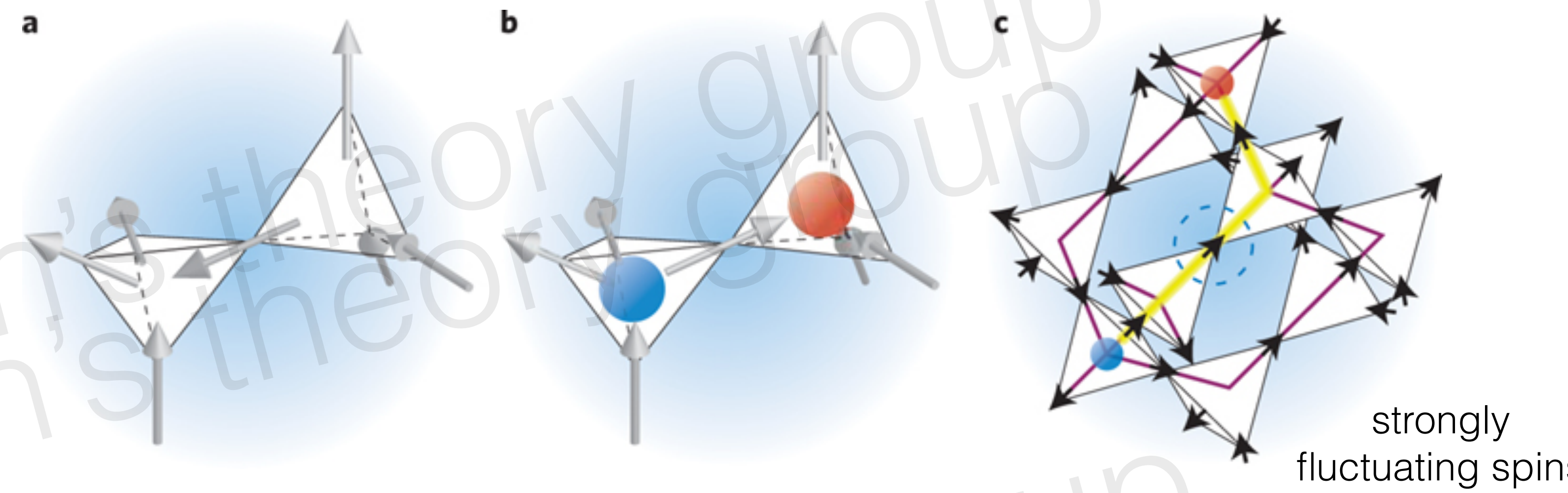


flip 6 spins on the hexagon

QSI is NOT a Landau phase


Landau





Figs from Moessner&Schiffer,2009

Spinon deconfinement

- Unlike CSI, QSI is a **novel phase of matter**. No LRO, no symmetry breaking, **cannot** be understood in Landau's paradigm!

- The right description is in terms of fractionalization and emergent gauge structure. It is an exotic phase of matter, like FQHE, and shares similar properties as topological order.

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Does QSI exist in experiments?

Realistic models

- Kramers' doublet

$$H = \sum_{\langle ij \rangle} \{ J_{zz} \mathsf{S}_i^z \mathsf{S}_j^z - J_{\pm} (\mathsf{S}_i^+ \mathsf{S}_j^- + \mathsf{S}_i^- \mathsf{S}_j^+) + J_{\pm\pm} (\gamma_{ij} \mathsf{S}_i^+ \mathsf{S}_j^+ + \gamma_{ij}^* \mathsf{S}_i^- \mathsf{S}_j^-) + J_{z\pm} [\mathsf{S}_i^z (\zeta_{ij} \mathsf{S}_j^+ + \zeta_{ij}^* \mathsf{S}_j^-) + i \leftrightarrow j] \},$$

S. H. Curnoe, PRB (2008).

- Non-Kramers' doublet

$$H = \sum_{\langle ij \rangle} \{ J_{zz} \mathsf{S}_i^z \mathsf{S}_j^z - J_{\pm} (\mathsf{S}_i^+ \mathsf{S}_j^- + \mathsf{S}_i^- \mathsf{S}_j^+) + J_{\pm\pm} (\gamma_{ij} \mathsf{S}_i^+ \mathsf{S}_j^+ + \gamma_{ij}^* \mathsf{S}_i^- \mathsf{S}_j^-)$$

- Dipole-octupole doublet

$$H = \sum_{\langle ij \rangle} J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_z S_i^z S_j^z + J_{xz} (S_i^x S_j^z + S_i^z S_j^x).$$

Y-P Huang, Gang Chen, M Hermele, PRL 2014

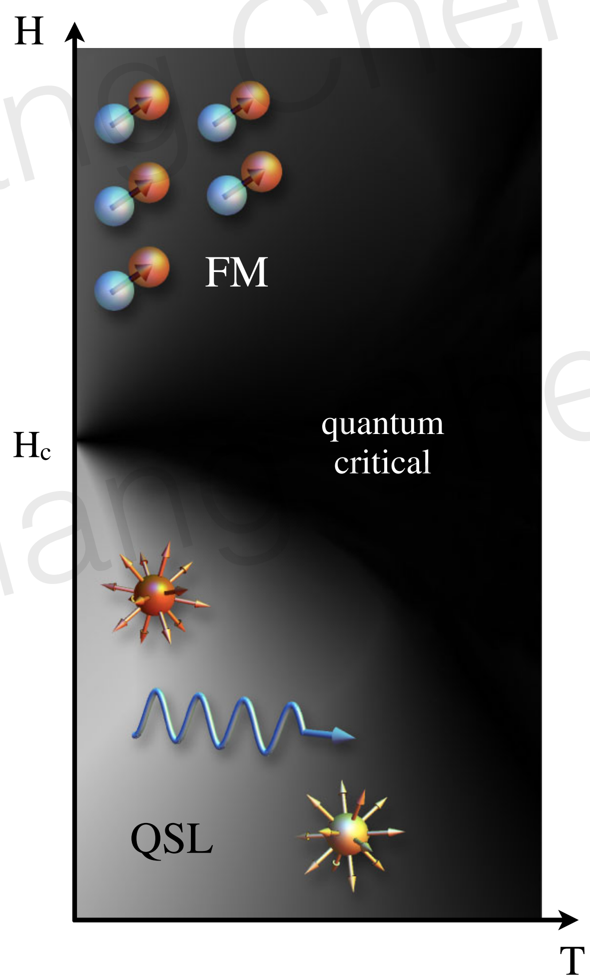
Experiments are sample dependent

PHYSICAL REVIEW X **1**, 021002 (2011)

Quantum Excitations in Quantum Spin Ice

Kate A. Ross,[1] Lucile Savary,[2] Bruce D. Gaulin,[1,3,4] and Leon Balents[5,*]

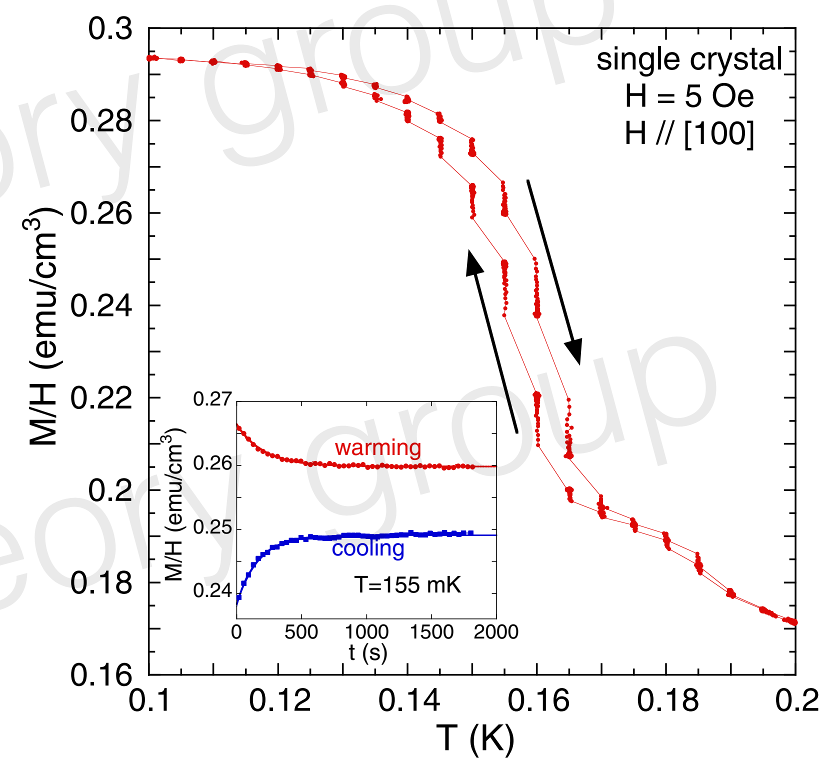
waves [14]. Although one neutron study [15] supported ferromagnetic order in $Yb_2Ti_2O_7$, intriguingly, the majority of neutron scattering measurements have reported a lack of magnetic ordering and the absence of spin waves at low fields in this material [16–18]. In a recent study,



PHYSICAL REVIEW B **89**, 224419 (2014)

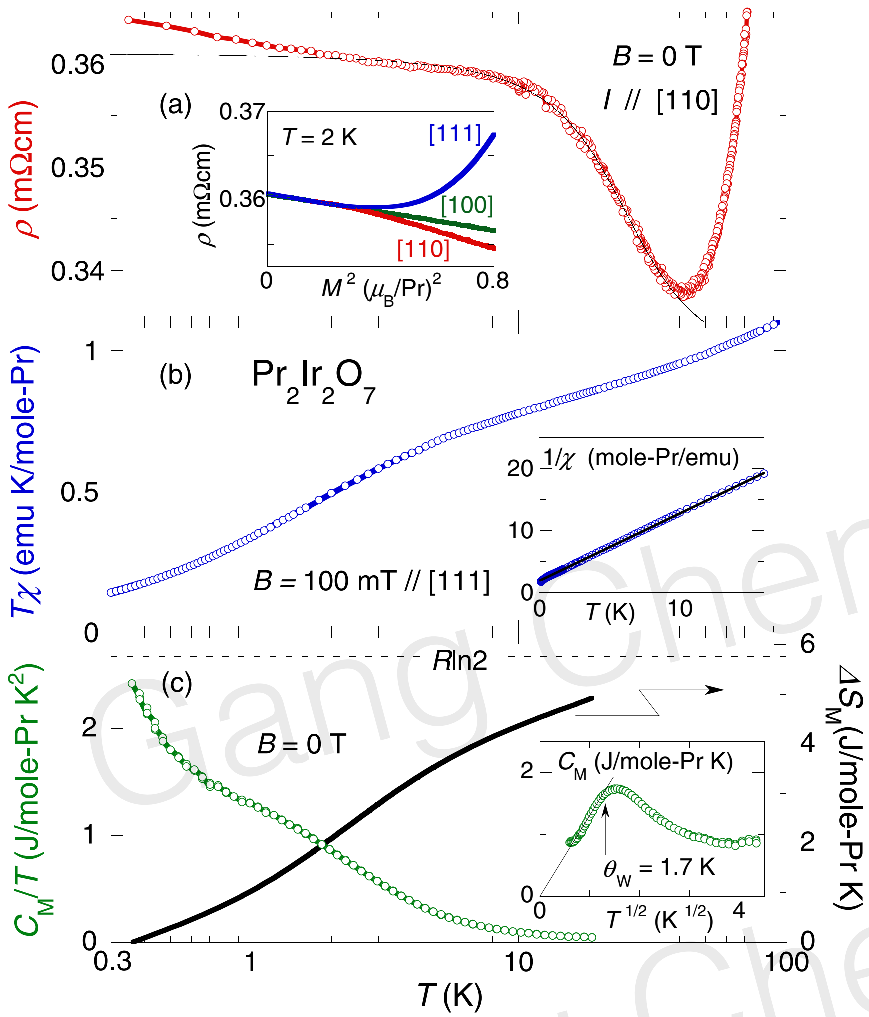
First-order magnetic transition in $Yb_2Ti_2O_7$

E. Lhotel,[1,*] S. R. Giblin,[2] M. R. Lees,[3] G. Balakrishnan,[3] L. J. Chang,[4] and Y. Yasui[5]



Some samples have FM LRO with 1st transition. Some samples do not have order.

Experiments: a featureless state near an ordered state

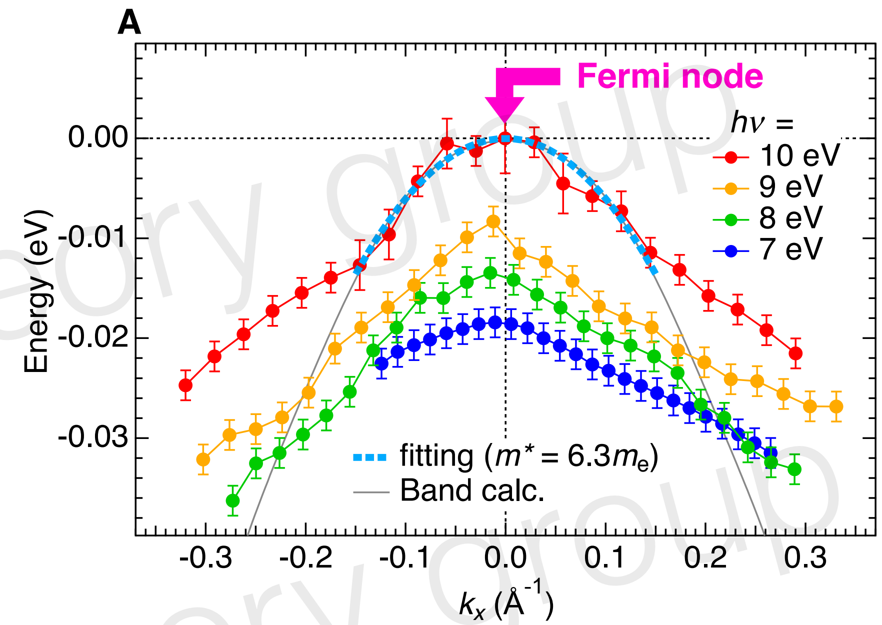


Nakatsuji, etc

PRL **96,** 087204 (2006)

$Pr_2Ir_2O_7$

ARPES: quadratic band touching of Ir d electrons



Valence band approaches the Fermi energy at few meV resolution

T. Kondo, S Shin, 2014 (unpublished)

Expts are sample dependent,
some samples are AFM ordered **(Prof. C. Broholm)**

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f-d exchange in R2Ir2O7, **Gang Chen**, M Hermele PRB 2012

Synthesis of experiments in a theory way

- What is the structure of the magnetic order?
- What is the relationship between the featureless disordered state and various magnetic states?
- What is the nature of the featureless disordered states? Is it **QSI**?

also see Prof Horoi's slides on Tb2Ti2O7

Insight from high-Tc superconductors

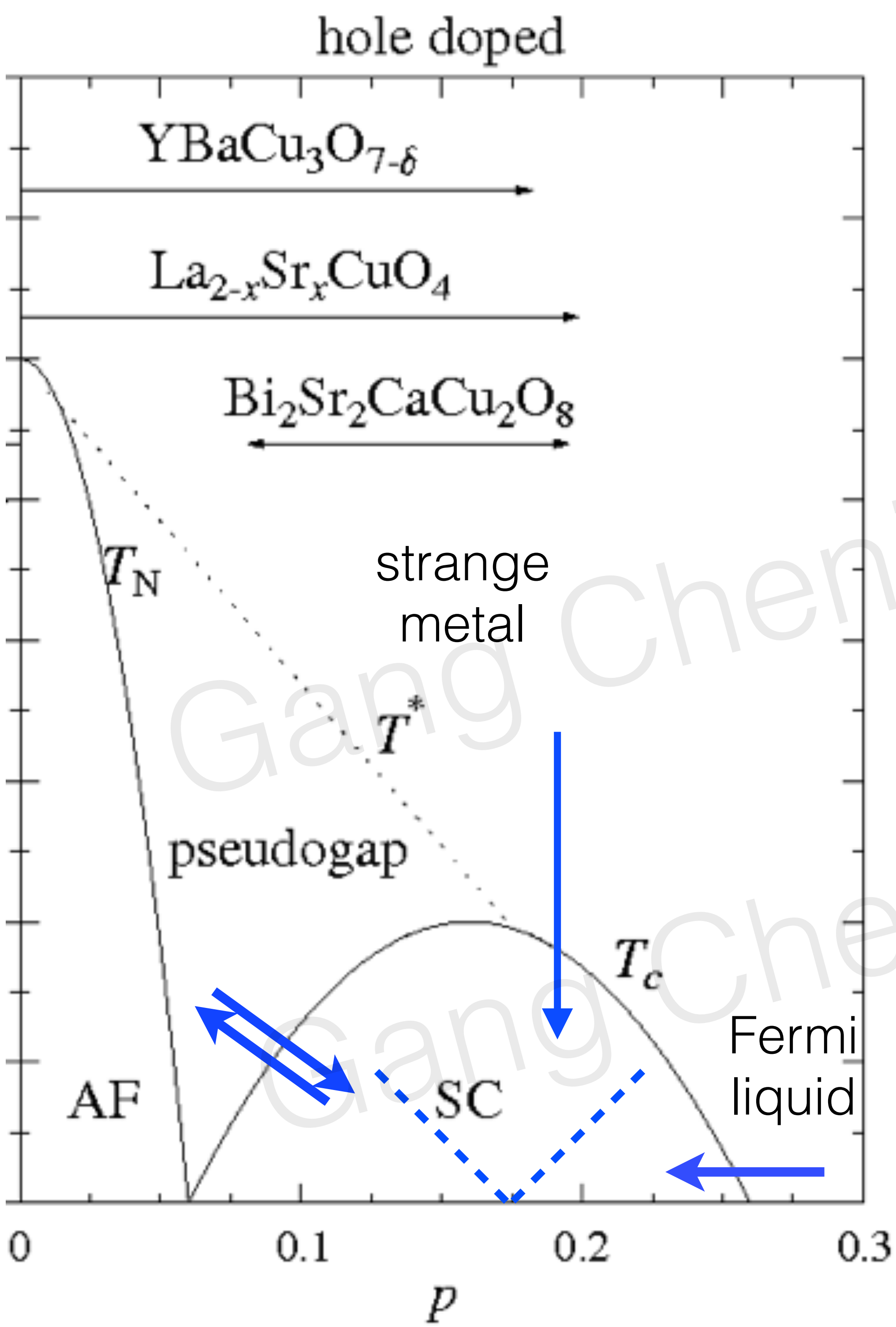


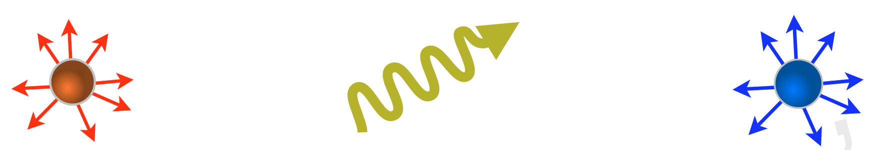
Figure from wiki

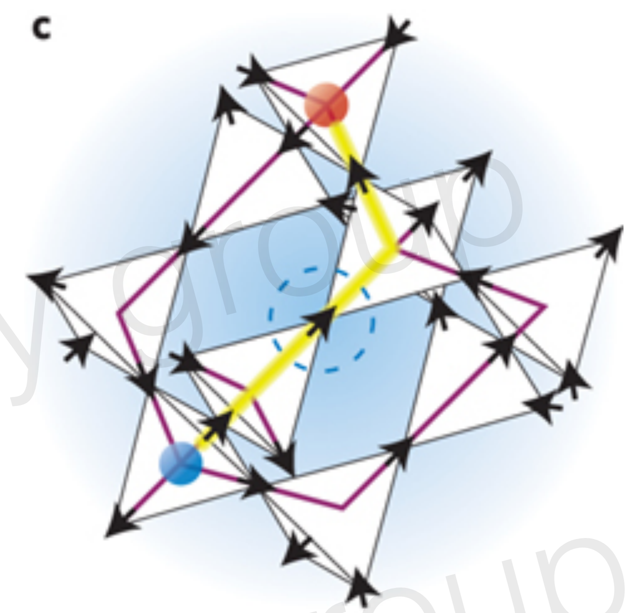
One important question is to understand the relationship between different phases (and/or orders)

1. Perturbative treatment (not interesting): instability of Fermi liquid;
2. Attack from top: instability of non-Fermi liquid;
3. Attack from bottom: some quantum criticality under the SC dome?
4. Attack from Left, attack from Right: what is PG (Z2 topological order?) ? (Senthil & Fisher 2000-2002);

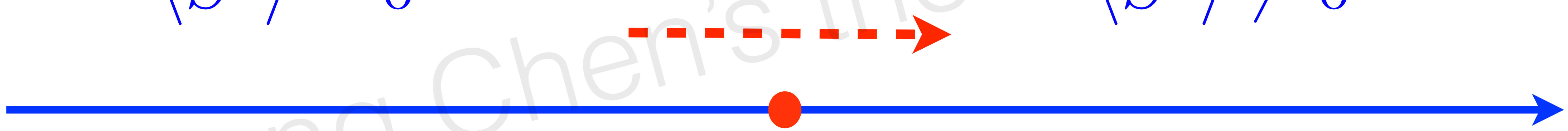
Attack from left (quantum spin ice)

spinon | gauge photon | spinon





$\langle S^z \rangle = 0$ | $\langle S^z \rangle \neq 0$



quantum spin ice | magnetic order

Spinons are deconfined. | **Spinons are confined !**

There is a subtle difference between Kramers' doublet and non-Kramers' doublet.

Outline

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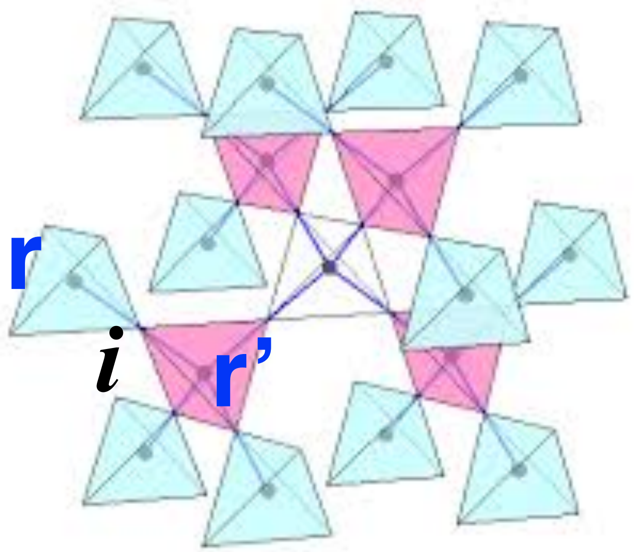
Lattice gauge theory formalism: technical part



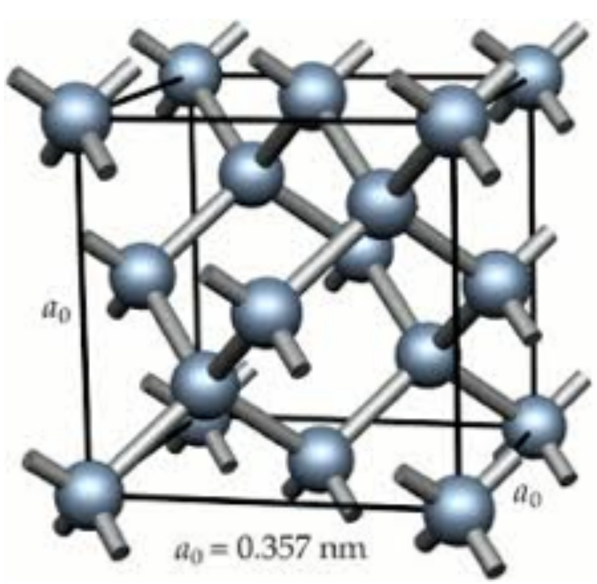
$$E_{\mathbf{rr'}} \sim S_i^z, \quad e^{iA_{\mathbf{rr'}}} \sim S_i^+.$$

Hermele etc, 2004

$$H_{\text{Ring}} \sim -K \sum_{hexagon} \left[S_1^+ S_2^- S_3^+ S_4^- S_5^+ S_6^- + h.c.\right]$$

$$H_{\text{LGT}} = \sum_{\langle \mathbf{rr'} \rangle} \frac{U}{2} (E_{\mathbf{rr'}} - \frac{\epsilon_{\mathbf{r}}}{2})^2 - \sum_{\hexagon_d} K \cos(curl\, A),$$

H_{LGT} captures the universal properties of QSI.



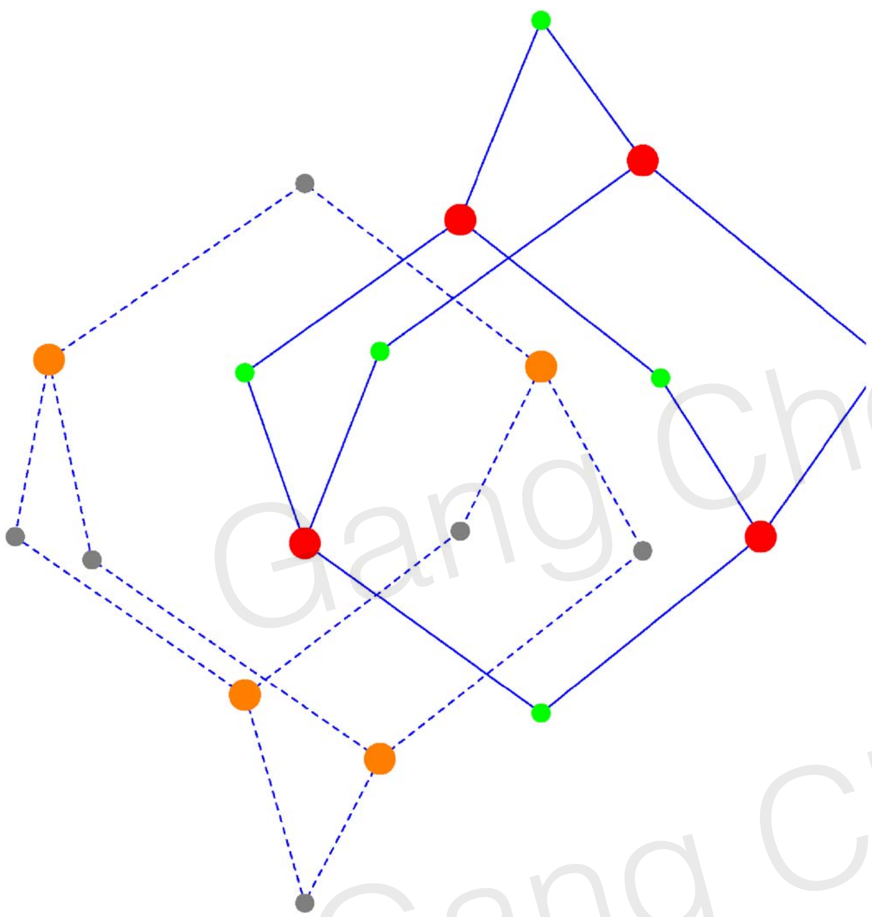
diamond lattice

- In an ordered state, <Sz>!=0, <S^+> is strongly fluctuating.
- In the gauge language, **E field** is static, **B magnetic field** is strongly fluctuating, the magnetic monopole (carrying magnetic charge) is condensed, which confines the electric charge carriers (spinons).

Electromagnetic duality

Monopole lives on dual diamond lattice, carry magnetic charge or dual U(1) gauge charge.

To study monopole physics, we need to use a technique called “duality” to make it explicit.



diamond (dotted) and
dual diamond lattice
(Bergman, etc, PRB 2006)

$$curl\, a \equiv \sum_{\mathbf{rr}' \in \hexagon_d^*} a_{\mathbf{rr}'} \equiv E_{\mathbf{rr}'} - E^0_{\mathbf{rr}'},$$ dual U(1) gauge

$$H_{\rm dual} = \sum_{\hexagon_d^*} \frac{U}{2}(curl\, a - \bar{E})^2 - \sum_{\mathbf{r},\mathbf{r}'} K \cos B_{\mathbf{rr}'},$$

insert monopole variables

$$H_{\rm dual} = \sum_{\hexagon_d^*} \frac{U}{2}(curl\, a - \bar{E})^2 - \sum_{\mathbf{r},\mathbf{r}'} K \cos B_{\mathbf{rr}'}$$
$$- \sum_{\mathbf{r},\mathbf{r}'} t \cos(\theta_{\mathbf{r}} - \theta_{\mathbf{r}'} + 2\pi a_{\mathbf{rr}'}).$$

- **B magnetic field** is strongly fluctuating, the fluctuation of dual U(1) gauge field is weak.

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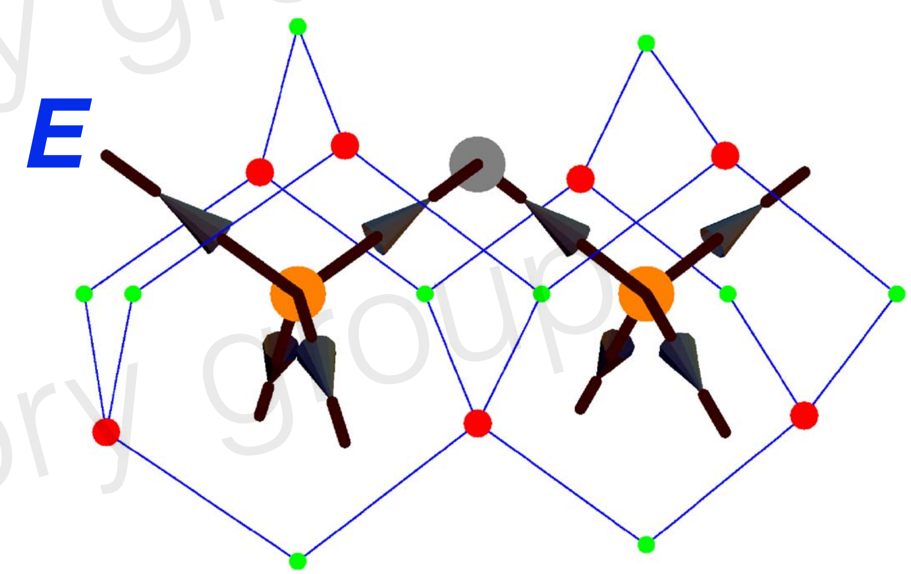
Physical observables are gauge invariant

- Monopole loop current defines the magnetic order

$$\text{curl}\ \vec{E} = -\frac{\partial \vec{B}}{\partial t} + \vec{J}_b,$$ **(Maxwell's equation)**

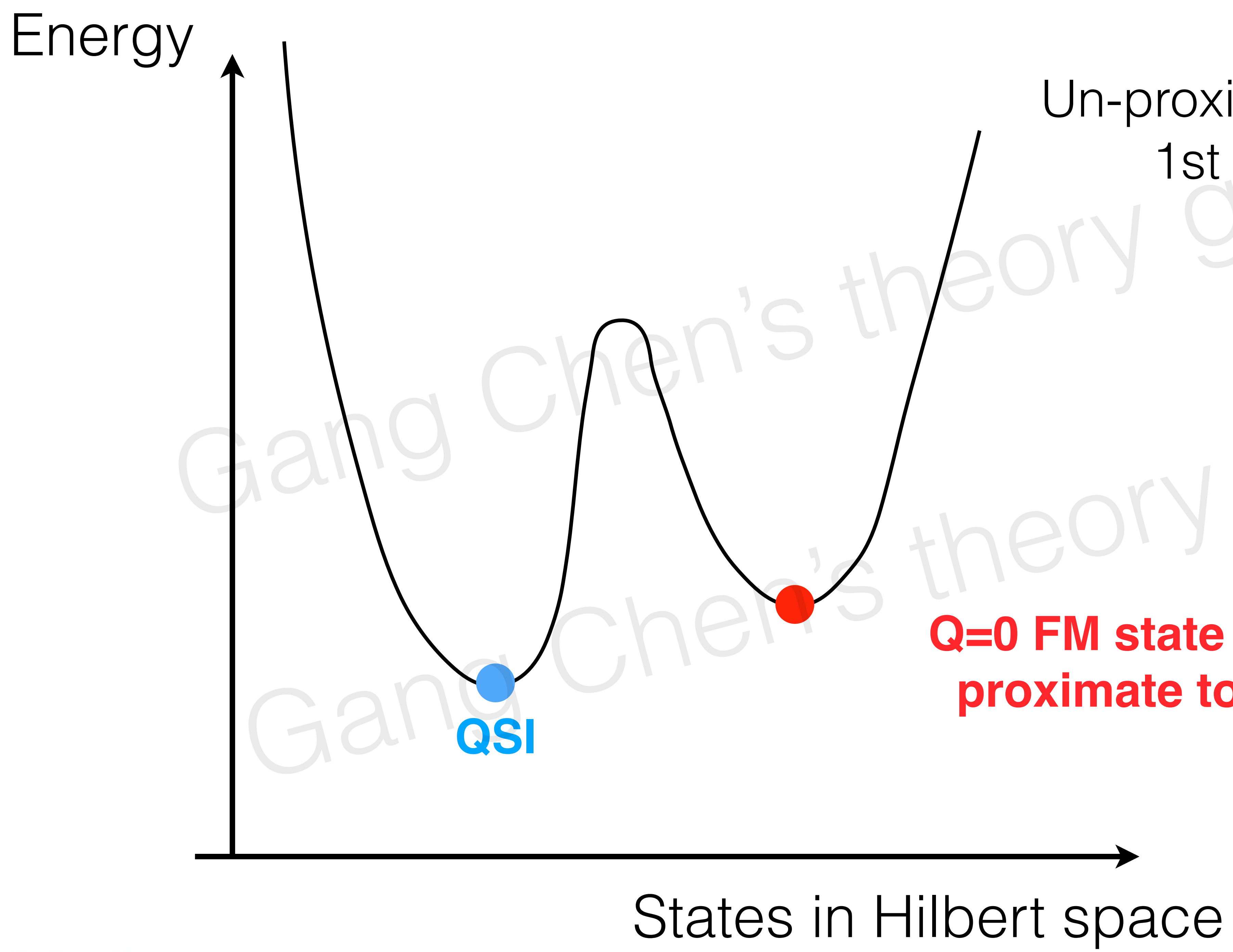
$$\oint_C \vec{J}_b \cdot d\vec{\ell} = \oint_C \text{curl}\ \vec{E} \cdot d\vec{\ell} = \int_S \vec{E} \cdot d\vec{A}.$$

$$S_i^z \sim E_{\mathbf{rr'}} \sim \sum_{\mathbf{rr'} \in \hexagon_d^*} \mathsf{J}_{\mathbf{rr'}}$$



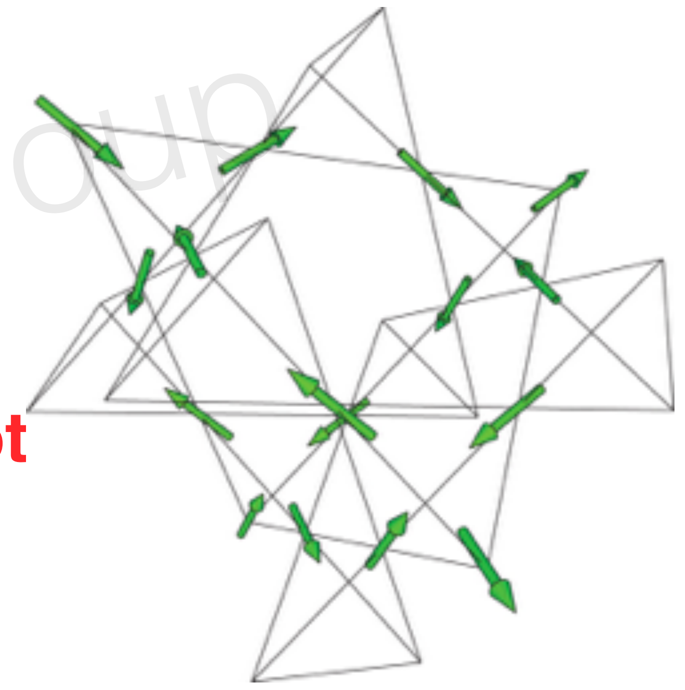
(Bergman, etc, PRB 2006)

Proximate and un-proximate magnetic states



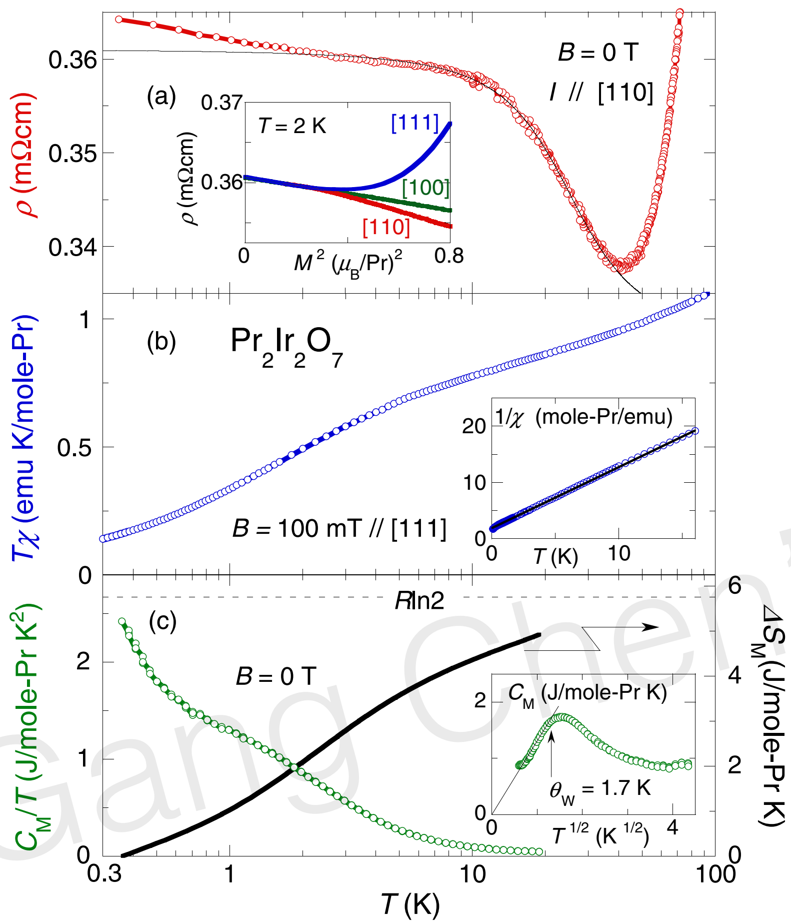
Un-proximity implies a strong 1st order transition!

Q=0 FM state is not proximate to QSI



Q=2Pi(001) AFM state is the simplest **proximate** state.

Implication for experiments



Nakatsuji, etc

PRL **96,** 087204 (2006)

Pr2Ir2O7

PHYSICAL REVIEW B **89**, 224419 (2014)

First-order magnetic transition in $Yb_2Ti_2O_7$

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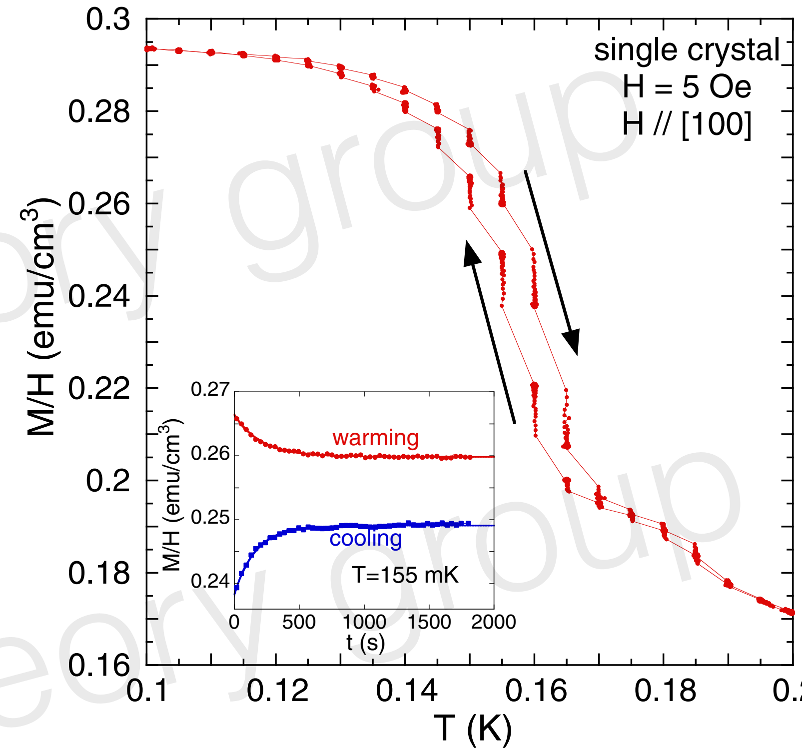


Yb2Ti2O7

PIO: different samples have different Fermi energy -> RKKY

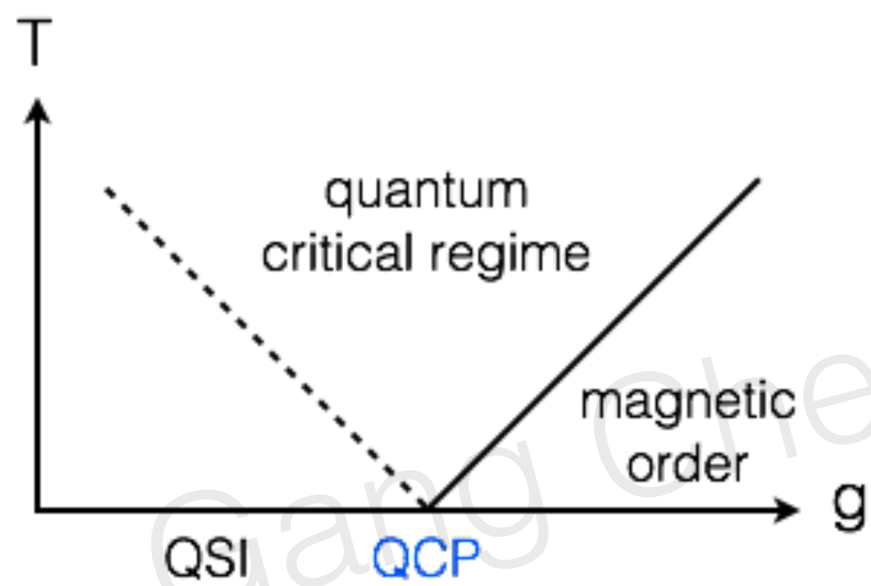
-> **magnetic order, Q= 2Pi(001)**

YTO: different samples might be trapped in some metastable states.

first order transition to **Q=0 FM state**.

Critical theory

Standard Landau-Ginzburg expansion in the monopole fields



$$\Phi_{\mathbf{r}} = \sum_{a=1}^{12} \varphi_a(\mathbf{r})\, \phi_a,$$

determined by **projective symmetry group**

$$L = \sum_a \left[|(\partial_\mu - i\tilde{a}_\mu)\phi_a|^2 + m^2 |\phi_a|^2 \right] + \frac{{F_{\mu\nu}}^2}{2}$$
$$+ u_0 \Big(\sum_a |\phi_a|^2\Big)^2 + \cdots,$$

The critical theory is described by multicomponent bosons coupled with a fluctuating U(1) gauge field in 3+1D.

a unusual weak divergence $\chi(Q) \sim -\ln T$ "subsidiary order"

Summary

- I have studied the phase diagram near quantum spin ice.

- Using field theoretic technique, I have obtained the structure of the magnetic states and the nature of the magnetic transition.

- I use my theoretical results to explain the puzzling experiments in $Pr_2Ir_2O_7$ and $Yb_2Ti_2O_7$. It implies the disordered phase is likely to be a QSI.

Thank you !

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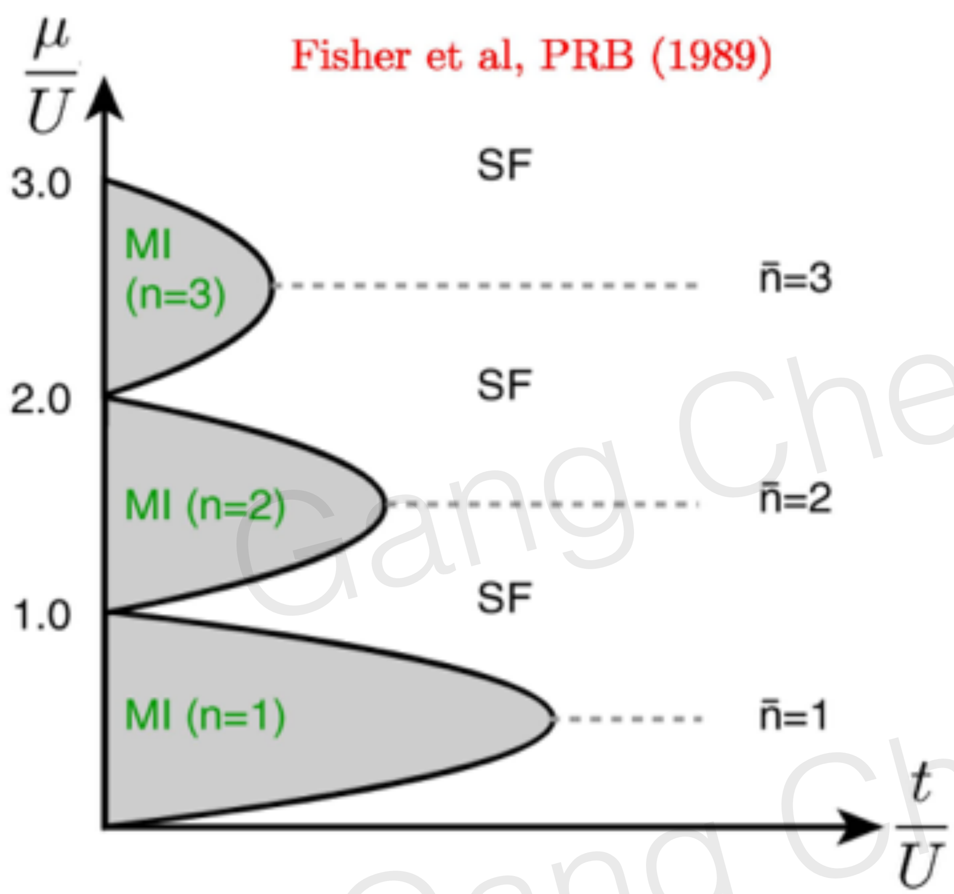
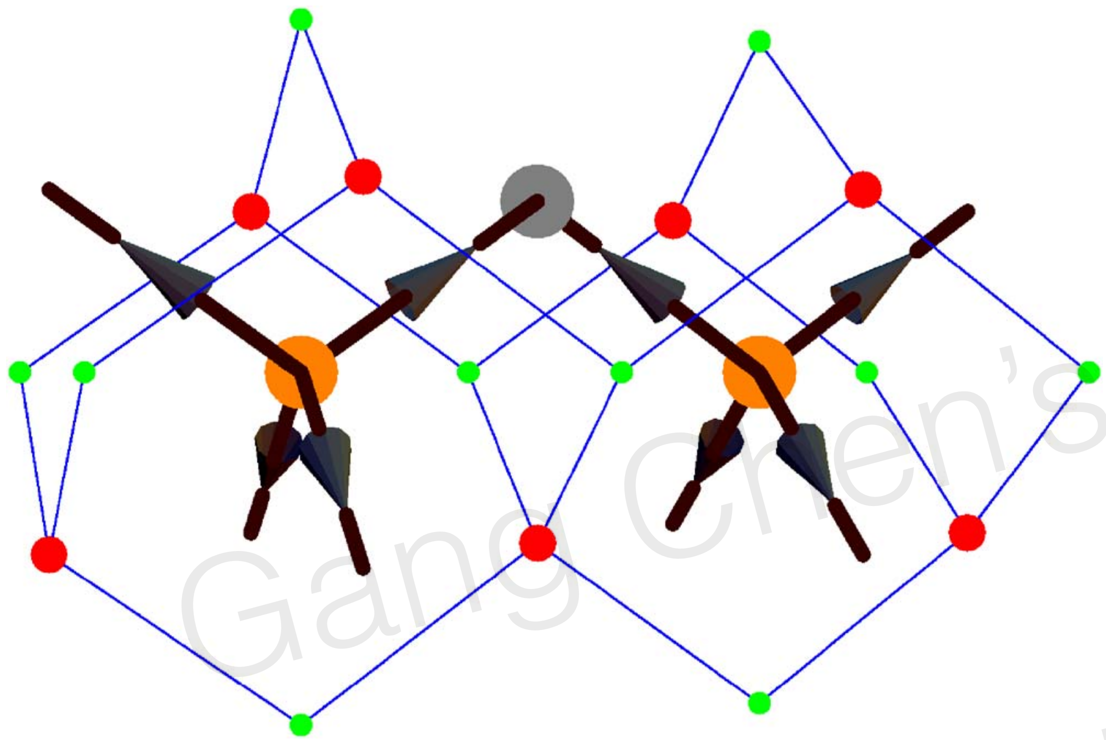
Analogy with Boson-vortex duality



SF	boson is condensed	vortex is gapped	n is strongly fluctuating
MI	boson is gapped	vortex is condensed	n is fixed
QSI	spinon is gapped	magnetic monopole is gapped	E field is strongly fluctuating
Sz order	spinon is confining	magnetic monopole is condensed	E field is fixed

Frustrated monopole bands



$$H_{\text{dual}} = \sum_{\hexagon_d^*} \frac{U}{2}(curl\, a - \bar{E})^2 - \sum_{\mathbf{r},\mathbf{r}'} K \cos B_{\mathbf{r}\mathbf{r}'}$$
$$- \sum_{\mathbf{r},\mathbf{r}'} t \cos(\theta_{\mathbf{r}} - \theta_{\mathbf{r}'} + 2\pi a_{\mathbf{r}\mathbf{r}'}).$$

Fixing gauge, $curl\, \bar{a} = \bar{E},$

$$H_m = - \sum_{\mathbf{r},\mathbf{r}'} t\, e^{-i2\pi \bar{a}_{\mathbf{r}\mathbf{r}'}} \Phi_{\mathbf{r}}^\dagger \Phi_{\mathbf{r}'}, \qquad \Phi_{\mathbf{r}} \equiv e^{i\theta_{\mathbf{r}}}$$
$$|\Phi_{\mathbf{r}}| \equiv 1.$$

The background electric field distribution creates a Pi flux experienced by the monopoles, this frustrates monopole hopping.

The monopole band minima have a line degeneracy in k space.

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