The spectral periodicity of spinon continuum in quantum spin ice

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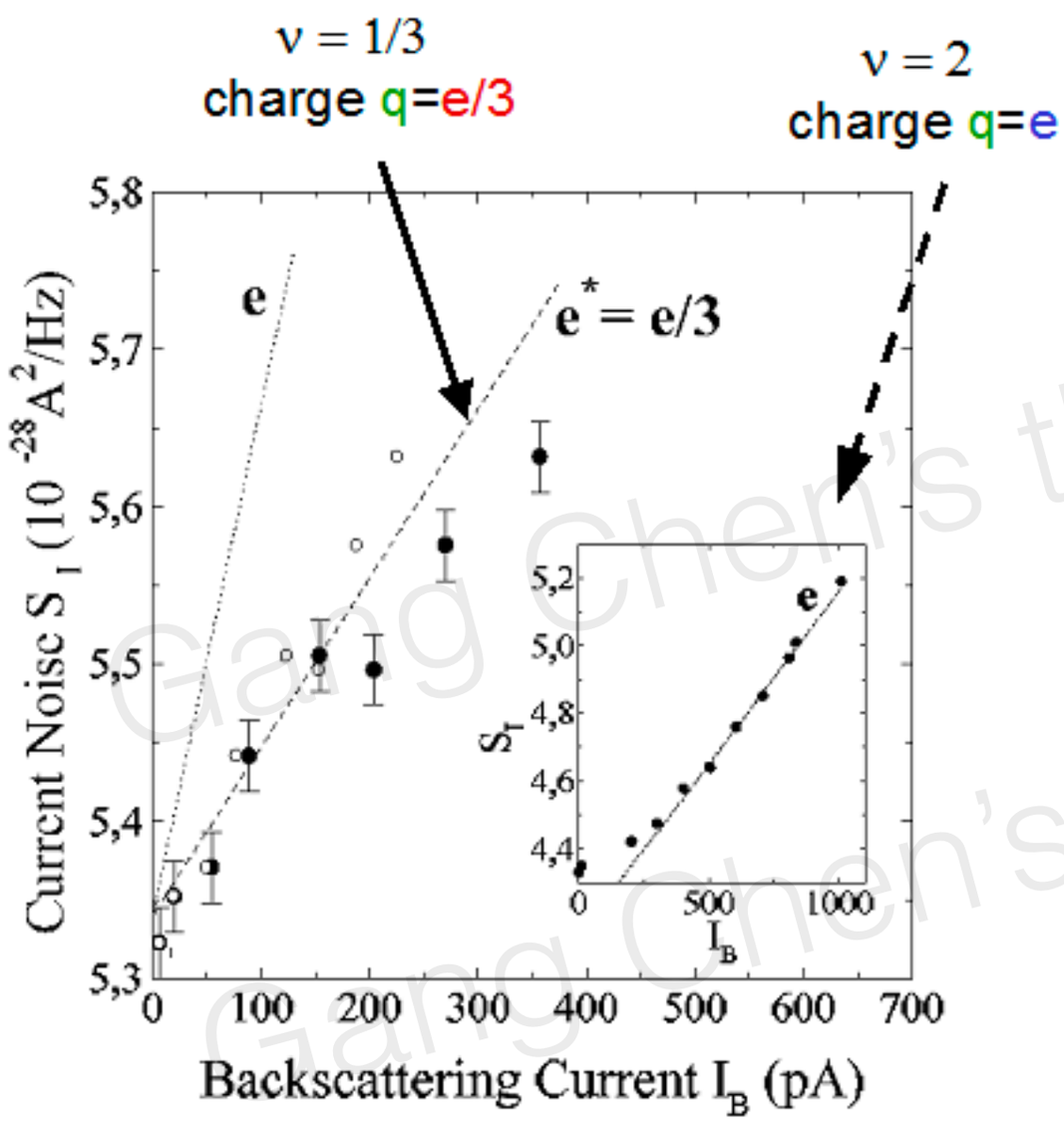
The spectral periodicity of spinon continuum in quantum spin ice

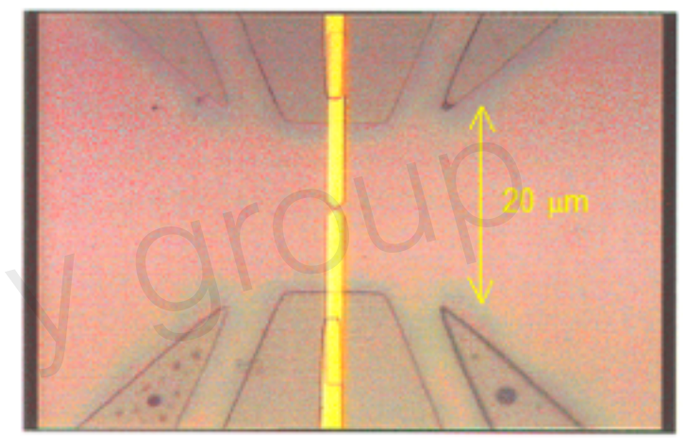
Gang Chen, arXiv 1704.02734

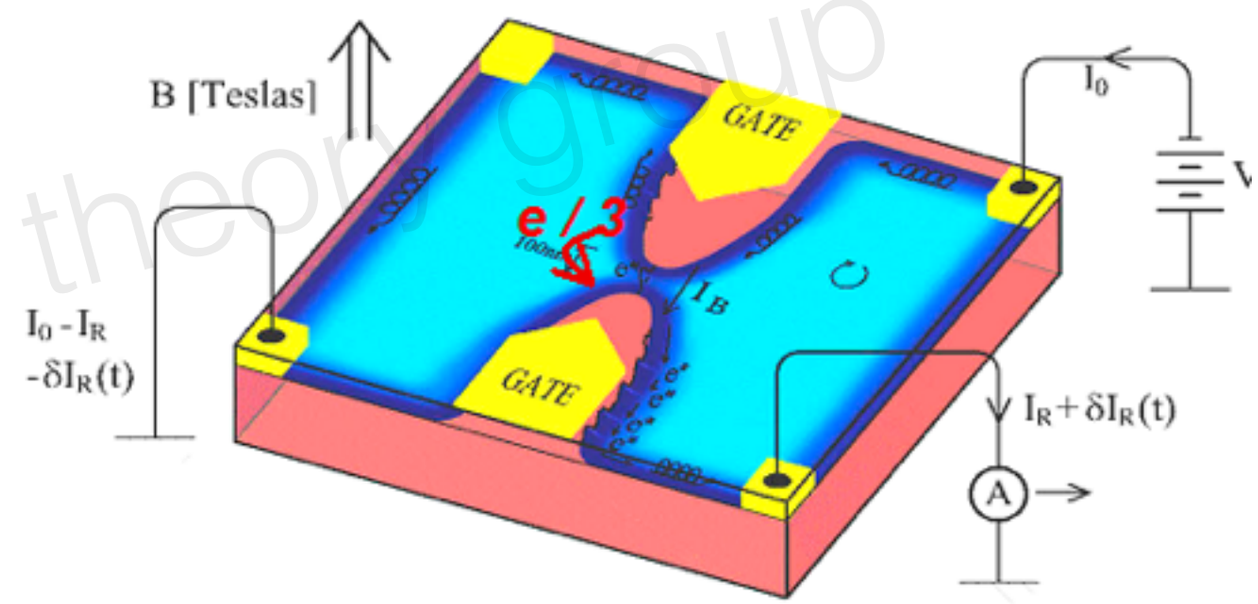
PhysRevB, **96**, 085136 (**2017**)

The field is not just about finding spin liquid candidate,
but is to confirm/prove spin liquids.
The most important feature is probably fractionalization.

Fractionalization in FQHE: shot-noise measurement







Etien et al, PRL 79, 2526 (1997)
also see Heiblum et al, Nature (1997)

FQHE is arguably the only existing topological order so far.

Chiral (Abelian) topological order

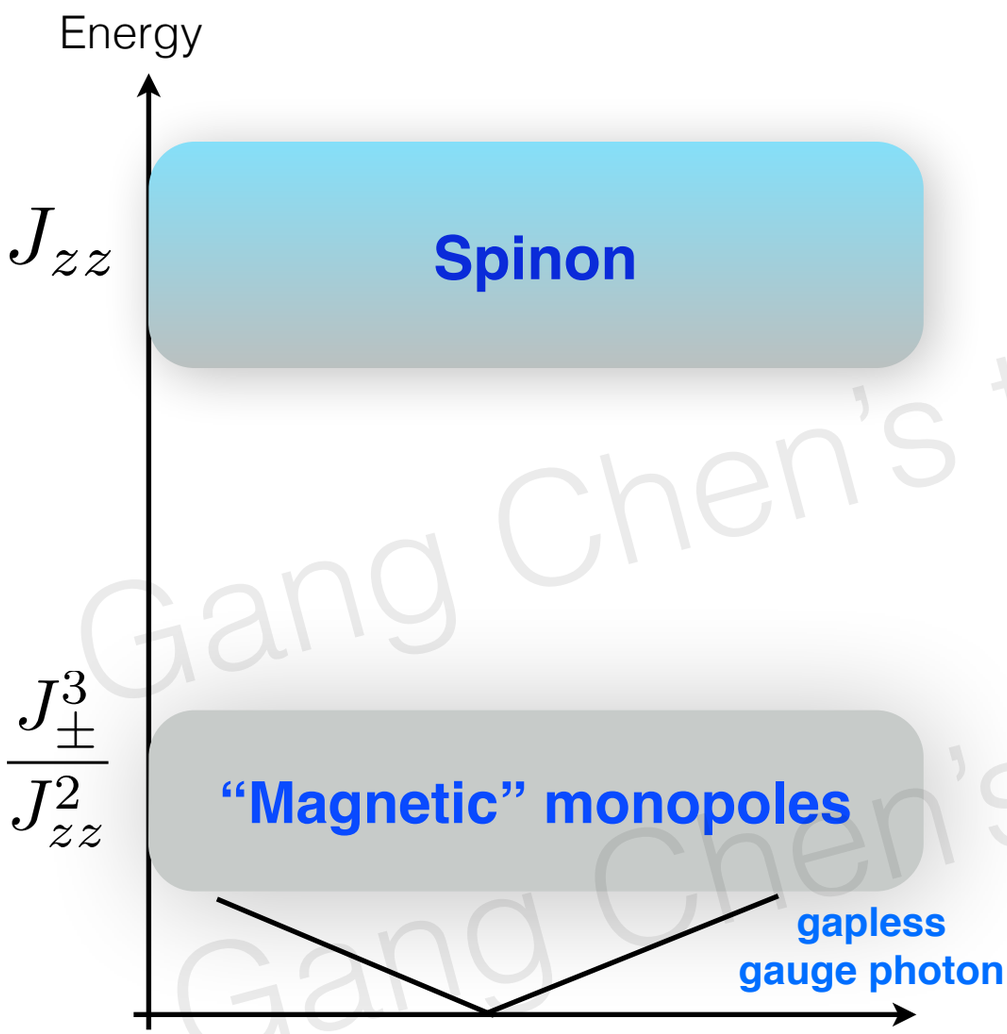
Fractionalization: fractionalized & deconfined excitation
Chern-Simon gauge structure

with charge U(1) **symmetry**:
charge conservation

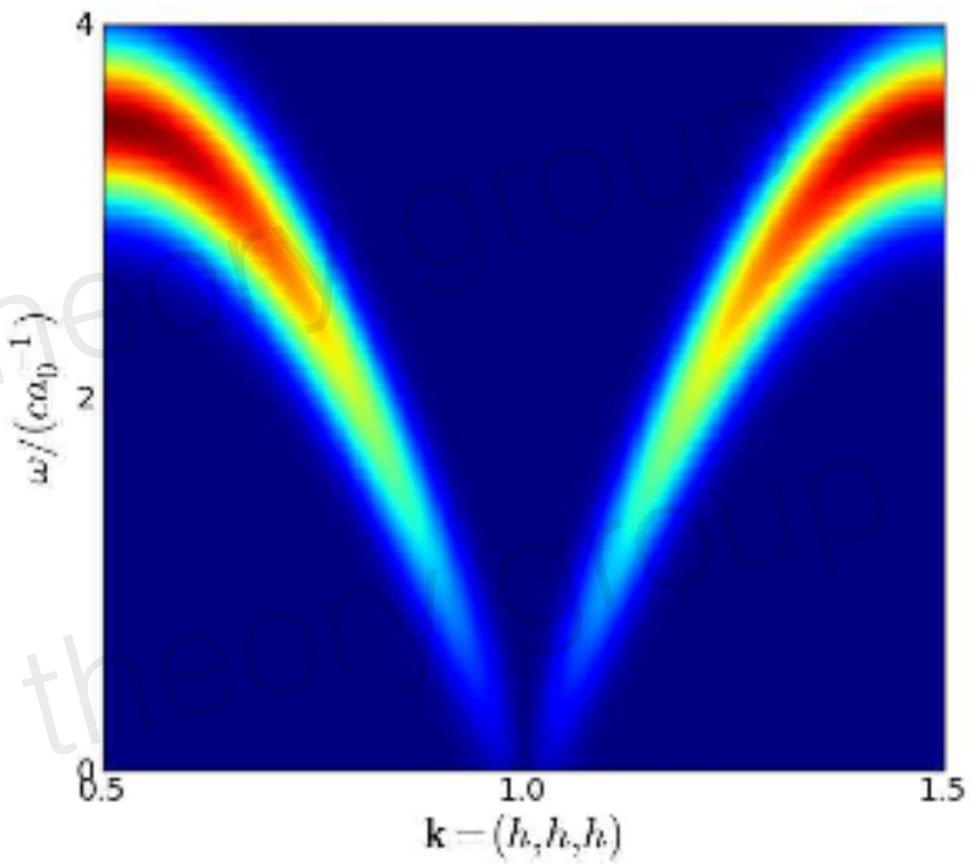
Fractionalized charge excitation

Symmetry makes topological order more visible in experiments.

What is the sharp physical observable for the U(1) QSL in quantum spin ice?



Hermele, Fisher, Balents 2004



$I(\omega) \sim \omega$

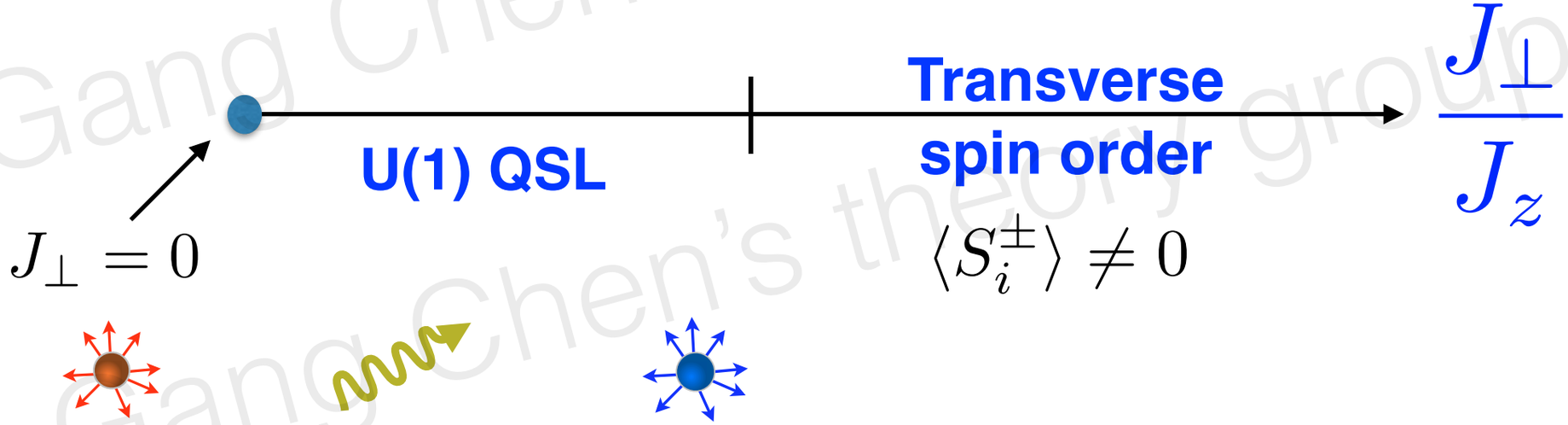
low energy scale
suppressed intensity

Nic Shannon, etc 2012,
Savary, Balents, 2012

heat capacity (Savary&Balents: 1000 times larger than phonon!)
and spinon continuum (higher in energy)

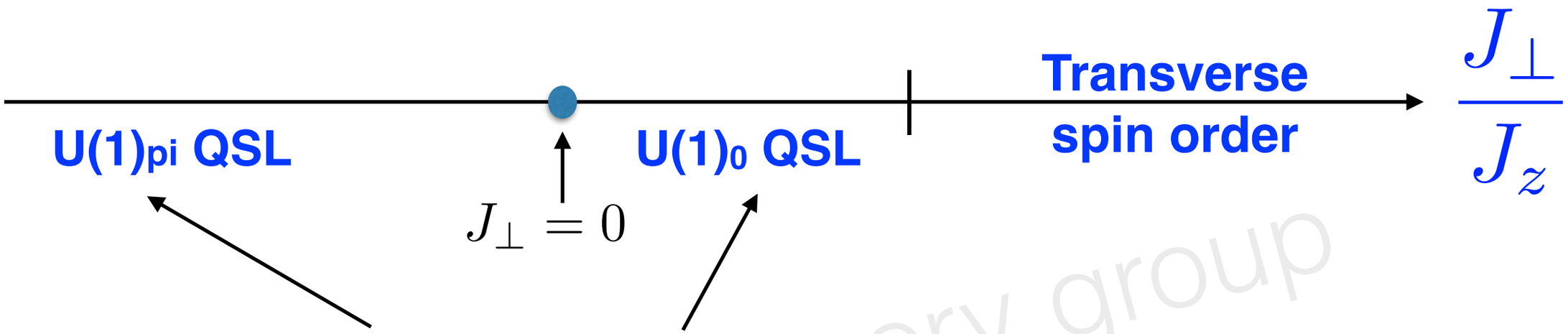
Use the XXZ model to illustrate the universal physics

$$\mathcal{H}_{\mathrm{XXZ}} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_\perp (S_i^+ S_j^- + S_i^- S_j^+),$$



Hermele, Fisher, Balents, 2004,
Banerjee, Isakov, Demle, YB Kim 2008
Savary, Balents, 2012
Kato, Onoda, 2015
N Shannon, et al, 2012

Frustrated regime



Related by unitary transformation (Hermele, Fisher, Balents 2004)

Sungbin Lee, S Onoda, L Balents, PRB 2012

Besides the quantitative differences, are there sharp distinctions between the $U(1)_{pi}$ QSL on the left and the $U(1)_0$ QSL on the right?

Lattice gauge theory

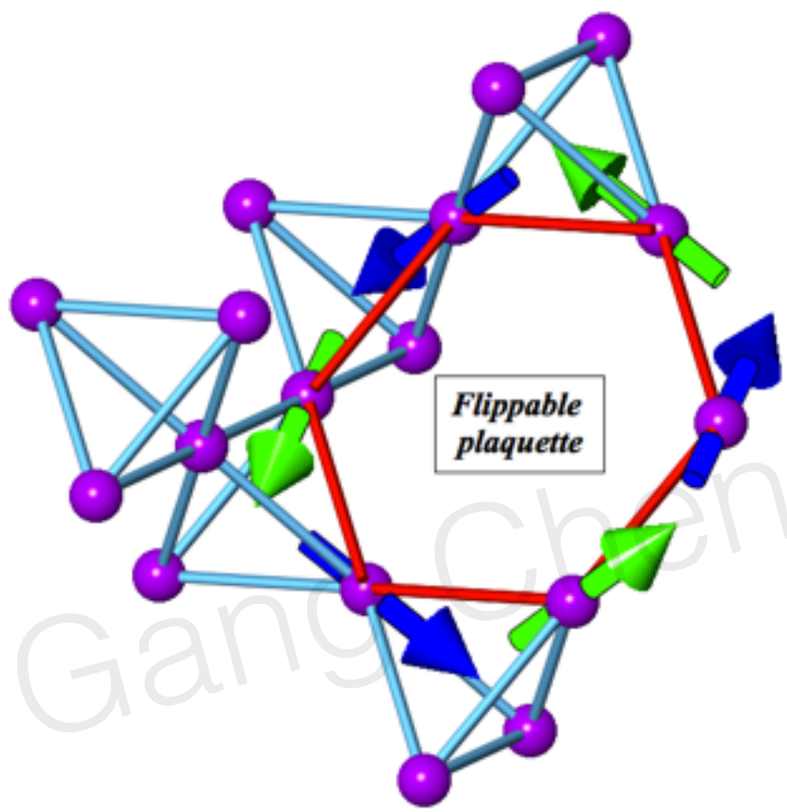


Figure from Michel Gingras' paper

$$\mathcal{H}_{\rm XXZ} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_\perp (S_i^+ S_j^- + S_i^- S_j^+),$$

3rd order degenerate perturbation
(Hermele, Fisher, Balents 2004)

$$\mathcal{H}_{\rm eff} = -\frac{12 J_\perp^3}{J_{zz}^2} \sum_{\hexagon_{\rm p}} (S_i^+ S_j^- S_k^+ S_l^- S_m^+ S_n^- + h.c.),$$

$$E_{\boldsymbol{rr'}} \simeq S^z_{\boldsymbol{rr'}}$$

$$e^{iA_{\boldsymbol{rr'}}} \simeq S^\pm_{\boldsymbol{rr'}}$$

Lattice gauge theory
on the diamond lattice

$$\mathcal{H}_{\rm LGT} = -K \sum_{\hexagon_{\rm d}} \cos(\mathrm{curl}\, A) + U \sum_{\boldsymbol{rr'}} (E_{\boldsymbol{rr'}} - \frac{\eta_{\boldsymbol{r}}}{2})^2$$

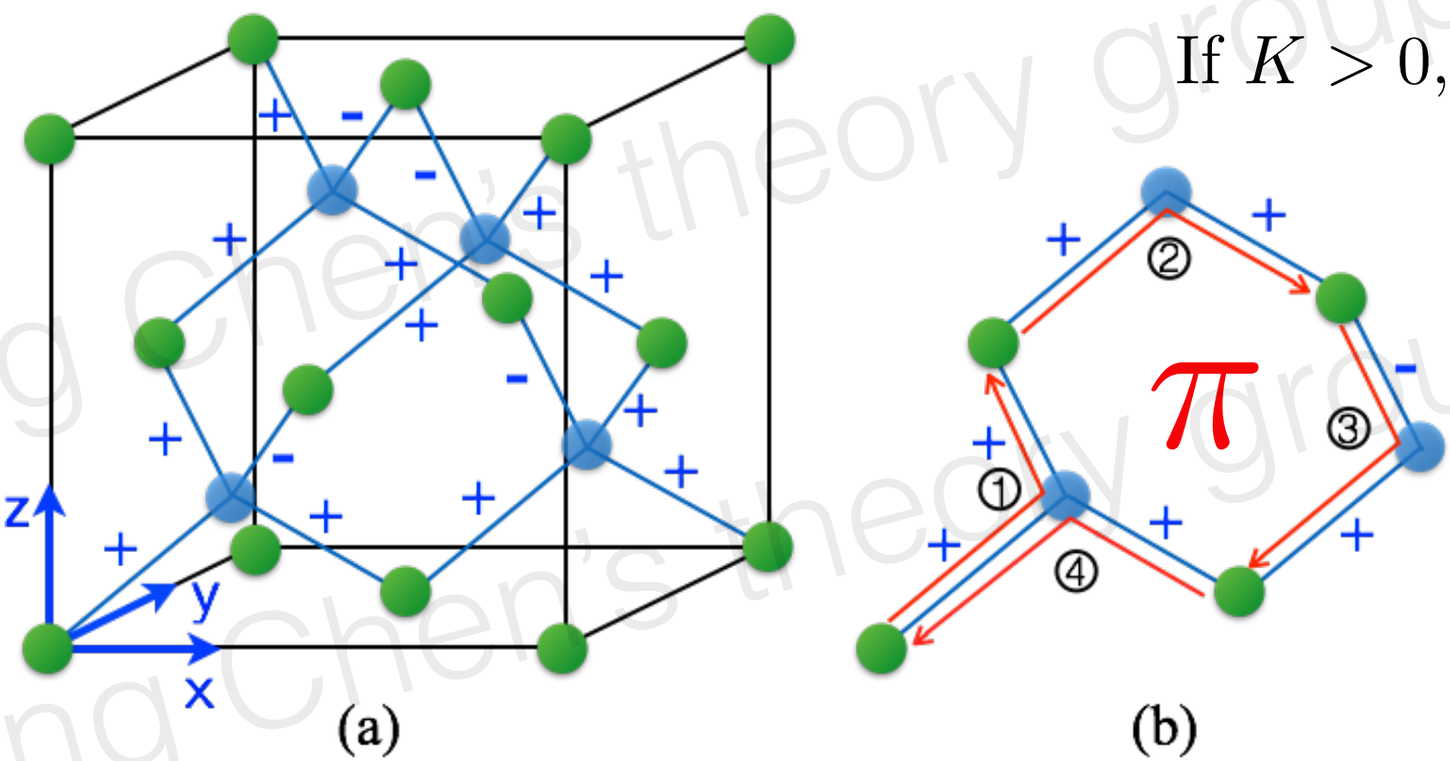
$$K = 24 J_\perp^3 / J_{zz}^2$$

Pi flux and the spinon translation

$$\mathcal{H}_{\mathrm{LGT}} = -K \sum_{\hexagon_{\mathrm{d}}} \cos(\mathrm{curl}\, A) + U \sum_{\boldsymbol{rr'}} (E_{\boldsymbol{rr'}} - \frac{\eta_{\boldsymbol{r}}}{2})^2$$

If $K < 0,\ curl A = \pi$

If $K > 0,\ curl A = 0$



$$T^s_\mu T^s_\nu (T^s_\mu)^{-1} (T^s_\nu)^{-1} = \pm 1$$

Aharonov-Bohm flux experienced by spinon via the 4 translation is identical to the flux in the hexagon.

Pi flux means crystal symmetry fractionalization

$$T^s_\mu T^s_\nu = -T^s_\nu T^s_\mu$$

2-spinon scattering state in an inelastic neutron scattering measurement

$$|a\rangle \equiv |\boldsymbol{q}_a; z_a\rangle,$$

construct another 3 equal-energy states by translating one spinon by 3 lattice vector

$$|b\rangle = T^s_1(1)|a\rangle, \quad |c\rangle = T^s_2(1)|a\rangle, \quad |d\rangle = T^s_3(1)|a\rangle$$

$$\begin{aligned}
T_1|b\rangle &= T^s_1(1)T^s_1(2)T^s_1(1)|a\rangle = +T^s_1(1)[T_1|a\rangle], \\
T_2|b\rangle &= T^s_2(1)T^s_2(2)T^s_1(1)|a\rangle = -T^s_1(1)[T_2|a\rangle], \\
T_3|b\rangle &= T^s_3(1)T^s_3(2)T^s_1(1)|a\rangle = -T^s_1(1)[T_3|a\rangle],
\end{aligned} \quad \Longrightarrow \quad \mathbf{q}_b - \mathbf{q}_a = 2\pi(100)$$

Xiao-Gang Wen, 2001, 2002,
Andrew Essin, Michael Hermele, 2014
Gang Chen, PhysRevB, **96**, 085136 (**2017**)

Spectral periodicity of the spinon continuum

spectral periodicity for the spinon continuum. The spectral periodicity can be reflected by the spectral intensity $\mathcal{I}(\boldsymbol{q}, E)$, the lower $\mathcal{L}(\boldsymbol{q})$ and upper excitation edge $\mathcal{U}(\boldsymbol{q})$ of the spinon continuum. For $\mathrm{U}(1)_\pi$ QSL, we have

$$\begin{aligned}
\mathcal{I}(\boldsymbol{q}, E) &= \mathcal{I}(\boldsymbol{q} + 2\pi(100), E) = \mathcal{I}(\boldsymbol{q} + 2\pi(010), E) \\
&= \mathcal{I}(\boldsymbol{q} + 2\pi(001), E), \\
\mathcal{L}(\boldsymbol{q}) &= \mathcal{L}(\boldsymbol{q} + 2\pi(100)) = \mathcal{L}(\boldsymbol{q} + 2\pi(010)) \\
&= \mathcal{L}(\boldsymbol{q} + 2\pi(001)), \\
\mathcal{U}(\boldsymbol{q}) &= \mathcal{U}(\boldsymbol{q} + 2\pi(100)) = \mathcal{U}(\boldsymbol{q} + 2\pi(010)) \\
&= \mathcal{U}(\boldsymbol{q} + 2\pi(001)).
\end{aligned}$$

But elastic neutron scattering will NOT see extra Bragg peak.

Xiao-Gang Wen, 2001, 2002,
Andrew Essin, Michael Hermele, 2014
Gang Chen, PhysRevB, **96**, 085136 (**2017**)

Calculation to demonstrate the above prediction

$$\mathcal{H}_{\mathrm{XXZ}} = \sum_{\langle ij \rangle} J_{zz} S_i^z S_j^z - J_\perp (S_i^+ S_j^- + S_i^- S_j^+),$$

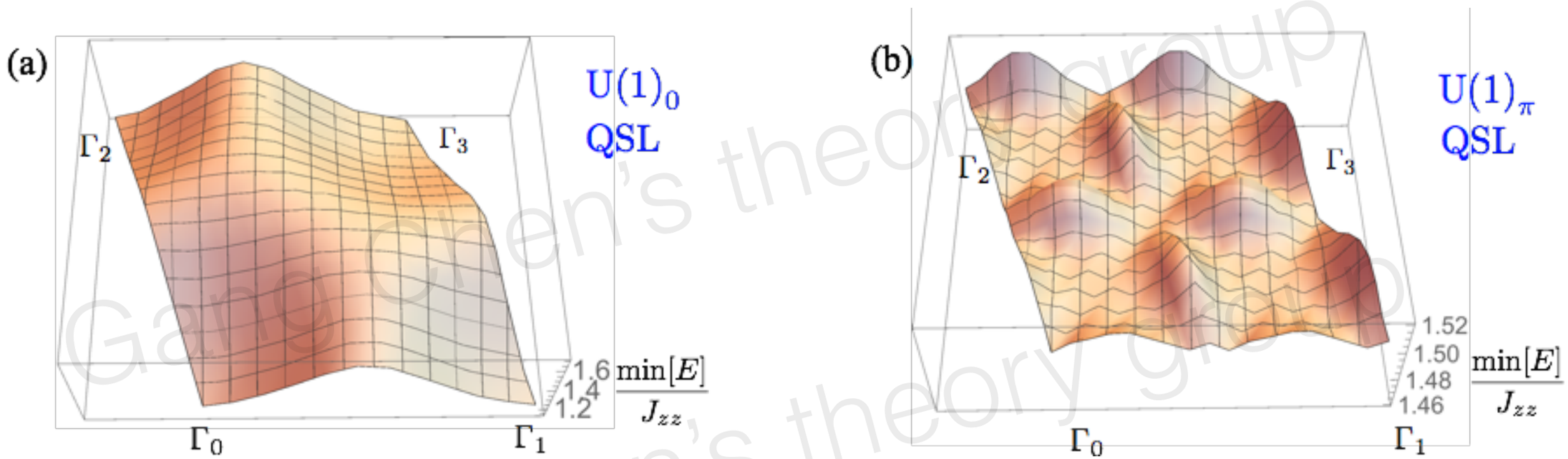


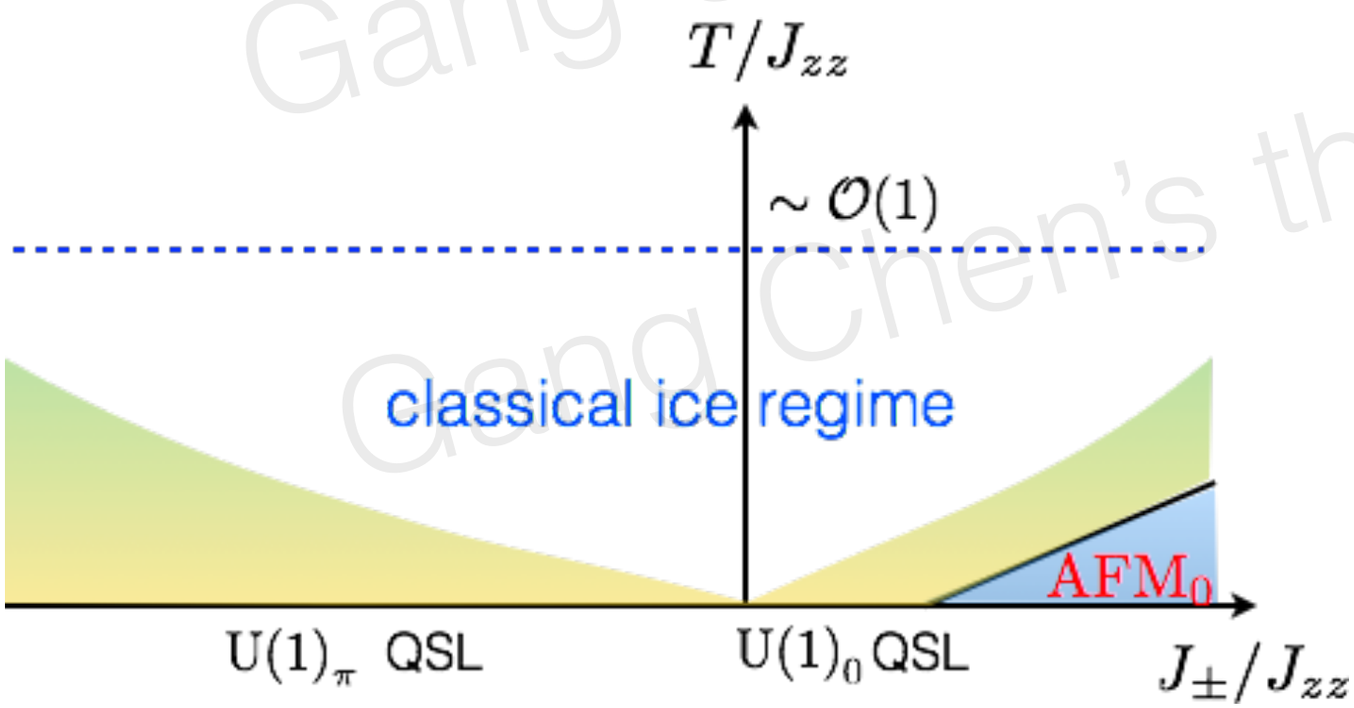
FIG. 3. (Color online.) The lower excitation edge of the spinon continuum in $\mathrm{U}(1)_0$ and $\mathrm{U}(1)_\pi$ QSLs. Here, $\Gamma_0\Gamma_1 = 2\pi(\bar{1}11), \Gamma_0\Gamma_2 = 2\pi(1\bar{1}1)$. We set $J_\perp = 0.12 J_{zz}$ for $\mathrm{U}(1)_0$ QSL in (a) and $J_\perp = -J_{zz}/3$ for $\mathrm{U}(1)_\pi$ QSL in (b).

Enlarged periodicity is like the fractional charge in FQHE.

Lower excitation edge of spinon continuum

Conclusion

U(1) QSLs	$U(1)_0$ QSL	$U(1)_\pi$ QSL
	> 0	$J_\perp < 0$
Background U(1) Flux	0 Flux	π Flux
Heat Capacity	$C_v \sim T^3$	$C_v \sim T^3$
Proximate XY Order	Keep Translation	Enlarged Cell
Spectral Periodicity	Not Enhanced	Enhanced



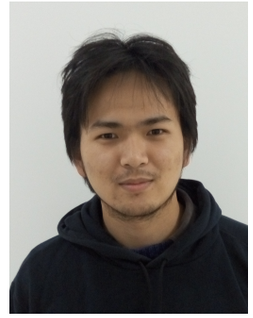
Lee, Onoda, Balents: Pi flux state is more robust. This is great !

This means it is more likely for a candidate material to have **spectral periodicity enhancement for the spinon continuum.**

Symmetry Enriched U(1) topological order on a pyrochlore lattice

- Dipole-octupole doublet

$$H = \sum_{\langle ij \rangle} J_x S_i^x S_j^x + J_y S_i^y S_j^y + J_z S_i^z S_j^z + J_{xz}(S_i^x S_j^z + S_i^z S_j^x).$$



Yao-Dong Li
(Fudan -> UCSB)

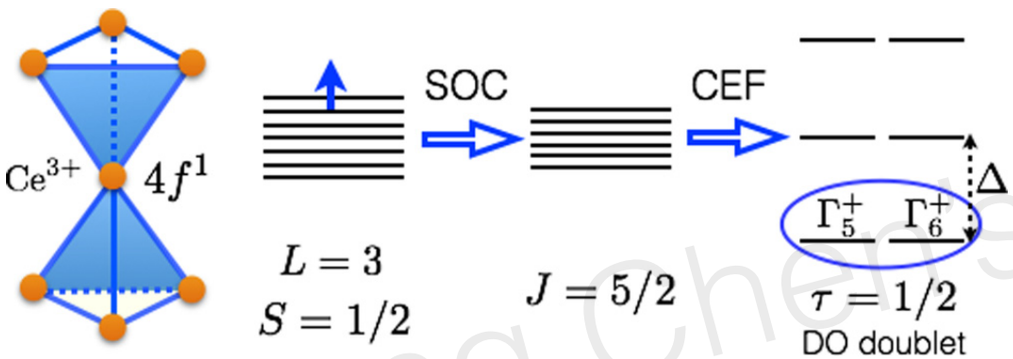
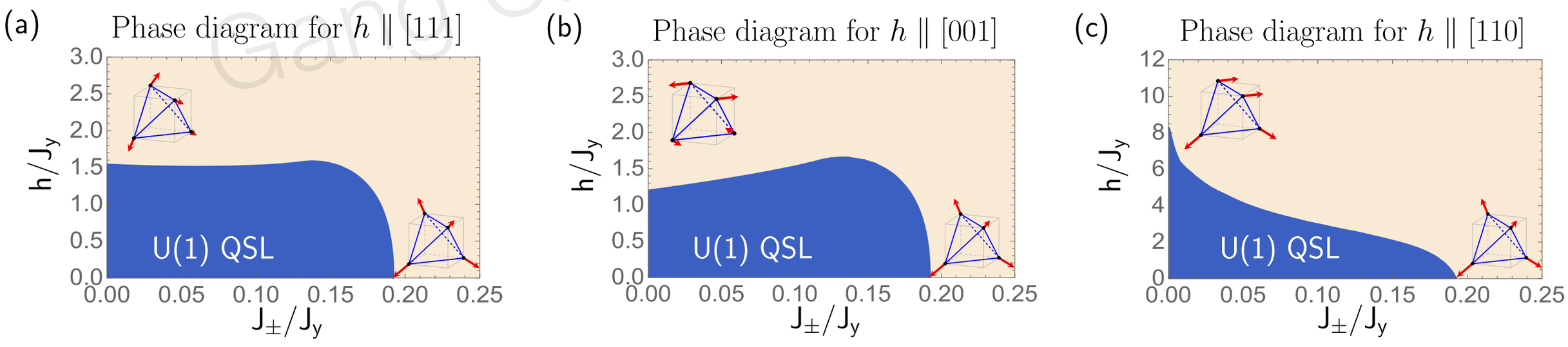


FIG. 1. The electron configuration and the D_{3d} crystal electric field (CEF) splitting of the Ce^{3+} ion in $Ce_2Sn_2O_7$. The CEF ground state wave functions are combinations of $J^z = \pm 3/2$ states [14], thus the CEF ground state is a DO doublet. Δ is the CEF gap and was fitted to be $\Delta = 50 \pm 5$ meV [14].

Y-P Huang, **Gang Chen**, M Hermele, PRL 2014
Yao-Dong Li, **Gang Chen**, PRB 2017



What does inelastic neutron scattering measure in quantum spin ice?

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Gang Chen, PhysRevB, 96, 195127 (2017)

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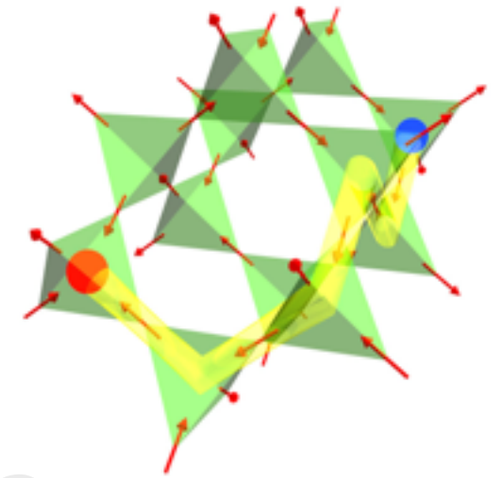
Equivalence of “notations”: three excitations



Excitations (notation 1)	Excitations (notation 2)
Spinon	Magnetic monopole
“Magnetic monopole”	Electric monopole
Gauge photon	Gauge photon

has classical analogue

purely quantum, no classical analogue

Gang Chen, PhysRevB, 96, 195127 (2017)

“Magnetic monopole” is probably closer in spirit to **Dirac’s monopole (1931).**
One has to confirm that “magnetic monopole” is emergent excitation,
rather than a fictitious particle.

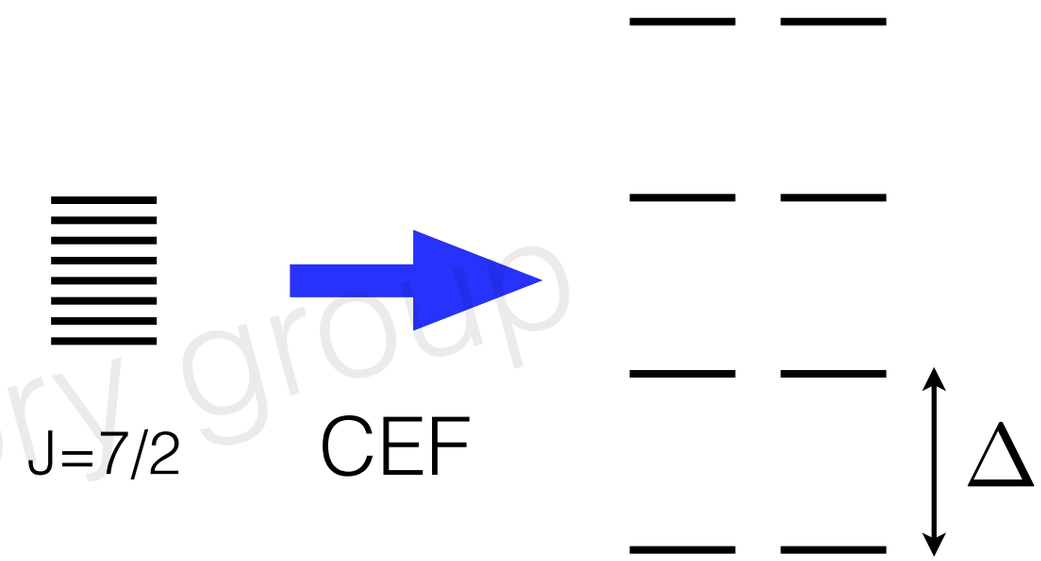
What piece of experimental info indicates these exotic and emergent particles?

Kramers vs Non-Kramers doublet

Kramers doublet: e.g. Yb ion in $Yb_2Ti_2O_7$

Yb^{3+} ion: $4f^{13}$ has J=7/2 due to SOC.

$$\mathcal{T}: S^x \to -S^x, S^y \to -S^y, S^z \to -S^z$$

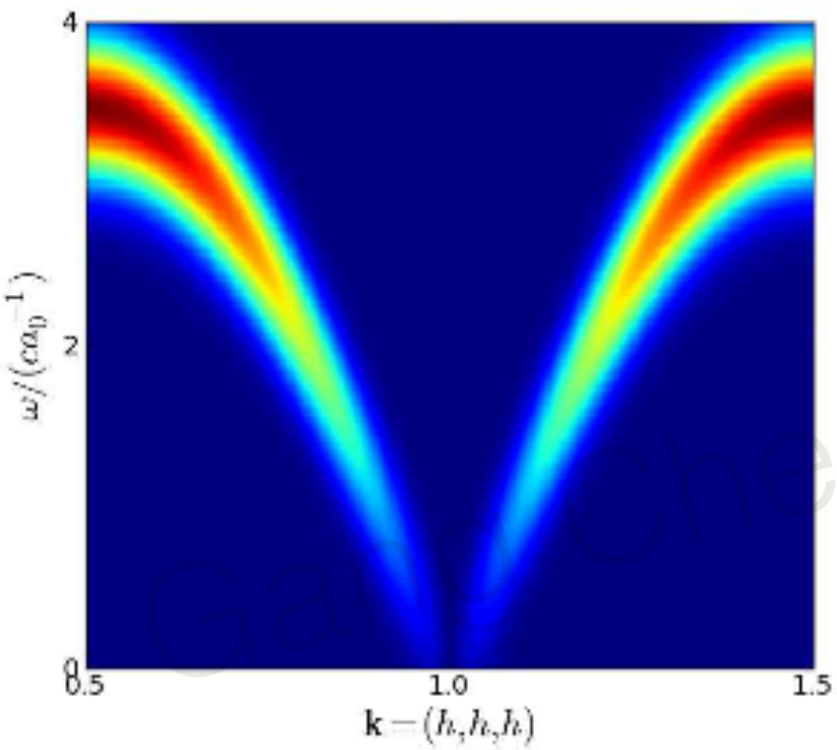


(unusual example is dipole-octupole doublet in $Ce_2Sn_2O_7$ and $Nd_2Zr_2O_7$),
YP Huang, GC, Hermele, PRL 2014; YD Li, GC, PRB2016, YD Li, GC, PRB 2017

In contrast, the Tb ion in $Tb_2Ti_2O_7$, Pr ion in $Pr_2Ir_2O_7$, $Pr_2Sn_2O_7$, $Pr_2Zr_2O_7$, etc, are **non-Kramers doublets**

$$\mathcal{T}: \quad S^{x,y} \to S^{x,y}, \quad S^z \to -S^z.$$

Emergent light: U(1) photon



$$I(\omega) \sim \omega$$

emergent U(1) photon in U(1) QSL

Hermele etc 2004
N Shannon etc 2012,
L Savary etc 2012

$$S_z \sim E \text{ (emergent electric field)}$$

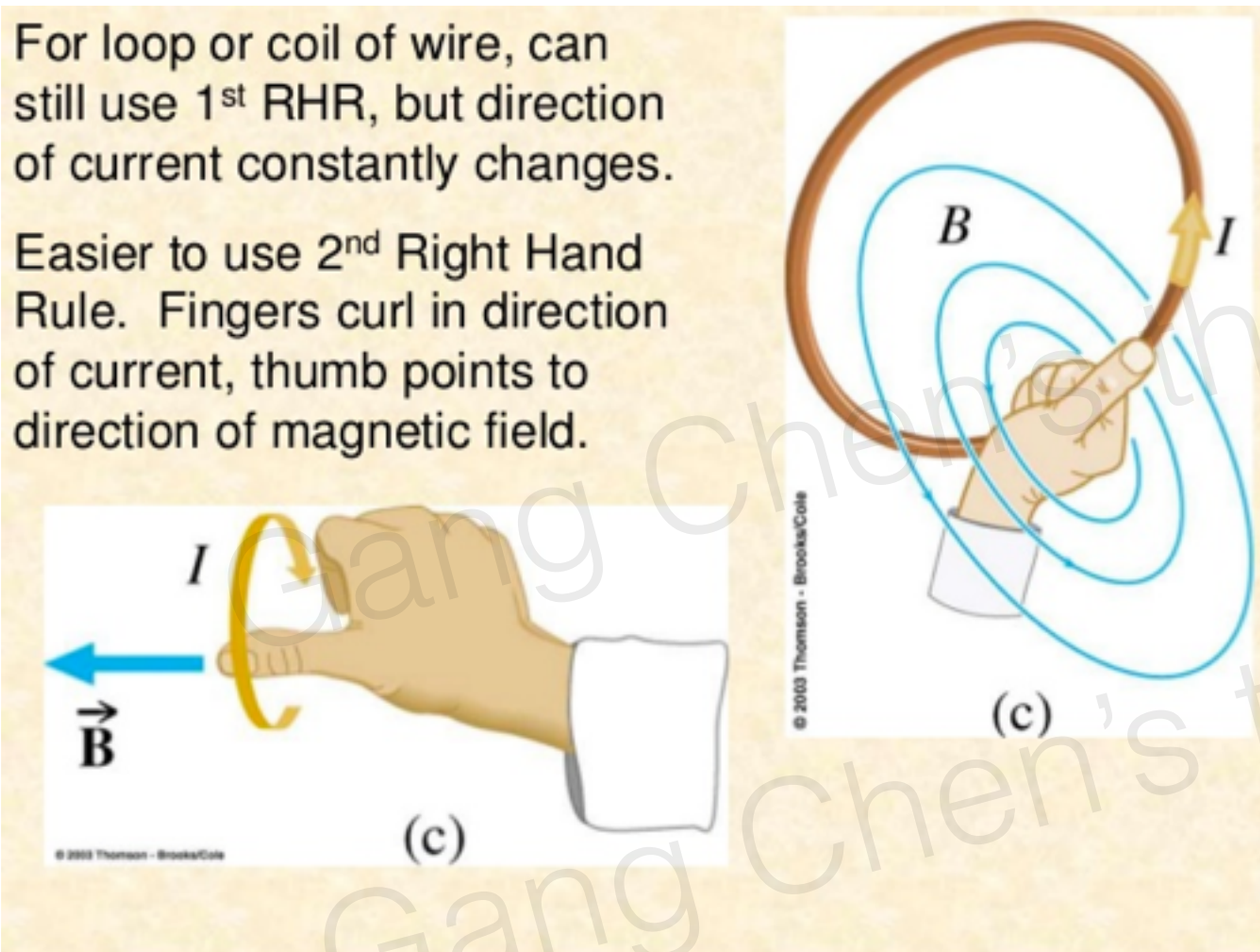
Low energy theory

$$\mathrm{Im}[E^{\alpha}_{-\mathbf{k},-\omega} E^{\beta}_{\mathbf{k},\omega}] \propto [\delta_{\alpha\beta} - \frac{k_\alpha k_\beta}{\mathbf{k}^2}]\, \omega\, \delta(\omega - v|\mathbf{k}|),$$

The well-known result of the photon modes in the INS measurement was obtained by considering the low-energy field theory that describes the long-distance quantum fluctuation within the spin ice manifold. The actual spin dynamics, that is captured by the S^z correlation in the INS measurement, operates in a broad energy scale up to the exchange energy and certainly contains more information than just the photon mode from the low-energy Maxwell field theory. What is the other informa-

Gang Chen, arXiv:1706.04333

Electromagnetic duality

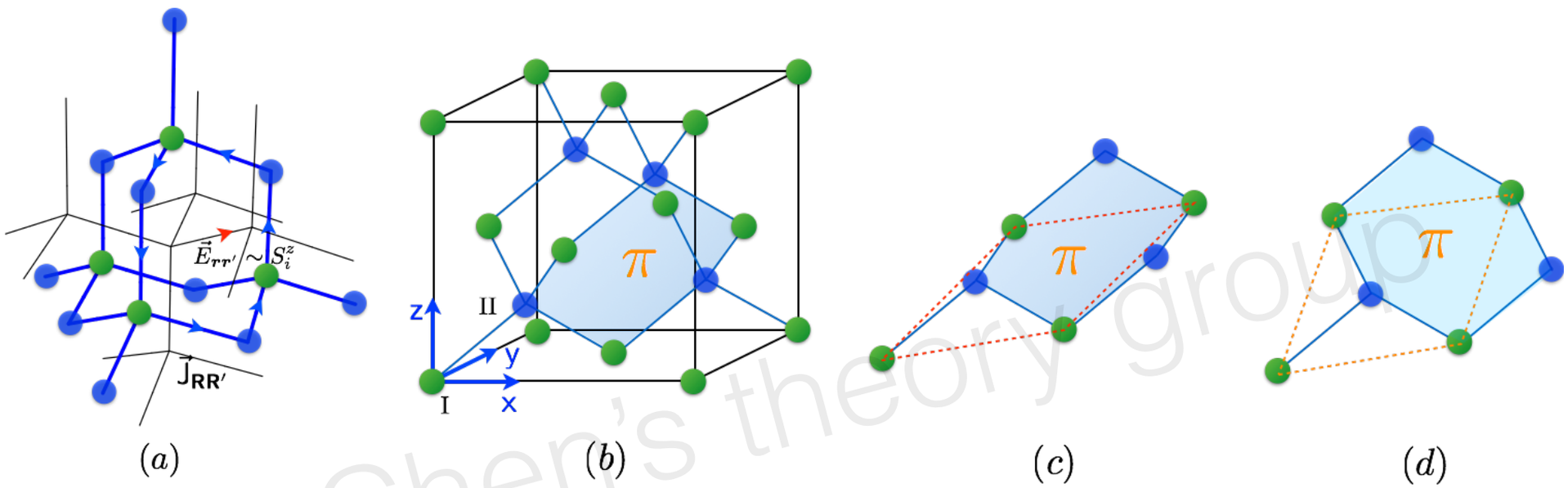


Duality

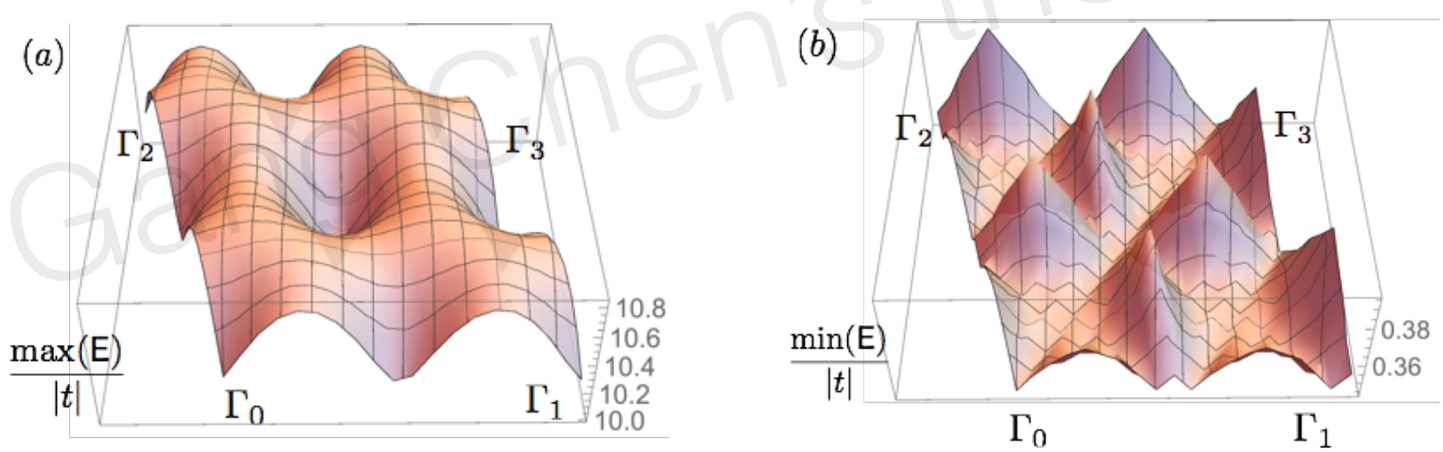
Electric loop current -> Magnetic field
Magnetic loop current -> Electric field

$S_z \sim E$ (emergent electric field)

Sz correlation = monopole loop current correlation



$$H_{\text{dual}} = -t \sum_{\langle \mathbf{RR'} \rangle} e^{-i2\pi\alpha_{\mathbf{RR'}}} \Phi^\dagger_{\mathbf{R}} \Phi_{\mathbf{R'}} - \mu \sum_{\mathbf{R}} \Phi^\dagger_{\mathbf{R}} \Phi_{\mathbf{R}} + \frac{U}{2} \sum_{\hexagon^*} (curl\alpha - \frac{\eta_r}{2})^2 - K \sum_{\langle \mathbf{RR'} \rangle} \cos B_{\mathbf{RR'}} + \cdots$$

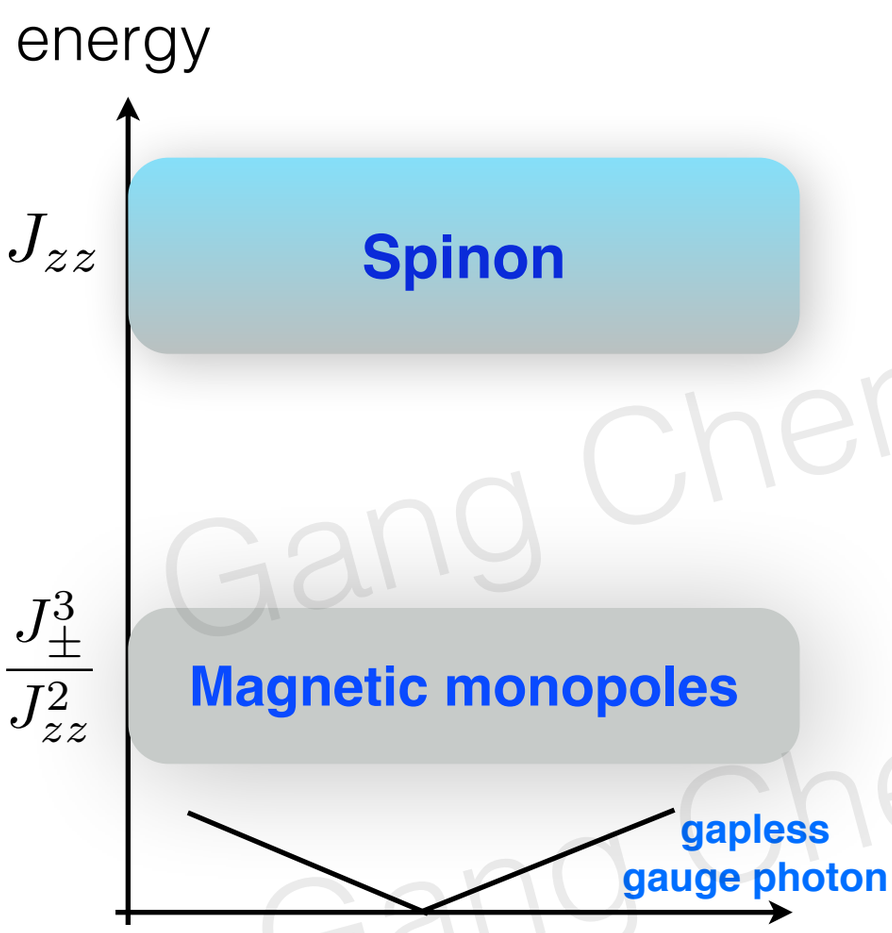


the upper edge

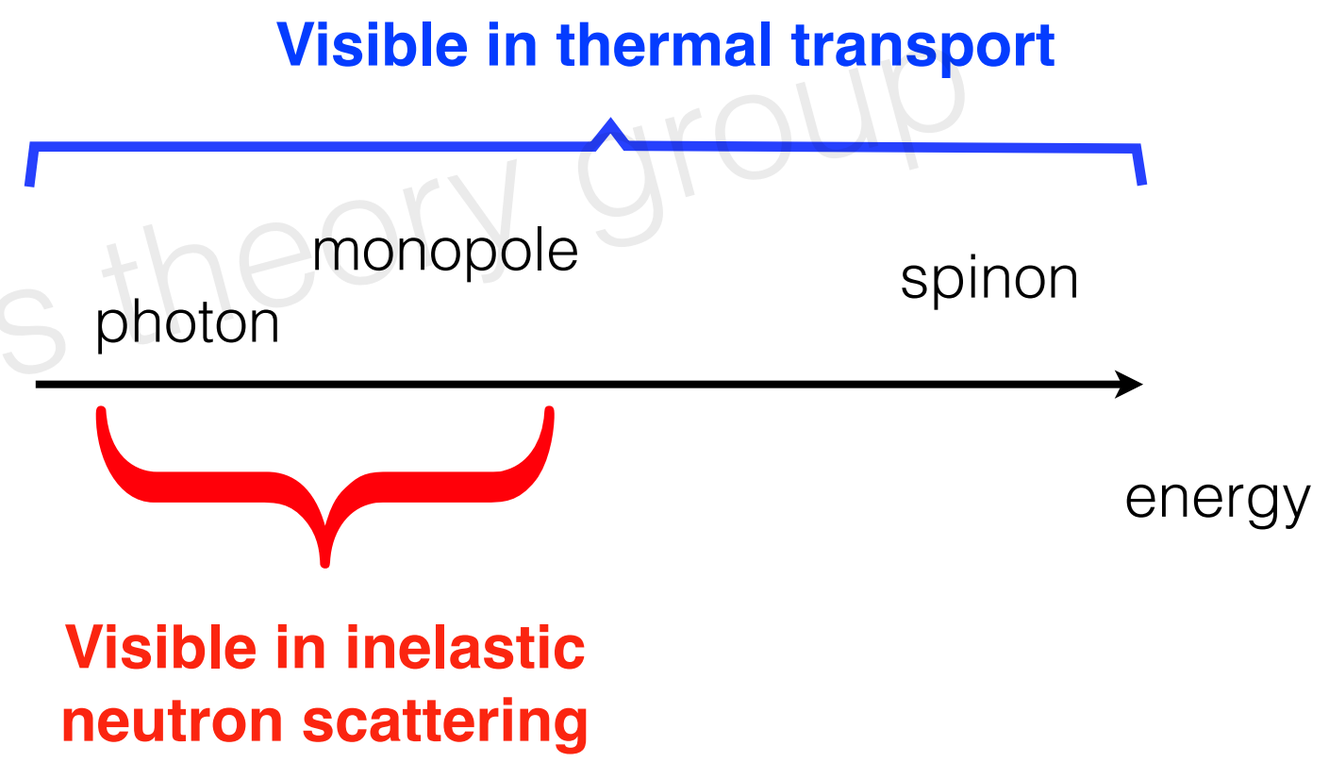
the lower edge

Monopole always experiences Pi flux

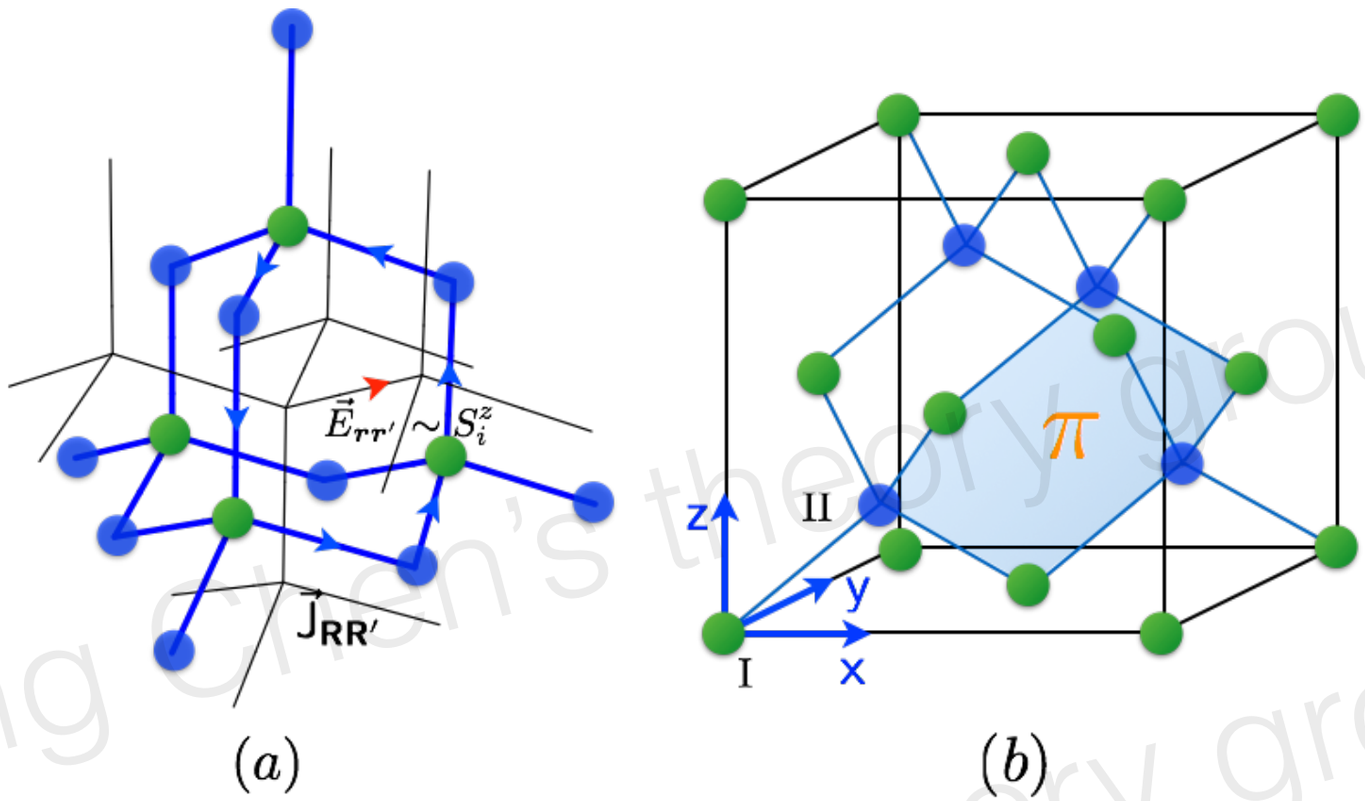
Suggestion 1: combine thermal transport with inelastic neutron



For **non-Kramers doublets** such as Pr ion in $Pr_2Zr_2O_7$ and Tb ion in $Tb_2Ti_2O_7$



Suggestion 2: effect of the external magnetic field



$$H_{Zeeman} = \vec{B} \cdot \sum_i S_i^z \hat{z}_i$$

The weak magnetic field polarizes Sz slightly, and thus modifies the background electric field distribution. This further modulates monopole band structure, creating "**Hofstadter**" monopole band, which may be detectable in inelastic neutron.

Summary

1. We point out the existence of “magnetic monopole continuum” in the U(1) quantum spin liquid, and monopole is purely **quantum origin**.

2. We further point out that the “magnetic monopole” always experiences a Pi flux, and thus supports enhanced spectral periodicity with **folded Brillouin zone**.

In fact, continuum has been observed in $Pr_2Hf_2O_7$ (R. Sibille, et al, arXiv 1706.03604).